ON THE MECHANISM OF SHRINKAGE AND DRYING IN CYLINDRICAL BODIES OF MIXTURES IN HYGROSCOPIC CASINGS

WITH SPECIAL REFERENCE TO THE DRYING OF MEAT MIXTURES AT LOW TEMPERATURE GRADIENTS

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1. Introduction. On the directed development of drying procedures in general

1.1. Drying is frequently the predominant process in the production technology of moisture containing materials. The operational conditions may greatly affect the internal and external characteristics of the product, the production time and the uniformity of quality. The deficiencies as encountered in the practice are often due to the insufficiently elucidated character of drying technics.

The optimum conditions for each product have to be determined separately. Taking these into consideration a drying program should be developed to ascertain favourable effects. This program is generally suitable for automated direction and thus the deficiences originating from subjective direction can be eliminated.

In order to establish the optimum drying program the mechanism of drying has to be known and substantiated mathematically for practical use. Further the initial external and boundary conditions, always present and for other reasons forming requisits, have to be taken into consideration. Finally the material characteristics, necessary for the computation have to be known and the results computed have to be compared with the procedure which occurs under actual working conditions.

However, in doing this, difficulties are frequently encountered. Difficulties may arise in respect to the theory — calculation technics — since this field is not sufficiently explored. For similar reasons, the shape of the product may also cause difficulties, for instance, in the study of the mechanism of drying.

Calculations are also hampered by the fact that the number of drying characteristics known as yet is insufficient. The manner — sometimes very intricate — in which this characteristics depend on the drying parameters is also unknown.

1.2. *The purpose of the present study* is to explore the field, as yet insufficiently known, of the drying mechanism of mixtures in casings, the shrinkage of cylindrical bodies, and — as practical interpretation of the subject — the drying procedure of different kinds of salami.

The aim of the research work was the development of the production of salami preserved by drying and the results were applied in this field.

2. The expedient division of the drying technical process in enveloped materials

2.1. In the theoretical procedure of calculating the drying process one of the basic goals is to determine the moisture distribution ($u = u(r, t)$) within the substance at a given time (t), the integral moisture content (W), and the equation of the drying curves ($W = W(t)$).

The kinetics of drying in bodies of homogeneous consistency and simple geometric form may be expressed by rather intricate correlations.

Taking into consideration the characteristics of the drying process of salami, which is the special subject of our investigations, such as: very low drying intensity at a relatively long drying period of appr. 2000 hours, the negligibility of the "filtering motion" caused by the inner pressure gradient, further the negligibility of thermodiffusion because of the extremly low inner temperature gradient — a certain simplification may be achived and the differential equation of moisture exchange [1,2] may be formulated as follows:

$$\frac{\partial u}{\partial \tau} = \text{div}\,(k \cdot \text{grad}\, u) \tag{1}$$

The Hungarian salami is a characteristic example of a meat mixture having a cylindrical shape in hygroscopic casing. The ratio of length/diameter in the salami generally falls between 6 and 9 and thus the effect of the ends of the stick on the radial moisture distribution in the cylindrical part is negligible and in calculating the moisture distribution, the salami may be considered as a cylinder of infinite length.

The salami mixture is statistically of isotropic character and its behaviour resembles that of a capillary, porous, colloidal body. During the process of drying, simultaneously, places of different moisture values will radially develope in the salami stick, and the moisture transfer in the mixture as a whole may be the resultant of the various ways in which moisture migrates in the layers of the salami. The k moisture transfer coefficient may enable us to take this resultant effect into consideration, provided it is correctly determined in experiments.

Thus equation (1) (r = radius; R = external radius; characteristic measurement) with transformation $\varrho = \frac{r}{R}$ for a cylinder of infinite length:

$$\frac{\partial^2 u(\varrho, t)}{\partial \varrho^2} + \frac{1}{\varrho} \frac{\partial u(\varrho, t)}{\partial \varrho} = \frac{R^2}{k} \frac{\partial u(\varrho, t)}{\partial t}. \tag{1a}$$

Initially some sort of moisture distribution is assumed:

$$u(\varrho, 0) = f(\varrho). \tag{2}$$

The boundary condition is expressed as follows

$$-\left.\frac{\partial u}{\partial \varrho}\right|_{\varrho=1} = \nu \cdot [u(1, t) - u_e]. \tag{3}$$

In equation (3) u_e is the equilibrium moisture content of the mixture concordant to the relative partial pressure in the drying environment, and ν is a complex moisture transfer criterion.

2.2. With the aid of the ν criterion the effect of the casing on the mixture may be taken into consideration, provided the following conditions are acceptable:

a) In the intensity value (i_b) of the moisture flow, escaping from the surface of the casing, and expressed by the mass transfer potential (Θ)

$$i_b = -\frac{1}{R} k_{mb} \nabla \Theta = \beta_{mb} [\Theta_1 - \Theta_e]_b \tag{3a}$$

the amount of moisture originating from the drying casing is negligible *i.e.* the moisture content of the casing compared to that of the emulsion is insignificant.

(In correlation (3a): $k_m = k \cdot \gamma_0 \cdot c_m$; $\beta_m = \beta \cdot \gamma_0 \cdot c_m$; $c_m = \frac{\partial u}{\partial \Theta}$, and γ_0 is the dry specific density.)

b) The thickness of the casing (v) is negligible as compared to the characteristic dimension (R) of the cylinder (less than 1%), thus the concordance of the consistency of the meat mixture and the external radius gives an acceptable approximation.

c) The moisture gradient in the casing — as a consequence of the condition according to b) — may be considered constant, *i.e.* the moisture distribution may be considered linear.

d) The condition (3a) — with the β_{mp} coefficient, characteristic of the moisture exchange between mixture and casing — can be made valid for the moisture flow (i_p) and according to a) $i_p = i_b$. The value β_{mp} is a function of the adhesion and shrinkage properties of the mixture and casing used, further the course of drying. (Under normal drying conditions the β_p values for Hungarian "winter" and "dessert" salami are shown in the diagram in Fig. 1. as plotted from experimental data.)

On the basis of these conditions, with potential values correlated to the mixture the boundary condition (3) is given and the moisture transfer criterion is the following

$$\nu = \frac{1}{\dfrac{k_p}{R\beta_p} + \dfrac{u_{pe}}{u_{b1}}\dfrac{\gamma_{op}}{\gamma_{ob}}\left[\dfrac{k_p}{k_b}\dfrac{v}{R} + \dfrac{k_p}{R\beta_b}\right]} \tag{3b}$$

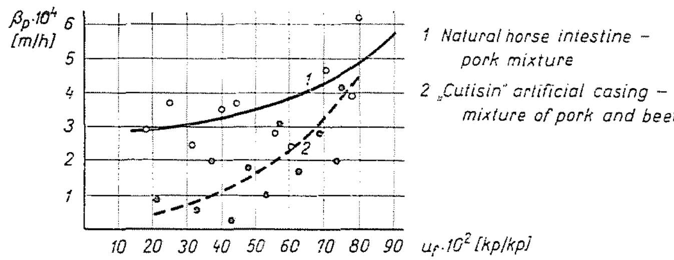

Fig. 1. The β_p values at normal drying conditions in Hungarian "winter" and "dessert" salami

Condition *a*) in the first phase of drying of wrapped up bodies can generally not be accepted. The natural casing (horse intestine) and certain protein based artificial casings contain a substantial amount of moisture. At the beginning of the drying process only the casing will lose moisture, therefore in this phase the value of ν should be calculated from the characteristics of the casing only:

$$\nu_I = \frac{\beta_b}{k_b} R. \tag{3c}$$

As can be seen, in this case ν_I corresponds to the Biot criterion of moisture transfer. By the time the mixture begins to dry, the hygroscopic casing will have lost the major part of its moisture content. If this phase of drying (Phase I) is considered separately, condition *a*) will be acceptable even in the case of casings of higher moisture contents. Thus, the full period of drying (t_{tot}) consists of two phases: t_I phase the drying of the casing and the second phase starting with the commencement of moisture loss from the mixture $t_{tot} = t_I + t$. (The moisture content in some of the artificial casings is so small, that phase *I* need not be taken into account.)

2.3. In the second phase of the drying process — in the *t* period — the values belonging to factors ν, *k*, *R*, etc., change, because — according to earlier investigations [3—6] — each factor is a function of the moisture content of the material. Thus the (1) differential equation and (3) boundary condition are not linear and their solution causes difficulties. If the factors enumerated are treated as constants — using these means — the approximation could not be acceptable as correct. Thus, in order to simplify the mathematical treatment of the problem it is our proposal to examine the dehydration process of the emulsion divided into several phases. Further the drying characteristics of the materials have to be treated as constants only within a single phase, by replacing them with the mean value within the given phase. The conditions of accession between phases are satisfied by considering the moisture distribution $f(\varrho)$ at the end of the previous phase as the starting condition — according to equation (2) — of the next phase.

3. The description and phases of the drying process

3.1. The mixture is filled into the hygroscopic casing by excluding the air. At the beginning of dehydration ($t = t_0 = 0$) the moisture distribution in the casing and the mixture is uniform, the initial moisture contents, however, are not equal (Fig. 2).

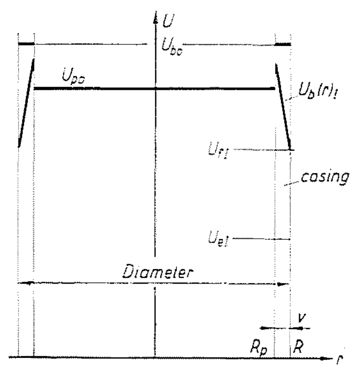

Fig. 2. The initial moisture distribution as developed at the end of drying phase I.

The initial moisture content ($u_b = u_b(\varrho, 0) = u_{b0}$ = constant) is affected by the kind of casing applied, the treatment previous to filling (brine) etc. The initial moisture content of the mixture ($u_p = u_p(\varrho, 0) = u_{p0}$ = constant) depends on its composition (in meat mixtures of the quality of meat used, the fat content, and the amount of additives).

If both the mixture and the casing contain sodium chloride and the salt concentrations differ, after filling the equalization of concentrations begins which may lead to moisture exchange. The salt present, according to earlier investigations [3, 5, 6] may also affect the moisture coefficients.

The condition of dehydration at the beginning is that at the given temperature the environmental relative partial pressure should be lower than the equilibrum relative humidity (*ERH*) belonging to the substance to be dryed.

Taking into consideration the initial ($u_{0p} = 1.0$ [kp/kp]) and the final moisture contents ($W_e = 0.3 - 0.35$ [kp/kp]) generally encountered in Hungarian salami and the characteristic dimensions ($2\ R_0 = 55 - 75$ [mm]) it is useful to divide the process of drying into four phases.

As was already mentioned, in the first phase of dehydration only the drying of the casing is taken into consideration.

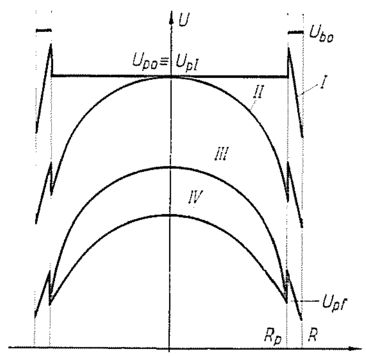

Fig. 3. Moisture distribution curves at the end of the drying phases in a cylindrical body in casing

After t_l time the mixture begins to lose moisture. The intensity of the moisture exchange between casing and meat mixture — at equal external conditions (u_e) — decreases, since the escaping water originates from deeper and deeper layers. The moisture migrating from one layer into the next one — towards the surface — consists of two distinct parts: the first part is the loss of moisture belonging to the layer itself mobilized by the difference of tension —, and the second part is the water transferred from the deeper layers.

The moisture transfer from the deeper layers begins, when the moisture content of the outer layers had already decreased, *i.e.* a proper moisture gradient has developed.

If the dehydration of the meat mixture is studied in three phases the question arises, how long should each phase last? Since the change in the surfacial moisture content is the greatest at the beginning of drying in the mixture, it is advisable to make this period (II. phase) shorter than the other two. Thus the period during which the drying effect reaches the geometrical mean line of the cylinder (Fig. 3, Diagram II), may be considered as II. phase.

According to the above in the dehydration of Hungarian winter salami the period between 300—400 hours may be considered as the second phase.

3.2. The mathematical treatment of phases I. and II. is concordant because of the initial condition

$$u(\varrho, 0) = u_0 = \text{constant} \tag{2a}$$

expressing the uniform initial moisture distribution. The solution — based on boundary condition (3) and the (2a) — may be in phase I. with constant (mean) casing characteristics (ν_I), within $\varrho_p = \frac{R_p}{R} \leq \varrho \leq 1$ limits $(0 \leq t \leq t_I)$, in phase II. with constant material characteristics (ν_{II}), averaged for the phase within $0 \leq \varrho \leq 1$ limits $(0 \leq t \leq t_{II})$ may be formulated as follows:

$$u(\varrho, t) = u_e + 2\nu[u_o - u_e] \sum_{i=1}^{\infty} \frac{J_0(\zeta_i \varrho)}{(\zeta_i^2 + \nu^2) \cdot J_0(\zeta_i)} e^{-\frac{\zeta_i^2 kt}{R^2}}. \tag{4}$$

(The — not specific — mathematical details of the solution are disregarded. Similar mathematical solutions are to be found in heat transfer problems [17, 22]).

In correlation (4) $J_0(\zeta_i)$ is a first degree, zero order Bessel function, and the ζ_i values are the roots belonging to

$$\zeta_i J_0'(\zeta_i) = J_0(\zeta_i) \tag{5}$$

self value equation.

3.3. In the III. and IV. phases of drying, when moisture distribution is calculated, the solution of phase II, and phase III resp. is considered as an initial condition

$$(t_{II} \leq t \leq t_{III}, \text{ i.e. } t_{III} \leq t \leq t_{IV}):$$

$$u(\varrho, t_{II}) = f(\varrho)_{II} \text{ i.e. } u(\varrho, t_{III}) = f(\varrho)_{III} \tag{2b}$$

The solution of (1a) differential equation based on (2b) initial and (3) boundary conditions is as follows

$$u(\varrho, t) = \sum_{k=0}^{\infty} A_k J_0(\zeta_k \varrho) \cdot e^{-\zeta_k^2 \frac{kt}{R^2}}. \tag{6}$$

Based on (6) from boundary condition (3) the self value equation pertinent to ζ_k is

$$-\sum_{k=0}^{\infty} A_k \zeta_k J_1(\zeta_k)\, e^{-\zeta_k^2 \frac{kt}{R^2}} = \nu \left[u_e - \sum_{k=0}^{\infty} A_k \cdot J_0(\zeta_k)\, e^{-\zeta_k^2 \frac{kt}{R^2}}\right] \tag{7}$$

where $J_1(3_k)$ is a Bessel function of the first degree and order.

If $\zeta_0 = 0$, then $A_0 = u_0$, thus at $k \geq 1$ the self value equation is:

$$\zeta_k J_1(\zeta_k) = \nu \cdot J_0(\zeta_k). \tag{5a}$$

From the above the solution (6) may be taken as follows:

$$u(\varrho, t) = u_e + \sum_{k=1}^{\infty} A_k J_0(\zeta_k \varrho) e^{-\zeta_k^2 \frac{kt}{R^2}}. \qquad (6a)$$

Based on (2b) initial condition in general:

$$f(\varrho) = u_e = \sum_{k=1}^{\infty} A_k \cdot J_0(\zeta_k \varrho). \qquad (7)$$

Multiplying both sides by $\varrho \cdot J_0(\zeta_k \varrho)$ and integrating for the ϱ

$$\int_{\varrho=0}^{1} f(\varrho) \cdot \varrho \cdot J_0(\zeta_k \varrho)\, d\varrho = u_e \int_{\varrho=0}^{1} \varrho \cdot J_0(\zeta_k \varrho)\, d\varrho = A_k \int_{\varrho=0}^{1} \varrho \cdot J_0^2(\zeta_k \varrho)\, d\varrho. \qquad (8)$$

Taking into consideration the orthogonality of the Bessel functions and (5(a)) the coefficient may be defined as

$$A_k = \frac{2 \int_{\varrho=0}^{1} f(\varrho) \cdot \varrho J_0(\zeta_k \varrho)\, d\varrho - 2u_e \frac{\nu}{\zeta_k^2} \cdot J_0(\zeta_k)}{J_0^2(\zeta_k) \cdot \left(1 + \frac{\nu^2}{\zeta_k^2}\right)}. \qquad (9)$$

Using (9) in (6a) equation:

$$u(\varrho, t) = u_e + \sum_{k=1}^{\infty} \frac{2 \int_{\varrho=0}^{1} f(\varrho) \cdot \varrho \cdot J_0(\zeta_k \varrho)\, d\varrho}{J_0^2(\zeta_k) \cdot \left(1 + \frac{\nu^2}{\zeta_k^2}\right)} J_0(\zeta_k \varrho) \cdot e^{-\zeta_k^2 \frac{kt}{R^2}} -$$

$$- 2 u_e \nu \sum_{k=1}^{\infty} \frac{J_0(\zeta_k \varrho)}{(\zeta_k^2 + \nu^2) \cdot J_0(\zeta_k)} e^{-\zeta_k^2 \frac{kt}{R^2}}. \qquad (10)$$

The solution (10) satisfies (1a) differential equation, (2) initial and (3) boundary condition, thus it is a real solution.

If solution (10) is used in phase III., $u(\varrho, t_{II})$ stands for the initial condition, and the solution according to (4) is applied, so the solution for phase III will be more intricate. It is evident, that the solutions may be expressed in terms more and more complex and intricate and so longer, and by increasing the number of phases the technical-calculation difficulties will arise.

The application of the drying curves of the layers (Fig. 14) is of advantage in moisture distribution determinations. These curves may be plotted from a few points, with a safety sufficient for practical use, and the moisture distribution at a given time may be determined from these with good approximation.

4. Further conditions of the drying process

4.1. As mentioned in the introduction, in construing the optimum drying program constraints and conditions originating from other circumstances have also to be taken into consideration.

The moisture coefficients, for instance, are functions of not only the moisture content of the materials, but of the temperature, the composition of the mixture, and environmental conditions. These factors are, on the other hand, frequently restricted by other external conditions.

The *temperature relations* — in meat mixture — are affected by the optimum conditions of the biochemical processes in relation to the ripening and the noble mould coating of the casing.

The *composition* of the mixture depends on the kind of product. The velocity and exchange of air, — their adjustability — the direction of air blast, on the other hand — depends on the apparatus used. *The relative humidity of the air is*, in certain periods of drying, conditioned by the growth of moulds.

4.2. Certain constraints may occur in relation to the parameters affecting one of the most important characteristics, the drying time, such as the permissible drying rate. The achievable highest drying rate is restricted partly by the time needed for ripening and colour development, and partly by internal fissures or indentations due to uneven moisture distribution. The latter one is of decisive importance because it can not be influenced by other means, (whereas ripening and colour development my be accelerated by other means) and it may lead to defects causing a substantial decrease in the product value.

Uneven moisture distribution effects uneven shrinkage, thereby causing stresses in the material.

Thus, in considering the changes of the characteristic dimensions for the calculations and the evaluation of optimum conditions, the study of the mechanism of shrinkage is inevitable.

5. The mechanism of shrinkage in cylindrical bodies of meat mixtures in hygroscopic casings

5.1. The shrinkage occuring during drying and the theoretical principles of calculating the tension caused thereby were investigated by several authors [1, 7—11]; there also is a number of data known, in relation to the drying of paste-like substances [12—15]. These investigations, however, elucidate cases much simpler than and different from the present task (the mixtures investigated were of lesser thickness and diameter, they were dried from a single direction, could be considered homogenious and were not filled in casings). Their chief purpose was the elimination of fissures on the surface.

The shrinkage in larger bodies filled into casing (70—80 mm diameter) is a function of quite different conditions having other results. In the present case a certain — perhaps substantial — part of the mixture consists in unshrinkable ballast material (fat, bacon particles), the presence of which affects the shrinkage characteristics of the material.

In the following the ideas formed on the basis of the theoretical and experimental work aim at elucidating this field and the calculation technics developed, are described. The discussion of the process of shrinkage is evolved with reference to the part procedures as given above.

5.2. In the first phase of drying shrinking occurs only in the casing. As it can be seen in the shrinkage curves [3, 4] belonging to the casings used in Hungary, the shrinkage of the casing is not uniform. At higher moisture contents it progresses more slowly, while with the advance of drying it becomes faster and between the critical moisture values it may be considered linear.

Since the shrinkage coefficient of the casing changes during drying, this process may be considered without substantial error only if the process of drying is divided into phases. Within each phase the shrinkage coefficient may be considered as constant.

In the first phase, the drying of the casing reaches the first critical point. Since the meat mixture is filled into the casing under overpressure p_0 a certain tension develops in the casing before the beginning of the drying process. The overpressure p_0 is caused by filling more mixture into the casing than its volume prior to filling permits, and as a result of the tension thus evolved the casing will stretch.

The principal stresses evolving in the casing, resulting from the internal overpressure — similarly to the behaviour of a tube of thin wall under internal overpressure [18] — may be formulated as follows:

$$\sigma_1 = \sigma_{t0} \frac{R_p \cdot p_0}{v} \tag{11}$$

$$\sigma_2 = \sigma_{a0} = \frac{R_p \cdot p_0}{2v} \tag{12}$$

$$\sigma_3 = \sigma_r = -p_0 \tag{13}$$

where R = the internal radius, v = the thickness of the wall, σ_1 = the tangential, σ_2 = the axial, σ_3 = the radial principal stresses. According to MOHR the characteristic reduced stress is

$$\sigma_{\text{red}} = \sigma_1 - \sigma_3 = \frac{R_p \cdot p_0}{v} + p_0 = \frac{R_p \cdot p_0}{v} \left(1 + \frac{v}{R_p}\right). \tag{14}$$

Since the thickness of the casing is of the order of 10^{-1} mm, the member $\frac{v}{R_p}$ is negligibly small.

It is evident that at too high p_0 pressure values the casing would frequently burst, on the other hand, at too low p_0 values the cylindrical shape of circular cross section would not develop and the mixture would not bind properly. Thus, the filling overpressure p_0 is necessary in spite of the prestress it gives to the casing, thereby overstraining it already before the beginning of the drying.

As a result of the stresses evolved the casing suffers elastic dimensional changes. The specific tangential change of dimension (ε_t) is a consequence of the stretch of the circumference due to the ΔR_0 increase in the radius, thus

$$\varepsilon_{t0} = \frac{2\pi(R_p + \Delta R_{p0}) - 2\pi R_p}{2\pi R_p} = \frac{\Delta R_{p0}}{R_p}. \tag{15}$$

Using Hooke's law and the principle of superposition (when σ_3 is negligibly small, [18]):

$$\varepsilon_{t0} = \frac{\sigma_{t0}}{E} - \frac{\sigma_{a0}}{m.E} = \frac{2m-1}{2m\,E}\,\frac{R_p\,p_0}{v} \tag{16}$$

where $1/m$ is the Poisson number.

From the above ΔR_{p0} may be derived.

The longitudinal specific change of dimension:

$$\varepsilon_{a0} = \frac{\Delta l_0}{l} \tag{17}$$

and similarly for the determination of Δl_0

$$\varepsilon_{a0} = \frac{\sigma_{a0}}{E} - \frac{\sigma_{t0}}{mE} = \frac{m-1}{2m\,E}\,\frac{rp_0}{v}. \tag{18}$$

Using Δr_0 and Δl_0 values the additional volume ΔV and the decrease of thickness Δv of the casing may be calculated. (In actual practice if the casing bursts the major part of the additional volume ΔV will extrude through the slit. Since the meat mixture must be considered practically incompressible this extrusion is the result of the elastically prestressed state of the casing.)

Simultaneously with the beginning of drying the casing would have to shrink in both directions, radially and lengthwise. The meat mixture, however, represents an unchanged volume and is practically incompressible, thus the shrinkage of the casing in this phase of drying is completely hindered. Because of the hindered shrinkage the casing must, while further decreasing its thickness, stretch. Therefore, further decrease in the thickness of the casing is caused not only by its radial shrinkage but by the stretching due to the hindered shrinkage along the circumference.

Using r_0 for the radius of the casing after filling — at a moisture content u_{b0} — and assuming that in phase I the mean moisture content of the casing decreased to u_{bI}, using a_{bI} shrinkage coefficient of the casing valid for this phase of drying, the restricted specific change of dimensions is:

$$\varepsilon_{tI} = \frac{\Delta r_I}{r_0} = \frac{a_{bI}(u_{b0} - u_{bI})}{1 + a_{bI} \cdot u_{b0}} . \tag{19}$$

A further tangential stress

$$\sigma_{tI} = \varepsilon_{tI} \cdot E \tag{20}$$

is formed to the limit of elastic deformation, without changing the character of stress in the casing.

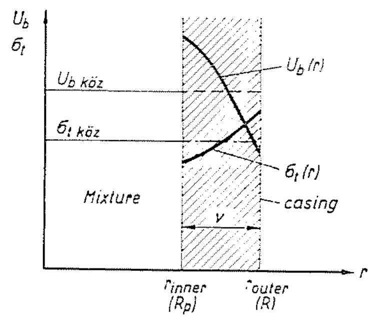

Fig. 4. The stress in the casing at the end of drying phase I.

(Owing to identical stress

$$\sigma_{aI} = \frac{\sigma_{tI}}{2} ; \qquad \sigma_{rI} = - p_I .)$$

Thus, the first phase of drying has to be directed so as to obtain

$$\sigma_{ttot} = \sigma_{t0} + \sigma_{tI} \leqq \sigma_{perm} \tag{21}$$

as the determinative stress. In order to prevent permanent stretch of the casing the value σ_{perm} has to be chosen at 20% below the elasticity limit. If the casing would suffer permanent deformation it would not be able to follow the mixture during its subsequent shrinkage. The casing would become detached, and the salami would lose its value. The expedient choice of σ_{perm} is advisable also to prevent the small local faults of the casing which causes splits.

Further it may be noted, that as a consequence of the moisture gradient developing in the casing during the first phase of drying the shrinkage, and therefore, the stress is greater on the surface of the casing than the mean value (Fig. 4). This effect may be taken into consideration with a safety factor, depending on the drying rate and having the value of 1.25 — based on average conditions.

Based on the above the maximum loss of moisture, which is permissible in the first phase of drying from the point of view of shrinkage, may be estimated.

5.3. *In the second phase of drying* proportionately to the loss of moisture the mixture begins to shrink.

5.31. The loss of moisture in the casing and therewith its shrinking soon becomes slower than that of the mixture and the overstrain gradually becomes reduced and thus the further phases of drying are not relevant as regards the maximum requirements in relation to the casing. (In the subsequent phases of drying, for instance, owing to their lesser shrinking capacity, the artificial casings cannot follow the dimensional reduction of the mixture. Such casings stick to the mixture at its most adhesive points — where the meat particles are in close connection with the casing. In other places the casing becomes loose and wrinkled.)

5.32. *The mixture, previous to the study of its shrinkage*, has to be graded on the bases of its composition and properties from the points of view of dynamics and shrinkage. It is obvious that the behaviour of emulsions composed of several components is quite different from that of meat emulsions, Italian pastes or meat mixtures consisting of large particles.

The grading of the latter is out as follows:

a) Meat mixtures used in salami manufacture consist of meat and fat particles of nearly uniform dimensions, when raw (similar to rice grains) and having cubic or prismatic shape.

b) No meat or fat particle occupies a distinct position in relation to the other or to the axis of the cylinder.

c) An adhesive connection exists only between the meat particles or between meat particle and casing, the fat particles are embedded between the meat particles.

d) Only the skeleton consisting of meat particles is capable of drying and shrinking. The fat particles can, at most, change their shape and place.

e) From the point of view of shrinkage and firmness the mixture may be considered statistically isotropic in every direction, or in other words, the firmness of the mixture is a statistical probability dependent on the particle size and shape, and the proportion of meat to fat.

5.33. In order to facilitate the study of the phenomenon it seemed useful to construct a model to examine shrinkage. Thus the cylindrical body may be considered as a series of concentric tubes of various diameters and the mechanism of shrinkage is studied from the point of view of the behaviour and mutual effect of these layers. Theoretically the finer this division of the cylinder is the greater the accuracy of results. In the practice, however, the overdoing of the division (for instance dividing below particle size) is not advantageous and by applying 5 to 6 layers satisfactory results may be achieved.

5.34. In the second phase of drying the shrinkage of the mixture occurs according to the moisture distribution (Fig. 3. Curve II.) i.e. the outer layers shrink more than the inner ones, or in other words the shrinkage of the outer layers is hampered by the inner ones. Thus, the outer layers of the cylinder, — similarly to tubes overstrained from the inside — are under stress in the second phase of drying. Paste-like substances, also meat mixtures, up to a certain moisture content (boundary moisture content) behave like pure plastic

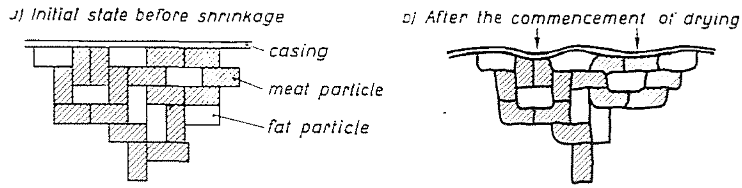

Fig. 5. The development of unevenness of the surface in a mixture containing meat and fat

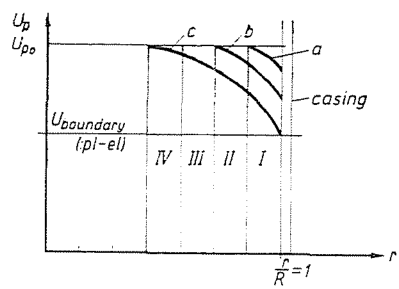

Fig. 6. Taking into consideration the gradual drying and shrinkage from layer to layer

materials, or in other words at the beginning of drying the outer layers (in meat mixtures: the skeleton consisting of meat particles) suffer, as a result of overstrain a permanent deformation. As a consequence of this a stretch relative to the shrunken dimensions, or a proportionate radial attenuation occurs.

5.35. In meat mixtures an adhesive binding occurs between the casing and the meat particles, and the casing will follow the radial shrinkage of the meat particles, while the fat particles relatively protrude more. This phenomenon leads to protrusions or unevenness of surface, particularly in goods filled into thin casings (such as "Tourist salami"). As a consequence of the deformation of the skeleton consisting of meat particles also the shape of fat particles becomes deformed (Figure 5).

5.36. Fig. 6 shows the gradual drying from layer to layer in the second phase. When the drying effect reaches the boundary of the first layer (*e.g.* $\frac{r}{R} = 0.9$) the moisture content of the outside is u_{Ia} (curve marked *a*). Marking the outside radius of each layer with the number of the layer, the outer radius of the first layer is r_{I}, and the inner one r_{II}. Since, during the shrinkage of the first layer in place $r_{\mathrm{II}} = 0.9\, r_{\mathrm{I}}$ no change of moisture content occurred, the dimension r_{II} is unchanged.

The question is, how the outside radius belonging to the first layer may be established, after the known extent of its shrinkage.

Using *a* shrinkage coefficient, valid in the range under examination, the dimension, as calculated on the basis of the r_d radius of the absolute dry material, in the case of free shrinkage, is, after reduction, formulated as follows:

$$r_{\mathrm{I}}^* = r_{d\mathrm{I}}\,(1 + au_{\mathrm{I}a}) = \frac{r_{\mathrm{I}}}{1 + au_{p0}}\,(1 + au_{\mathrm{I}a}). \tag{22}$$

Owing to the limiting effect of the given dimension of the inner layer, the above calculated dimension cannot come into being. In order to establish the actual dimension the extent of the limiting effect has to be determined.

Assuming that the thickness of layer is small and the moisture distribution in the layer linear, the thickness, uniformly shrunken may be calculated according to (22)

$$v_{\mathrm{I}}^* = v_{d\mathrm{I}}\,(1 + au_{\mathrm{I}ka}) = \frac{v_{\mathrm{I}0}}{1 + au_{p0}}\,(1 + au_{\mathrm{I}ka}). \tag{23}$$

The unlimited outer radius belonging to the cylinder may be considered as the sum of r_{II} and the thickness of layer after shrinking:

$$r_{\mathrm{I}}^{**} = r_{\mathrm{II}} + v_{\mathrm{I}}^* = 0{,}9\, r_{\mathrm{I}} + \frac{r_{\mathrm{I}} - r_{\mathrm{II}}}{1 + au_{p0}}\,(1 + au_{\mathrm{I}ka}). \tag{24}$$

By simply comparing the right sides of equations (22) and (24), it becomes apparent that $r_{\mathrm{I}}^{**} > r^*$ and thus, the outside diameter of the salami stick cannot shrink to the desired extent. The limited specific deformation

$$\varepsilon_{p\mathrm{I}} = \frac{r_{\mathrm{I}}^{**} - r_{\mathrm{I}}^*}{r_{\mathrm{I}}^{**}} = 1 - \frac{r_{\mathrm{I}}^*}{r_{\mathrm{I}}^{**}}. \tag{25}$$

The stress occurring — reduced with time and with the beginning of flow — [15] is:

$$\sigma = \frac{\mathrm{d}\varepsilon_{p\mathrm{I}}}{\mathrm{d}\tau} \cdot \eta_p \tag{26}$$

where η_p is the viscosity of the mixture. The flow of the substance forced to reach the "prescribed" larger dimension — assuming the volume to be constant — causes a changed thickness:

$$v_{\mathrm{I}}^{**} = v_{\mathrm{I}}^* \frac{r_{\mathrm{I}}^*}{r_{\mathrm{I}}^{**}} = v_{\mathrm{I}}^*\,(1 - \varepsilon_{p\mathrm{I}}) \tag{27}$$

or the change of thickness caused by hindered shrinkage:

$$\Delta v_{\mathrm{I}}^{**} = \varepsilon_{p\mathrm{I}} \cdot v_{\mathrm{I}}^{*} . \tag{28}$$

Thus the actual outer diameter is formulated according to:

$$r_{\mathrm{I}v}^{**} = r_{\mathrm{II}} + v_{\mathrm{I}}^{**} = 0{,}9\, r_{\mathrm{I}} + \frac{r_{\mathrm{I}} - r_{\mathrm{II}}}{1 + au_{p0}} (1 + au_{\mathrm{I}ka})(1 - \varepsilon_{p\mathrm{I}}) . \tag{29}$$

With the advancement of drying — e.g. when the moisture distribution as shown in Fig. 6, curve marked b is attained — the calculation has to be formulated as before, considering r_{III} radius as a given dimension. First the changed radius belonging to layer II is formulated:

$$r_{\mathrm{II}vb}^{**} = r_{\mathrm{III}} + v_{\mathrm{II}}^{*} (1 - \varepsilon_{p\mathrm{II}}) \tag{30}$$

then to compute the changed radius belonging to layer I the value $r_{\mathrm{II}vb}^{**}$ is considered as constant.

However, the permanent deformation of the 1st layer as discussed above, must not be left out of consideration. Because of the permanent deformation, the so called "dry" dimensions change also, therefore, in step b, when the outer radius is computed

$$r_{\mathrm{I}vb}^{**} = r_{\mathrm{II}v}^{**} + v_{\mathrm{I}b}^{*} (1 - \varepsilon_{p\mathrm{I}b}) \tag{31}$$

the values $r_{\mathrm{II}vb}^{**}$ and $v_{\mathrm{I}b}^{*}$ are determined from the "dry" values as changed after step a ($v_{d\mathrm{I}a}$, $r_{d\mathrm{I}a}$). The dry dimensions are modified according to the following:

$$v_{d\mathrm{I}a} = \frac{(v_{\mathrm{I}}^{*} (1 - \varepsilon_{p\mathrm{I}})}{1 + au_{\mathrm{I}ka}} = \frac{v_{\mathrm{I}}^{**}}{1 + au_{\mathrm{I}ka}} \tag{32}$$

$$r_{d\mathrm{I}a} = \frac{r_{\mathrm{I}v}^{**}}{1 + au_{\mathrm{I}a}} . \tag{33}$$

5.37. The theory and practice of computation changes only when the outer surface of the mixture (r_{I}) reaches the boundary moisture content (u_{p-e}) belonging to plastic deformation (Fig. 6. Curve marked "c"). From this point the stretch caused by the hindered shrinkage is partly plastic and only the other part is permanently deformed. The boundary of the plastic deformation is, naturally, not distinct. The material goes over gradually at first from the realm of plastic deformation to the elastoplastic, then into the domain of elastic deformation. Meat mixtures, when statically investigated, proved to behave in a similar manner (Fig. 7). Assuming in the case of $u > u_{p-e}$ pure permanent and in the case of $u < u_{p-e}$ completely elastic deformation the calculations may be simplified. For — as it will be later seen (Fig. 15) — the actual computation proves that the errors made in either direction are small and eventually cancel each other. Thus, with moisture values below u_{p-e} there is no need to correct the "dry" dimensions.

When the boundary moisture content u_{p-e} is reached, a certain tension will occur in the mixture. This tension may be obtained, on the basis of the specific hindered shrinkage values (ε), calculated according to the above, with the aid of the load diagrams as seen in Fig. 7. There is, however, no reason why the tensions should not be calculated on the basis of the linear deformation moduli E_{p-e} and E_e (elastoplastic and elastic) with correlation $\sigma = \varepsilon \cdot E(\varepsilon)$. Fig. 8. shows the moisture and tangential stress distribution at the end of drying phase II.

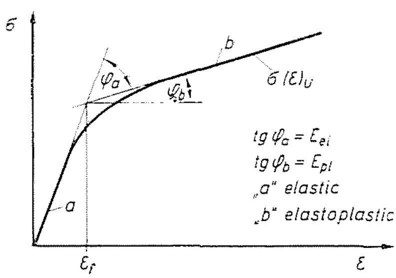

Fig. 7. A typical loading diagram $\sigma = \sigma(\varepsilon)$ in an elastoplastic material (e.g. meat mixture)

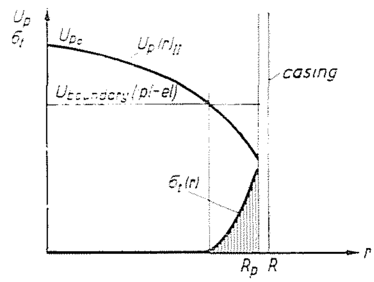

Fig. 8. The tangential stress distribution at the end of phase II applying normal drying rate

5.38. In the starting phase the stress is the greatest in the outer layers. In spite of this — due partly to the possibility of plastic deformation and to the prestretched state of the casing, there is no danger of the splitting of the mixture surface.

5.39. Shrinkage begins not only along the diameter, but also lengthwise. The longitudinal shrinkage of the outer layers is practically limited by the viscosity, i.e. the extent of binding. The greater the shrinkage is the shorter the layer becomes and the stick attains a characteristically peaked shape at the top. In casings of greater stretch characteristics there is an increase of diameter observable — due to gravity — at the lower end of the salami.

5.4. *In the third phase of drying,* as previously discussed in detail, the character of moisture distribution may be considered, closely approximating, as unchanged. On the other hand, neither the stress distribution, nor the over-strain of the layers can be considered as unchanged characteristically.

5.41. Owing to the withdrawal of the outer layers the difference between the extent of shrinkage in the outer and inner layers becomes more and more prominent. In other words the shrinkage of the outer layers is limited less and less by the inner layers and thus the release of the outer layers begins (Fig. 9).

The outer layers suffered permanent deformation during the previous phases of drying, the extent of which was dependent on the moisture gradient distribution in the initial phase of drying, *i.e.* on the drying intensity responsible for the distribution. The more rapid the emptying of the outer layer, the greater its permanent deformation, and the sooner it becomes released.

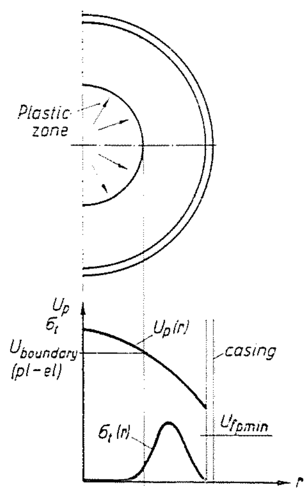

Fig. 9. The release of the surfacial layers

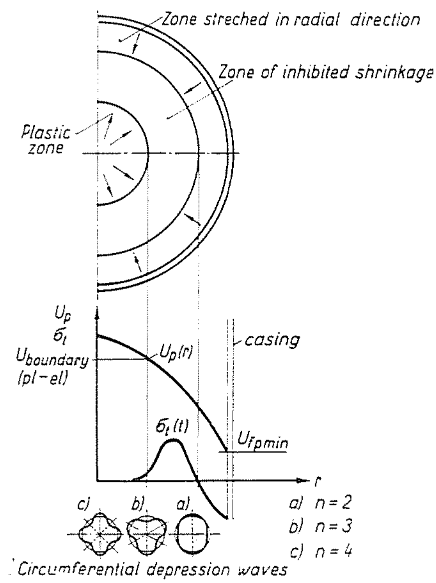

Fig. 10. The formation of radial stretching force (depression) and the cross sections corresponding to circumferential indentation wave numbers ($n = 2$—4)

5.42. The drying of the inner layers, however, continues after the surface layer, or layers are no longer under stress, and from then on the outer layers can not follow the inner ones in shrinking. Since, as a consequence of the internal binding between meat particles, the outer layers hinder further shrinkage of the inner layers, a radial stretch originates (Fig. 10).

Thus the statical overstrain in the mixture inevitably changes, and the outer layers behave like cylinders under outside pressure. As is known, this state may cause *the danger of indentation*. However, as a consequence of the drying mechanism of the mixture a further danger of damage exists: this is the danger of inner fissures. For the static characteristics belonging to the mixtures — compositions being otherwise equal — are greatly dependent on the moisture content of the mixture, or, the boundary stress — moisture distribution considered — will increase toward the center.

Let us consider in detail the causes and conditions of the phenomenon leading to these two types of damages.

5.43. The primary cause of indentation lies in the application of too high drying intensity in the initial phase. The critical depression value causing indentations and the number of indentation waves depends, to a large extent, on the thickness of the layer. The thinner this layer (the rougher the drying conditions) the smaller is the critical depression value.

The incidence of indentation is accelerated by the slightest divergence from the circular circumference. With mixtures in casings this is almost inevitable, partly because the salami stick, owing to unevenness in the wall thickness of the casing, or for other reasons, always differs from the circular cross section, and partly because of the cord tied around the stick to strengthen it and release the casing from overstrain. In the case of meat mixtures the unevennes of the surface, as developed after the beginning of drying and illustrated in Fig. 5, is also disadvantageous.

Thus, according to the experiment, it is advisable to increase the safety 3—5-fold in calculating the critical value.

The indentation — as a specific case of buckling — is from the viewpoint of statics difficult to treat, particularly under the special conditions of the present case.

First of all it had to be established whether the results of experiments hitherto carried out in relation to the indentation of tubes were applicable in the case of indentation caused by shrinkage and at what conditions were. It was established, that, in the case of meat mixture fillings, the number of indentation waves along the generating line has to be chosen $m = 1$, since on one hand this value is definite in relation to the smallest depression causing indentation, and on the other, because the value m is greatly affected by the unevennesses in the surface and cross section of the salami. The theory of Flügge [19] may be used complemented by two conditions: that the stiffening effect of the casing, and the fact that salami cannot be identified with a tube, have to be considered. The stiffening effect of the casing may be considered by adding the thickness of the casing to that of the depressive layer, and by ordering the firmness characteristics to the least moisture content of the depression layer.

The form differing from the tube has to be considered because the indentation is limited to a certain extent by the inner part. This may be done by adding the thickness of the neutral layer to that of the depression layer.

Using r for the mean radius of the depressible layer, v for its thickness, and l for the length of the stick, the Berger correlation [20] gives an approximation for the minimum critical depression value: when moisture distribution and E values of desirable accuracy are available.

$$p_{k\,\min} = 0{,}91 \cdot E \frac{r}{l} \left(\frac{v}{r}\right)^{\frac{5}{2}} \left[1 + 0{,}418 \left(\frac{\pi r}{l} - \frac{l}{2\pi r}\right) \sqrt{\frac{v}{r}}\right]. \tag{34}$$

Essentially this equation is that of the so called "girland" curve expressing the change of the exact values and thus offering the safety needed.

The number (n) of indentation waves occurring along the circumference may also be calculated from the series of curves of JUREČKA [21]. In the case of very rapid drying this may be considered at $n = 3$, otherwise at $n = 2$.

From the viewpoint of the indentation danger the length and diameter of the filled stick is not indifferent. By increasing the length, the value of p_k will decrease. From the viewpoint of statics the decreasing of the radius has a similar effect. However in the present case it does not hold, for with decreasing radius the differences in the moisture distribution decrease and with the appearance of the depressed zone, at constant drying rate, its probability greatly diminishes.

5.44. The cause of inner splitting is also to be sought for in the formation of depressed layers. If the stretch — to be calculated from the shrinkage hindered radially — exceeds the boundary stress, fissure occurs.

In the origination of fissures an important part is played by the so-called "weak points" having much smaller boundary stresses than the rest of the mixture. During the linear deformation a hindered slippage is caused, which in its turn, causes shear stress. In the case of a local fissure this shear stress may easily exceed the shear boundary stress and the local cleavage may turn into a fissure.

The weak points can be reckoned with mainly in mixtures, where ballast materials of small bearing capacity are added. Meat mixtures containing fat particles are of such a character. The firmness of a meat mixture is a statistical probability dependent on the size of meat and fat particles, on their shape, their numerical distribution and the extent of mixing. Subsequently there are, of necessity, points where more, and in others where fewer meat particles are side by side and thus binding is firmer in one place and looser in the other.

This assumption is supported by the diagram shown in Fig. 11. The diagram gives the results of the stress studies carried out with the mixture sample containing $W = 17\%$ integral moisture. An interesting change in the stress as a function of the permanent stretch is shown in the diagram. Up to a given stress the permanent stretch is constant. Above this the permanent stretch suddenly increases, again to become constant in a relatively large stress interval. With growing stress this phenomenon reoccurs and it may be explained by the breaking down of the bonds, first the fewer, later the more numerous ones. Since the probabilities belonging to certain bounds (because of the regularity of the form and distribution of the particles) are distinguished values, therefore, ranges of permanent stretch may be found, each characteristic of a "weakness grade".

In view of this the boundary of the beginning of inner fissures is extremely difficult to calculate. There is no doubt that inner fissures may be reckoned with already at small depression values. The inner fissures release locally the radial stretch between layers, thus – in case the indentation did not begin –

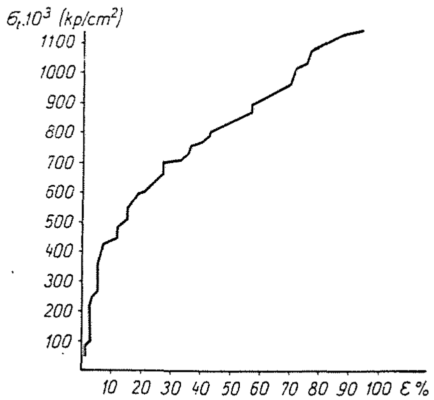

Fig. 11. The changing of the stress as a function of the permanent stretch in meat mixtures, to prove the postulate of the existence of distinguished bond numbers

the existence of a cavernous but not indented salami stick may be presumed (and actually found).

If, however, drying is ruthless both indentations and fissures occur at the same time. Thus, the theoretical basis underlying the computation of the critical states of cleavage and indentation are identical. It is therefore advisable to evade the formation of radial stretch in the inside of the salami stick during drying.

However in samples of large diameter this would be a too strict constraint. Sufficiently supported by experience, — in the case of salami mixture, for instance, the application of 4-fold safety is expedient — or knowing the boundary stress in the control calculations, only a quarter is permitted. In Fig. 12 the boundary stress belonging to various meat mixtures, in Fig. 13 the diagrams of stress plotted against specific stretch are shown and these may be used in the calculations.

The dynamical and shrinkage characteristics of mixtures are greatly affected by the quality of the meat. These effects manifest themselves in the shrinkage coefficient and in dynamical properties. It is further affected by the amount of fat particles, their manner of bounding and number of bonds.

If less ballast material (fat) is used the firmness of the mixture increases, but at the same time the shrinkage coefficient increases too. Since the latter effect is of greater significance, goods free of deformation may not be manufactured of pure meat, not even at the lowest permissible drying rate. (The high significance of meat quality was strongly supported by the experiments. It was proved, for instance, that meat mixtures prepared of beef posses a much higher loading capacity, the composition being otherwise equal than those prepared of pork. Thus, for the drying of beef based mixtures a much higher drying rate may be applied.)

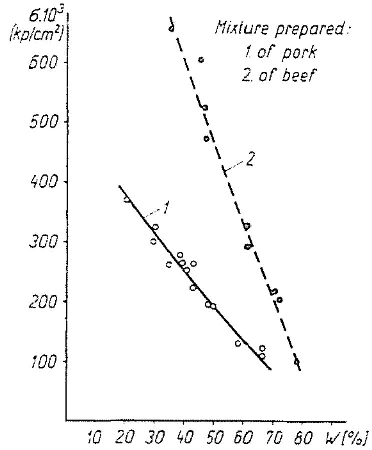

Fig. 12. The boundary stress in meat mixtures of various moisture contents, as experimentally determined

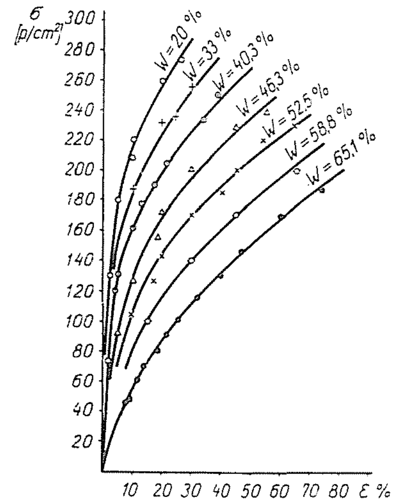

Fig. 13. Loading diagrams belonging to meat mixtures of varied moisture contents

6. Practical applications

6.1. When applying the calculations in actual practice the first step is the division of the stick into layers, the second to determine the drying curves belonging to each layer (Fig. 14).

Subsequently the transformation of the dimensions and the hindered shrinkage deformations are calculated with the aid of the initial data. The next step is to calculate the values belonging to the stresses thus evolved.

6.2. Under conditions, as seen in Fig. 14, the calculations resulted in $\sigma_t = 190$ p/cm^2 tangential stress after $\tau = 330$ hours drying (at the end of the initial phase). This stress remained constant for $\tau = 735$ hours, whereupon the release of the stress began. Drying was carried out under normal (mild) conditions, thus the release of stress followed slowly. After $\tau = 1565$ hours the stress remaining in the first layer amounted to $\sigma_t = 82.5$ p/cm^2 and 99.0;

103; 38; 24; respectively, in the others towards the center of the stick. $\sigma_t = 0$ for the first layer was reached within $\tau = 1920$ hours almost simultaneously with the following layer. At the end of the drying process ($\tau = 2305$ hours) a slight depression occurred in the first two layers, which, however, was of

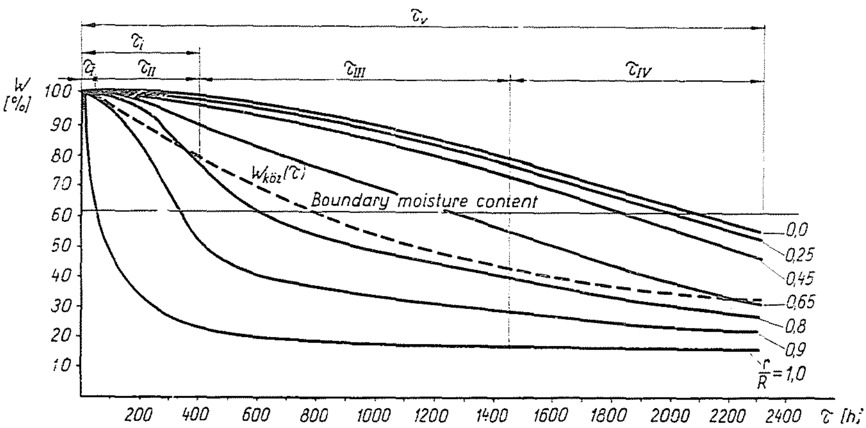

Fig. 14. Drying curves belonging to the layers of a cylindrical salami stick

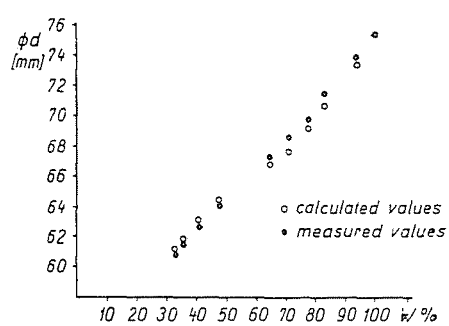

Fig. 15. The comparison of dimensions after shrinkage as calculated on the basis of the theory given and actually measured

a permissible extent ($\sigma_r = 25$ p/cm^2), and, since drying had ended by that time, no damage occurred. This stick of salami was faultless, neither deformation, nor inner fissure occurred. The shrinkage values belonging to the stick examined as plotted from data measured and calculated may be seen in Fig. 15. The concordance was found to be satisfactory.

6.3. In the case of extremely rapid drying moisture distribution developed as shown in Fig. 16. In the outer layer of the stick in $\tau = 100$ hours the tangential stress amounted to $\sigma_t = 200$ p/cm^2 which began to be released only after $\tau = 1000$ hours and at the same time a circumferential indentation wave of $n = 2$ started to develop. Apart from the indentations, also inner fissures occurred (Fig. 17).

6.4. In a short stick of salami having a great diameter an extremely high drying intensity was experimentally developed in silica gel. In this case $n = 3$ depression wave was formed in accordance with the calculations. In the

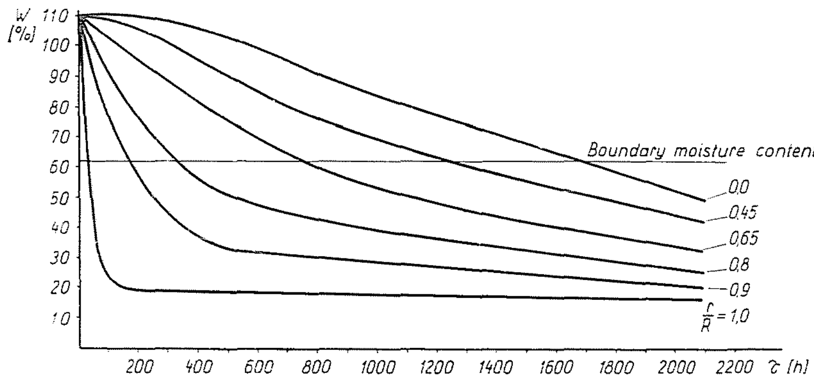

Fig. 16. The drying curves belonging to the layers of a salami stick initially exposed to ver rapid drying

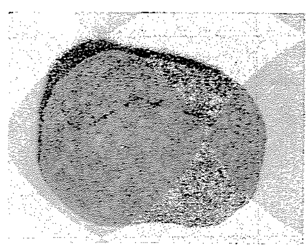

Fig. 17. Indentation and fissure caused by an extreme drying rate

central part of the stick, because of the complete initial indentation, inner fissures did not develop, for the depression zone was concealed by the excessive indentation (Fig. 18). The depression waves, however, advanced only to the conically shaped ends — developing as a result of the relatively large specific surface — therefore near to the ends large inner fissures occurred (Fig. 19).

6.5. The effect of increased shrinkage coefficient is very similar to that caused by increased drying rate. This may be achieved by the addition of a reduced amount of ballast material (fat). Considering the 30% fat content beside $W_0 = 100\%$ moisture content as normal, in order to verify the shrinkage theory, experiments were carried out with mixtures of reduced fat content. The test samples used were of 70 mm average diameter.

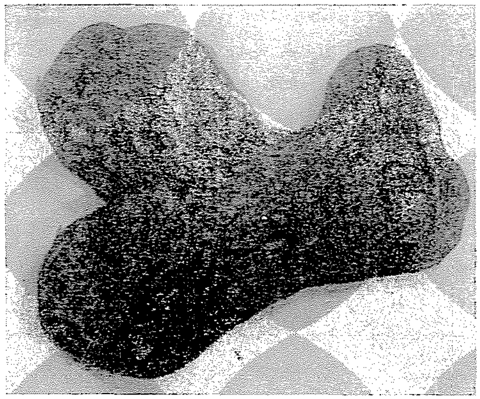

Fig. 18. The $n = 3$ wave number developed at extremely high initial drying rate

Fig. 19. The shape of a salami stick at very high initial drying rate

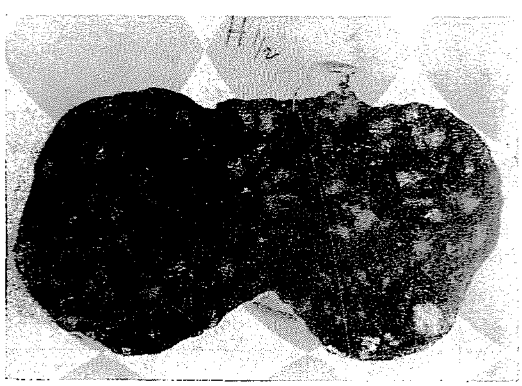

Fig. 20. The intermediate cross section of a salami stick dried at normal drying characteristics but containing a reduced amount (20%) of ballast material (fat)

The conditions occurring on the increase of the shrinkage coefficient proved these assumptions. In a stick, prepared from a mixture containing 20% fat, with parameters pertinent to a normal period of drying ($\tau = 2400$ hours) $n = 2$ circumferential depression waves and inner fissures developed (Fig. 20).

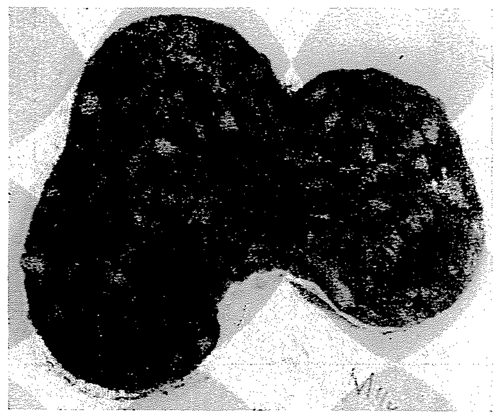

Fig. 21. The intermediate cross section of a salami stick at high initial drying rate and 20% ballast material

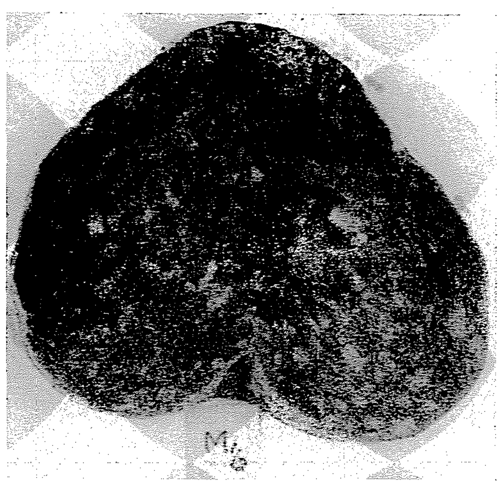

Fig. 22. The shrinkage cavities developed at the end of the stick (the same stick shown in the previous figure)

The fissures prove the presence of a stretch in the direction of the longer dimension. It may be observed that the fissures start from the so-called "weak points", *i.e.* from the proximity of fat particles, where bonding is weeker. In another case, with similar fat content but a higher initial drying rate, which however was later reduced to the same level as in the previous case, $n = 3$

depression wave was formed (Fig. 21). The inner fissures appeared as local cavities in the proximity of "weak points". At the top end of the salami stick indentation was reduced by the significant enlargement of the inner fissures, but the $n = 3$ number of depression waves remained unchanged (Fig. 22).

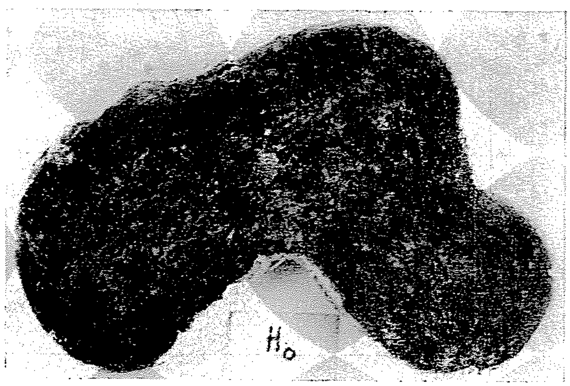

Fig. 23. The intermediate cross section of a salami stick dried at normal rate but containing only 10% ballast material

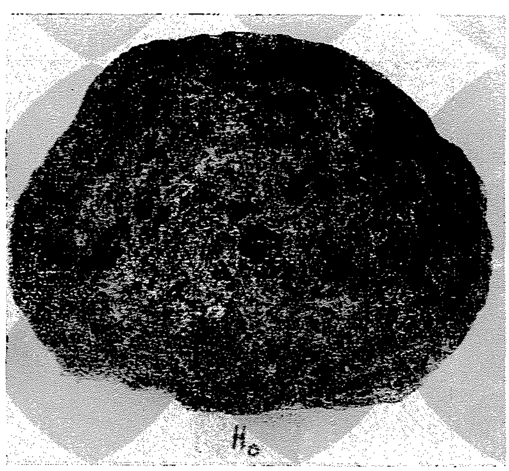

Fig. 24. The cross section at the end of the same stick as seen in the previous figure

A further reduction in the fat content led to a further increased shrinkage coefficient. Drying at normal parameters $n = 3$ indentation waves were formed and the central part of the stick was absolutely cavernous (Fig. 23) while at the ends large inner fissures were observable (Fig. 24). By the acceleration of the initial drying rate — later applying normal drying rate — $n = 3$ inden-

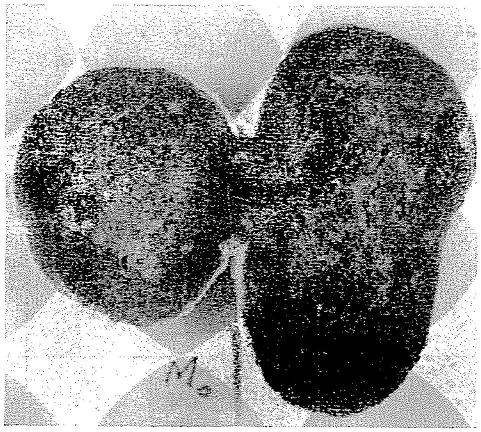

Fig. 25. The intermediate cross section of a stick with very high initial drying rate and 10% ballast material

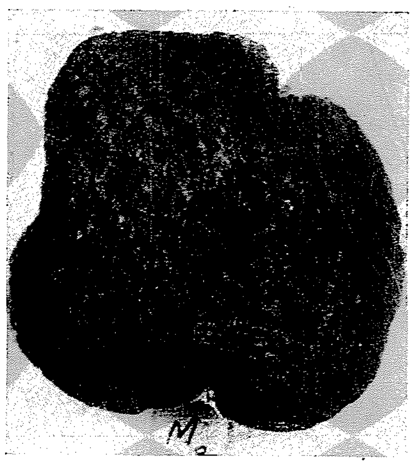

Fig. 26. The cross section at the end of the same stick as shown in Fig. 25

tation waves developed in such a deformed way that the stick was nearly divided into two (Fig. 25). At the ends extremely enlarged fissures were observable (Fig. 26).

Signs

W	[%]	integral moisture content
u	[kp/kp]	moisture content of the substance
v	[m]	thickness
r	[m]	radius
R	[m]	characteristic dimension

$\varrho = \frac{r}{R}$

l	[m]	length
t, τ	[h]	time
k	[m/h]	moisture conduction coefficient
β	[m²/h]	moisture exchange coefficient
ν		moisture transfer criterion
Θ	[M°]	mass transfer potential

$c_m = \frac{\partial u}{\partial \Theta}$ [kp/kpM°] specific mass capacity (mass transfer potential coefficient)

i	[kp/m²h]	drying intensity
γ_0	[kp/m³]	dry specific density
σ	[p/cm²]	tensile stress
p	[p/cm²]	pressure
ε		specific deformation
m		reciprocal of Poisson number
E	[p/cm²]	elasticity modulus
V	[cm³]	volume
n, m		number of indentation waves
η	[ps/m²]	dynamic viscosity
ξ		self value equation roots
α		shrinkage coefficient

Indices

p	value pertinent to the mixture
o	initial value
e	equilibrium value
v	actual value
b	value pertinent to the casing
m	value pertinent to mass exchange
1	ϱ value related to place 1
I—IV	markings of the drying phases, or layers
tot	total
i, k	serial number of roots
t	tangential
a	axial, the mark of distribution curve
r	radial
$perm$	permissible
d	dry

Summary

In order to consider the drying characteristics, dependent on the moisture content it seems useful to divide the drying procedure of cylindrical bodies of meat mixtures in hygroscopic casings into at least four phases. In the first phase drying is limited to the casing, thus the first phase lasts until the mixture begins to dry.

The drying of the mixture as affected by the casing may be considered as included in the boundary condition of the differential equation of moisture transfer inscribed on the infinite cylinder. The initial condition of the differential equation belonging to each drying phase is the moisture distribution developed at the end of the previous phase.

When meat mixtures in casing are dried the deformations and inner fissured developing during shrinkage form a limiting condition of moisture distribution and by this the drying rate.

The maximum stress, according to the assumption pertinent to shrinkage mechanism of meat mixtures in cylindrical casings, occurs in the first phase of drying.

In the outer layers of the mixture tangential stretch will develop (phase II.), and with the advancement of drying the maximum stress value appears in deeper and deeper layers, while the surfacial layers became released. In the case of rapid drying the release is followed by radial stresses, which in their turn cause the danger of indentation and inner fissures.

With the aid of the method developed for the computation of the dimensional changes caused by drying, the stresses occurring in the substance may be calculated. For the determination of the minimum depression causing indentation the theory and correlations known may be applied under certain conditions.

At various drying rates the deformation of the salami stick could be observed by the indentation waves corresponding to the theoretical data.

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