DIAGRAMS FOR THE DIMENSIONING OF HEAT TRANSFER IN COMPACT PLATE FIN HEAT EXCHANGERS

By

L. Szücs and Cs. Tasnádi

Department of Energetics, Polytechnical University, Budapest

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The fast development of chemistry and power engineering and the increasingly stringent demand for compact power machines (gas turbines, nuclear drives, etc.) urgently call for high-efficiency compact heat exchangers.

Research work in this field aims, among other things, at the designing and construction of high-efficiency heat exchangers with plate fins and laminar flow.

These heat exchangers are generally characterized by the more or less laminar flow of the medium along plate fins over a relatively long path, without mixing.

Knowing the heat transfer coefficient which in laminar flow is relatively easy to determine, heat exchange taking place in plate fins is readily calculable with the fin efficiency [1]. When computing fin efficiency, it is the phenomena of heat transfer and heat conductance arising along the fin simultaneously and mutually determine each other's boundary conditions that are taken into consideration, however, with the following neglections:

1. the heat conductance of the fin in the flow direction;
2. the warming up of the medium along the fin;
3. the temperature variations in the fin base in the flow direction.

(These neglections are not admissible if heat exchange takes place with the medium in laminar flow over a long path without mixing — as mentioned above.)

In deriving the relationships to be used in the computation of fin efficiency, it is always assumed that the fin material conducts heat towards the fin base only. But this assumption applies only if the changes in the temperature of the medium flowing along the fin is substantially less than the temperature difference which produces heat conductance in the fin proper. Should however the heat transfer coefficient be very good and the water equivalent of the flowing medium very small, or the fin length in the direction of flow relatively large — i.e. should the fin be very "deep" — then the temperature of the medium along the fin will undergo a considerable change and cause a corresponding temperature drop to arise in the fin not only in the direction of the base but also in that of the flow.

A further complicating factor is the non-uniformity of the temperature change that takes place in the medium: the fin base will cause a greater change in the temperature of the medium flowing adjacent to it than the fin tip which has a smaller effect on the temperature pattern, due to the thermal resistance of the fin.

A third change — similarly neglected in literature — may set in at the value of fin efficiency, on account of the changes in the temperature of the fin base in the flow direction.

It should be pointed out that the above neglections, as will also be proved numerically, are not admissible if the fins are relatively long and the water

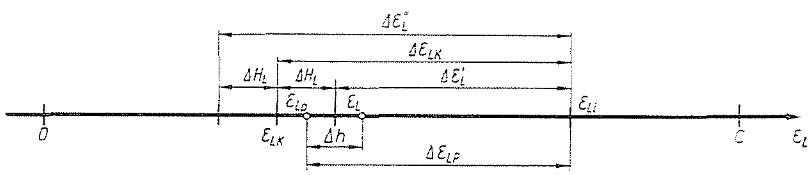

Fig. 1

equivalent of the flowing medium low in comparison with the heat transfer coefficient. This is often the case for instance, in tubes with longitudinally arranged fins (which are increasingly applied in modern heat exchangers).

For the accurate dimensioning of the heat exchangers of the above outlined type authors [2,3,4] have elaborated a number of relationships suitable for the computation of the efficiency of plate fins, taking into consideration in so doing also the above mentioned effects. These relationships were given in the form of infinite series.

The correction factor related to the temperature difference as measured on the inlet edge of the fin (similar to fin efficiency — its definition can be in detail seen in equation (50) can also be calculated, at constant fin base temperature, in the following manner:

$$\frac{\varepsilon_L}{C} = 1 - \sum_{n=1}^{\infty} \frac{2e^{\varepsilon_{3n}}}{\omega^2} \frac{1 - \left(\frac{\varepsilon_{3n}}{\varepsilon_{2n}}\right)^2 - e^{\varepsilon_{1n}-\varepsilon_{2n}}\left[1 - \left(\frac{\varepsilon_{3n}}{\varepsilon_{1n}}\right)^2\right] + e^{\varepsilon_{1n}-\varepsilon_{3n}}\left[\left(\frac{\varepsilon_{3n}}{\varepsilon_{2n}}\right)^2 - \left(\frac{\varepsilon_{3n}}{\varepsilon_{1n}}\right)^2\right]}{1 - \left(\frac{\varepsilon_{3n}}{\varepsilon_{1n}}\right)^2 - e^{\varepsilon_{1n}-\varepsilon_{2n}}\left[1 - \left(\frac{\varepsilon_{3n}}{\varepsilon_{2n}}\right)^2\right] - e^{\varepsilon_{3n}-\varepsilon_{2n}}\left[\left(\frac{\varepsilon_{3n}}{\varepsilon_{2n}}\right)^2 - \left(\frac{\varepsilon_{3n}}{\varepsilon_{1n}}\right)^2\right]}, \quad (1)$$

where

$$\varepsilon_{1n}, \ \varepsilon_{2n} \text{ and } \varepsilon_{3n}$$

are three radicals of an equation of the third degree:

$$s^3 + \frac{1}{C} s^2 - \frac{1 + \omega^2 A_u}{A_v} s - \frac{A_u \omega^2}{A_v C} = 0. \tag{2}$$

It should be noted that the relationship of ε_L to fin efficiency is also specified in the quoted papers, likewise at constant fin base temperature:

$$\varepsilon = C \ln \frac{C}{C - \varepsilon_L}. \tag{3}$$

The present paper gives those diagrams for dimensioning which were given by the above relationships, by digital computer. The diagrams refer to unchanged base temperature.

The relationship for changing fin base temperature is already available [4]. Its processing by electronic computer is in progress.

The starting equations for computer calculation

According to what has been mentioned before, the ε_L value — similar to that of the fin efficiency — can be found in literature [2], and is given in (1).

For the sake of brevity, the "n" from among the indices (which refer to the fact that to each member of the infinite series a third degree equation and three roots are pertaining have always been omitted.

(As to symbols, see the Nomenclature)

In that which follows, a brief outline will be given of the process followed in limiting the errors to within a given range. The relationships were programmed by Mr. S. Eszenyi on the 803-type National Elliot computer of the Hungarian Ministry of Heavy Industry.

Computer calculations; Assessment of errors

Our calculations were built up in such a way as to keep errors of the final result within one tenth of a per cent.

The errors were assessed by the separate examination of each member of the series and their factors.

Let us assume that series (1) was derived from (4)

$$\sum_{n=1}^{n} \frac{2}{\omega^2} = \frac{8}{\pi^2} \sum_{n=1}^{n} \frac{1}{(2n-1)^2} \tag{4}$$

having its members multiplied by an appropriate factor.

It can be proved that series (4) is convergent and monotonously increasing; its sum is:

$$\sum_{n=1}^{\infty} \frac{2}{\omega^2} = 1. \tag{5}$$

Furthermore it is obvious that the series is rapidly converging, so that even a few members ensure a high degree of accuracy. As a characteristic example, let us give two values here.

$$\sum_{n=1}^{1} \frac{2}{\omega^2} \cong 0{,}81, \tag{6}$$

$$\sum_{n=1}^{3} \frac{2}{\omega^2} \cong 0{,}94. \tag{7}$$

Moreover equation (1) indicates those members of (1) series which have been arrived at by the correction, first with the factor

$$e^{\varepsilon_3} \tag{8}$$

and subsequently by another, highly complex, factor.

Literature [6] on the other hand proves that the third root of the equation of the (2) $-\varepsilon_3$ — progresses monotonously decreasing towards $-1/C$, which proves that:

$$\lim_{n\to\infty} e^{\varepsilon_3} = e^{-\frac{1}{C}}. \tag{9}$$

The first multiplying factor, used in the correction of the members of series (1), proceeding towards a value which depends on one of the calculation parameters thus, the maximum departure of each member, can be computed.

Let us now examine the second correction factor, viz. the value of the fractional number in equation (1):

$$S_n = \frac{1 - \left(\frac{\varepsilon_3}{\varepsilon_2}\right)^2 - e^{\varepsilon_1-\varepsilon_2}\left[1 - \left(\frac{\varepsilon_3}{\varepsilon_1}\right)^2\right] + e^{\varepsilon_1-\varepsilon_3}\left[\left(\frac{\varepsilon_3}{\varepsilon_2}\right)^2 - \left(\frac{\varepsilon_3}{\varepsilon_1}\right)^2\right]}{1 - \left(\frac{\varepsilon_3}{\varepsilon_1}\right)^2 - e^{\varepsilon_1-\varepsilon_2}\left[1 - \left(\frac{\varepsilon_3}{\varepsilon_2}\right)^2\right] - e^{\varepsilon_3-\varepsilon_2}\left[\left(\frac{\varepsilon_3}{\varepsilon_2}\right)^2 - \left(\frac{\varepsilon_3}{\varepsilon_1}\right)^2\right]}. \tag{10}$$

The value of this fraction will at first decrease with a growing n, then, attaining a minimum, proceed towards $+1$, increasing monotonously.

Our investigations have furthermore shown that the value of the fraction in general rapidly converges towards $+1$.

To save computer time in calculating the value of S_n, we have included the following three commendments in the program:

$$\text{If}\ \left\{\begin{array}{l} n \geqq 3 \\ S_n > S_{n-1} \\ S_n > 0.999 \end{array}\right\}\ \text{then}\ \ S_{n+1} = 1. \tag{11}$$

These commendments ensure that the computer will assess the value of the fraction only until it attains 0.999 with increasing trend and then take it as $+1$.

Another fundamental criterion of the computation was that the first three members will always be accurately calculated. As is indicated by the numerical values of the (6) and (7) relationships — the correcting factors in the provinces under examination being always below 1 — after the first three members have been determined, the error will never exceed 6 per cent. After the determination of the first three members, the programmed restriction given in (11) set with respect to the fraction S_n will apply.

Test calculations have shown that the value of S_n calculated according to the above prescribed approximation, already from the third member onward, may be safely taken to be $+1$ in most parts of the field under examination.

Should this not be so, the computation shall be divided into two parts. While with the first i member the value of S_n must be considered with its real value, in the subsequent members it may be regarded as $+1$.

Denoting with ε_{Lp} the value of ε_1 where considering the fraction with the above-mentioned approximation, but for an infinite number of members, the following formula would yield.

$$\varepsilon_{Lp} = C\left[1 - \sum_{n=1}^{i} \frac{2}{\omega^2} e^{\varepsilon_3} S_n\right] - C \sum_{n=i+1}^{\infty} \frac{2}{\omega^2} e^{\varepsilon_3}. \tag{12}$$

Although the so calculated value will differ from the ε_L (in which the accurate value of S_n was considered throughout, but, there being no neglections up to the fourth member and the neglections even then being below 1 per mil (as per the comendment in (11), it may be safely written down as the relative error of the whole series due to this approximation in the calculation of S_n will never exceed

$$\frac{\Delta h}{\varepsilon_L} < 6 \cdot 10^{-5}. \tag{13}$$

Let us now introduce the following symbols:

$$\varepsilon_{Li} = C\left[1 - \sum_{n=1}^{i} \frac{2}{\omega^2} e^{\varepsilon_3} S_n\right], \tag{14}$$

and

$$\Delta\varepsilon_{Lp} = C \sum_{n=i+1}^{\infty} \frac{2}{\omega^2} e^{\varepsilon_3}, \tag{15}$$

whence:

$$\varepsilon_{Lp} = \varepsilon_{Li} - \Delta\varepsilon_{Lp}. \tag{16}$$

The calculated values may be illustrated in the manner as shown in Fig. 1.

Fig. 1 is eminently suitable to check the calculation of error and the various error limits. Therefore, it should always be borne in mind without any further special reference to it.

According to what has been stated with respect to the limit value of the e^{ε_3} series, the value of $\Delta\varepsilon_{Lp}$ can be restricted by the (9) equation between two well definable limits. One of the two limiting values is yielded by substituting $-1/C$ for ε_3 (the former always being less in value) and the other by substituting the value pertaining to $n = k + 1$ and denoted with ε_{3k+1} for ε_{3k} (here the former always being higher in value). Denoting these two limits by $\Delta\varepsilon'_L$ and $\Delta\varepsilon''_L$, respectively, we arrive at the following equation:

$$\Delta\varepsilon'_L < \Delta\varepsilon_{Lp} < \Delta\varepsilon''_L \tag{17}$$

respectively

$$C \sum_{n=k+1}^{\infty} \frac{2}{\omega^2} e^{-\frac{1}{C}} < \Delta\varepsilon_{Lp} < C \sum_{n=k+1}^{\infty} \frac{2}{\omega^2} e^{\varepsilon_{3,k+1}}. \tag{18}$$

According to what has been mentioned of the sum of series $2/\omega^2$ (5), it may be written that:

$$C\left(1 - \sum_{n=1}^{k} \frac{2}{\omega^2}\right) e^{-\frac{1}{C}} < \Delta\varepsilon_{Lp} < C\left(1 - \sum_{n=1}^{k} \frac{2}{\omega^2}\right) e^{\varepsilon_{3,k+1}}. \tag{19}$$

After having finished the calculation of the members of the series, the computer appplied the arithmetic mean of the $\Delta\varepsilon'_L$ and $\Delta\varepsilon''_L$ as correction factors. Denoting the correction by $\Delta\varepsilon_{Lk}$, we arrive at:

$$\Delta\varepsilon_{Lk} = \frac{\Delta\varepsilon'_L + \Delta\varepsilon''_L}{2} = C\left(1 - \sum_{n=1}^{k} \frac{2}{\omega^2}\right) \frac{e^{-\frac{1}{C}} + e^{\varepsilon_{3,k+1}}}{2}. \tag{20}$$

Accordingly, the maximum departure of the so computed $\Delta\varepsilon_{Lk}$ from $\Delta\varepsilon_{Lp}$, will be the half of the difference between the two limits:

$$|\varepsilon_{Lk} - \varepsilon_{Lp}| = \Delta H_L < \frac{\Delta\varepsilon''_L - \Delta\varepsilon'_L}{2} = C\left(1 - \sum_{n=1}^{k} \frac{2}{\omega^2}\right) \frac{e^{\varepsilon_{3,k+1}} - e^{-\frac{1}{C}}}{2}, \tag{21}$$

whereby the absolute value of the maximum error of calculation is given. Since, however, we intend to set a limit to even relative errors, these percentual errors should also be calculated.

Relative errors due to the application of the approximate instead of the accurate value of the S_n and the consideration of a finite number of members instead of infinite, might be summed up as the worst possible case.

The mathematical formulation of our criterion — i.e. to keep the relative error below one tenth of one per cent appears in the following from

$$\frac{\Delta H_L + \Delta h}{\varepsilon_L} < 10^{-3}. \tag{22}$$

Transforming the inequality, and taking the maximum error into consideration, due to the permissible approximation in the value of S_n is $6 \cdot 10^{-5}$, and may be written down as:

$$\frac{\Delta H_L}{\varepsilon_L} < 10^{-3} - \frac{\Delta h}{\varepsilon_L} = 0.00094\,. \tag{23}$$

Substituting now ε_{Lp} for the accurate ε_L value, we shall commit an error of a magnitude of $6 \cdot 10^{-5}$ in the calculation of the error. This, however is very small and consequently negligible.

Furthermore considering that the difference of ε_{Lp} (which—although with the rounding up of S_n but calculable from an infinite number of members) and ε_{Lk} (the approximate value) is $\Delta\, H_L$, our equation may be put into the following form:

$$\frac{\Delta H_L}{\varepsilon_L} \cong \frac{\Delta H_L}{\varepsilon_{Lp}} < \frac{\Delta H_L}{\varepsilon_{Lk} - \Delta H_L} < 0.00094, \tag{24}$$

which gives the following relationship for the permissible relative error:

$$\frac{\Delta H_L}{\varepsilon_{Lk}} < 0.00093. \tag{25}$$

The computer derived the relationship from the following formula:

$$\frac{\Delta H_L}{\varepsilon_{Lk}} = \frac{\left(1 - \sum\limits_{n=1}^{k} \frac{2}{\omega^2}\right) \frac{e^{\varepsilon_{3,k+1}} - e^{-\frac{1}{C}}}{2}}{1 - \sum\limits_{n=1}^{i} \frac{2}{\omega^2} e^{\varepsilon_3} S_n - \sum\limits_{n=i+1}^{k} \frac{2}{\omega^2} e^{\varepsilon_3} - \left(1 - \sum\limits_{n=1}^{k} \frac{2}{\omega^2}\right) \frac{e^{\varepsilon_{3,k+1}} + e^{-\frac{1}{C}}}{2}}\,. \tag{26}$$

This shows that the command whose fulfilment ensures the required accuracy is the following: the computer must derive the subsequent members until the value yielded by the (26) relationship is below $9{,}3 \cdot 10^{-4}$.

This will have given an idea of how the number of members necessary to ensure a min. 0,1 per cent coincidence of the approximate sum of the infinite series and its accurate value, can be determined.

As will be seen from the References [2], [3], [4] and the Nomenclature annexed to this paper, ε_L is not identical with the so-called fin efficiency known from practice. The two coefficients have much in common in that each is the value of the quotient of the heat flux and the heat transfer coefficient referred to some characteristic temperature differences, with the only difference that ε_L is referred to the temperature difference measured at the inlet and not to the mean difference between the temperatures of the fin base and the flow.

Fin efficiency — as referred to the mean temperature difference between fin base and the medium — is denoted by ε.

From the value of ε_L, ε is readily calculable as has been proved in the literature quoted in [3].

With uniform fin base temperature, the calculation may be carried through the already quoted simple relationship (3).

Since in the denominator of (3) is the difference of two values — the error in ε_L if the difference $(C - \varepsilon_L)$ is small — will cause a substantially greater error in the value of ε. It is therefore expedient to assess the error in ε (the fin efficiency) as well.

Let us introduce the following denotations:

$$\Delta H = |\varepsilon_k - \varepsilon|, \text{ and} \tag{27}$$

$$\Delta h + \Delta H_L > |\varepsilon_{Lk} - \varepsilon_L| \tag{28}$$

where the index k denotes (as above) the approximate value of the quantity represented by the letter for which it stands. ε_k is thus the approximate value of the fin efficiency, while ε_{Lk} the approximate value of the above examined ε_L.

Let us calculate from the approximate value of ε_L (ε_{Lk}) the approximate value of ε (ε_k), using the (3) equation:

$$\varepsilon_k = C \ln \frac{C}{C - \varepsilon_{Lk}} . \tag{29}$$

The difference between (3) and (29) will appear as:

$$\varepsilon_k - \varepsilon = C \left(\ln \frac{C}{C - \varepsilon_{Lk}} - \ln \frac{C}{C - \varepsilon_L} \right) . \tag{30}$$

After rearranging, and using the denotations of (27) and (28), we arrive at:

$$\Delta H < C \ln \left(1 + \frac{\Delta h + \Delta H_L}{C - \varepsilon_{Lk}}\right). \tag{31}$$

The relative error of the fin efficiency may also be calculated, setting the inequality of:

$$\frac{\Delta H}{\varepsilon_k} < 10^{-3} \tag{32}$$

as a criterion for calculation accuracy.

The fact that the accurate value of ε is unknown and only the approximate is available, may create some difficulty. Let us therefore first determine the value of

$$\frac{\Delta H}{\varepsilon_k} = \frac{\ln \left(1 + \dfrac{\Delta h/\varepsilon_{Lk} + \Delta H/\varepsilon_{Lk}}{C/\varepsilon_{Lk} - 1}\right)}{\ln \dfrac{C}{C - \varepsilon_{Lk}}}. \tag{33}$$

This may be written as:

$$\frac{\Delta H}{\varepsilon} < \frac{\Delta H}{\varepsilon_k - \Delta H} < 10^{-3}, \tag{34}$$

whence it follows that

$$\frac{\Delta H}{\varepsilon_k} < \frac{1}{1001} = 0.999 \cdot 10^{-3}, \tag{35}$$

which ensures that the error in ε will not exceed one tenth of one per cent.

To maintain this accuracy, we had the computer work out $(\Delta H/\varepsilon_k)$ and as a last commendment the computer was made to calculate further members of the series until the value of $\Delta H/\varepsilon_k$ dropped below the limit given in (35).

Calculating the efficiency of slotted ribbing in a computer

Slotted ribs are a special kind of high-efficiency plate fins. They consist essentially of plate fins appropriately slotted in flow direction, to improve heat transfer coefficient. The problems that tend to arise in coonnection with slotted ribs was treated in the [3] in some detail. From the point of view of programming for computer calculations, it is doubtless that the infinite series given for the ε_L of such fins will depart from the infinite series for continuous fins only in that the S_n is absent from the series of the slotted fins, and the values of ε_3 are easier to calculate

$$\frac{\varepsilon_L}{C} = 1 - \sum_{n=1}^{\infty} \frac{2}{\omega^2} e^{\varepsilon_3}, \tag{36}$$

respectively

$$\varepsilon_3 = -\frac{1}{C} \frac{2}{\omega^2 + (mh_x)^2}. \tag{37}$$

(see below the definition of mh.)

Fin efficiency ε may be calculated from the calculated value of ε_L in a similar manner, with equation (3). The relative error of ε and ε_L may also be estimated according to the given formulas.

Accordingly, the integration of the above special problem concerning slotted ribs into the program of the computer, causes no difficulty whatsoever.

The sequence followed in the computation

The computation process was performed in the following way:

Up to the i^{th} member ($i \geq 3$), the computer worked out all members with complete accuracy, also taking the values of S_n into consideration. Having fulfilled the criteria of (11) it calculated each member, taking the value of S_n to be 1.

On the basis of the (26) relationship, the computer took the value of $\Delta H_L/\varepsilon_{Lk}$, too, and when this dropped below 0.00093, it began to calculate the $\Delta H/\varepsilon_k$ error from the (33) relationship. When this value dropped below 0.999 per mille, no further members were calculated, but the computer added the correction given in (20). Finally it printed out the so computed value.

Chart parameters

With a view to satisfying practical demands, the following values were chosen for the parameters of the charts:

The first:

$$C = \frac{C_k}{2ah_y}. \tag{38}$$

The second:

$$\frac{1}{\sqrt{A_u}} = h_x \sqrt{\frac{2a}{\lambda_x v_0}} = m_x h_x = (mh)_x, \tag{39}$$

of which $m_x h_x$ will, in what which follows, appear in this form $(mh)_x$ or, abbreviated even, as mh_x, where the index x indicates that when calculating the „mh" dimensionless value, both the longitudinal fin dimension and the heat conductance of the fin in x direction must be taken into consideration.

And finally the third characteristic value is as follows:

$$\sqrt{\frac{A_v}{A_u}} = \sqrt{\frac{\lambda_y v_0}{2a}}\,\frac{1}{h_y}\sqrt{\frac{2a}{\lambda_x v_0}}\,h_x = \frac{h_x}{h_y}\sqrt{\frac{\lambda_y}{\lambda_x}}\,. \tag{40}$$

Calculation of fin efficiency with slotted ribs

The so-called slotted-fin heat exchanger is a special construction of high-efficiency heat exchangers. It applies fins densely slotted in the flow direction for better heat exchange.

Due to the fact that slots perpendicular to flow cut as it were the heat conductance of the material in y direction, thus fulfilling the criterion whereby $\lambda_y = 0$, the value of $\frac{h_x}{h_y}\sqrt{\frac{\lambda_y}{\lambda_x}}$, with slotted fins, is equal to zero.

Description of the enclosed charts

The values arrived at in the computations have been plotted in charts Nos. 1, 2, 3, 4, 5 and 6. On their abscissae the C and on their ordinates the fin efficiency — has been indicated, while the parameter of the set of curves is the value of mh_x.

The value of $\frac{h_x}{h_y}\sqrt{\frac{\lambda_y}{\lambda_x}}$ is constant within each chart.

Accordingly, the entire area under examination has been encompassed in six charts, while fin efficiencies pertaining to $\frac{h_x}{h_y}\sqrt{\frac{\lambda_y}{\lambda_x}}$ values for which no charts are available, can readily be determined by interpolation.

It should be noted that the charts naturally enable the determination of fin efficiencies as generally calculated, viz. those that disregard heat conductance of the fin in flow direction and the warming up of the medium, since this way of determination may be considered as the accurate fin efficiency, related to the value of $C =$ infinite.

Denoting the so calculated fin efficiency with ε_R, in conformity with the well known formula:

$$\varepsilon_R = \frac{\operatorname{th} mh_x}{mh_x}\,. \tag{41}$$

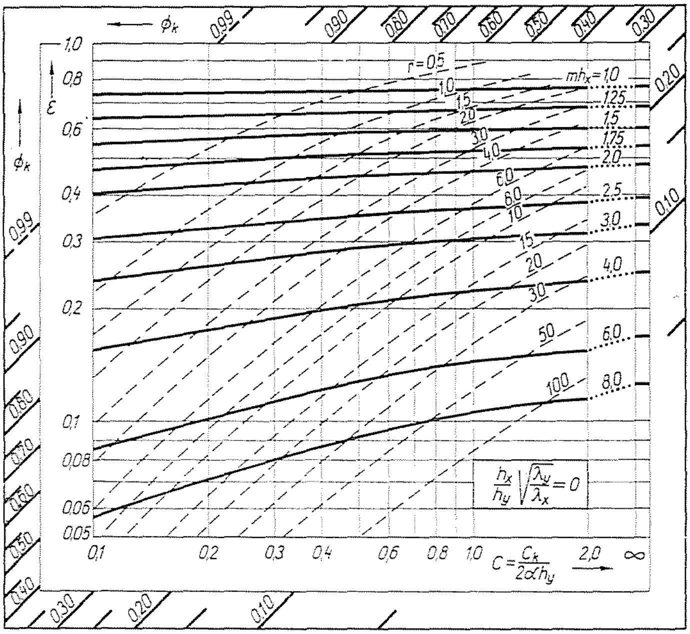

Chart 1. Accurate plate fin efficiency which takes into consideration the heat conductance of the fin in flow direction and the non-uniform changes in the temperature of the flow medium

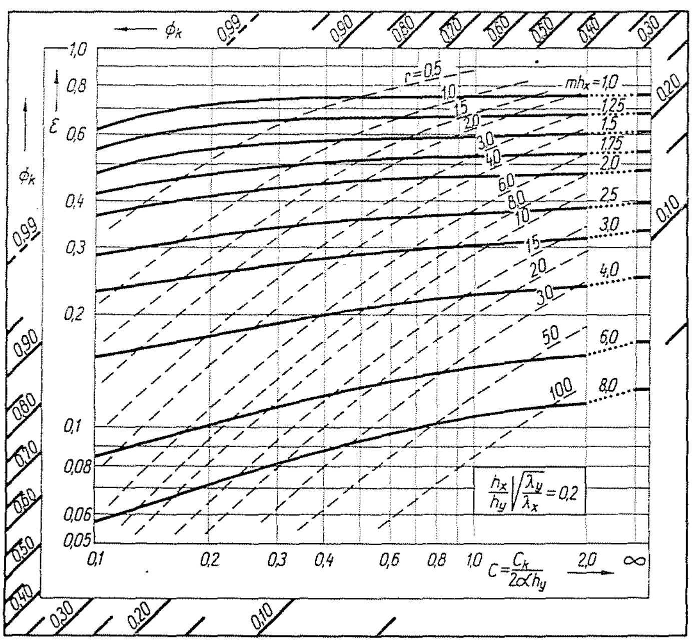

Chart 2. See 1.

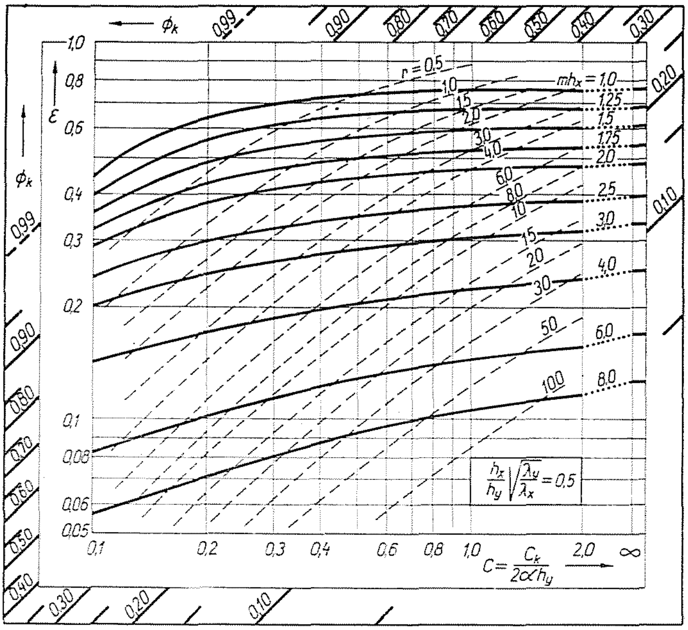

Chart 3. See 1.

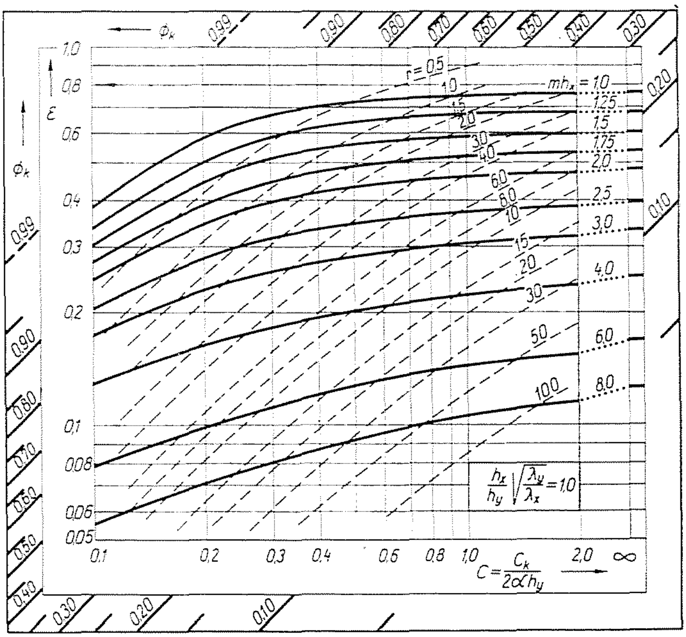

Chart 4. See 1.

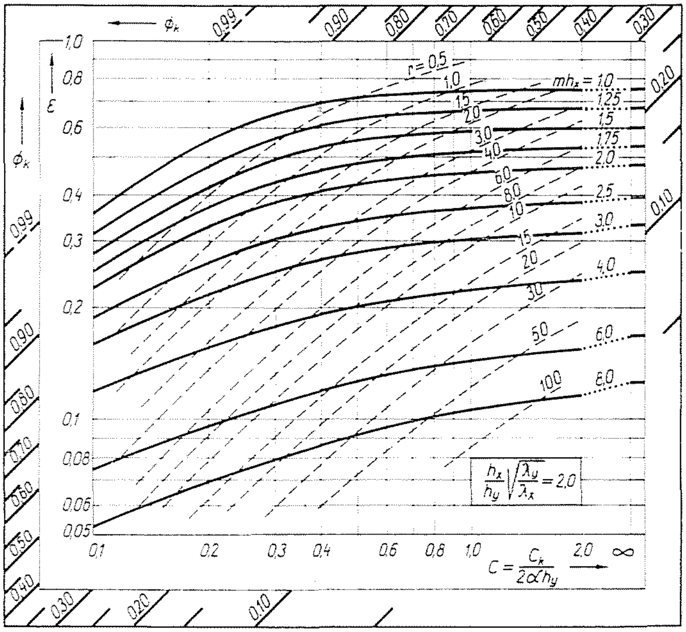

Chart 5. See 1.

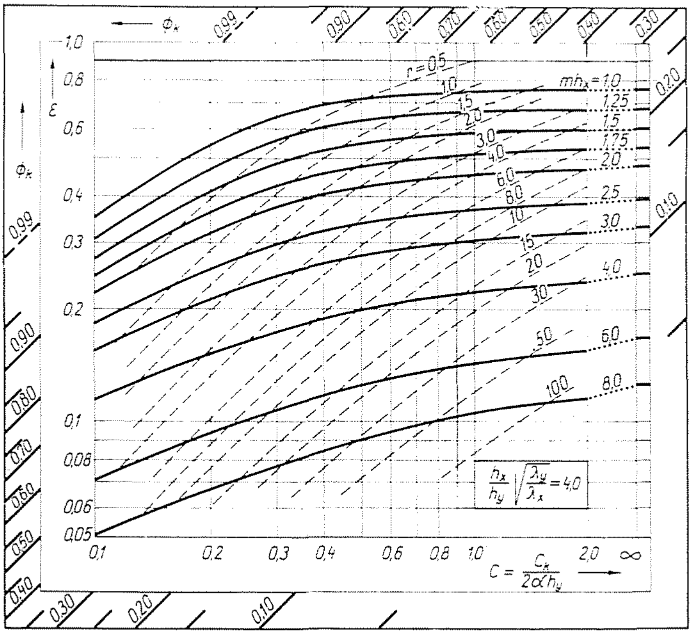

Chart 6. See 1.

Thus, according to the charts given above, in possession of the fin geometry (h_x length, h_y depth and v_0 thickness);

fin material (one having a heat conductance of λ_x perpendicular to the flow and λ_y in flow direction);

the water equivalent of the heat transfer medium (C_k); and

the heat transfer coefficient (a),

the accurate fin efficiency can be determined.

It appears from the charts that a given fin design will have different fin efficiencies corresponding to the different modes of operation. The points pertaining to these different modes of operation are plotted on a continuous curve.

With a given fin design, namely, the geometry and material characteristics of the fins which remain unchanged, the heat transfer coefficient must still in most cases be the continuous function of the mass rate of flow and, with it, of the water equivalent to the flowing medium:

$$h_x \; h_y \; v_0 \text{ and } \lambda \text{ are const; } a = f(C_k). \tag{42}$$

Consequently, in the enclosed fin efficiency charts, each fin design its own characteristic curve for a given heat transfer medium.

The introduction of fin effectivity (Φ_k)

There is no difficulty whatsoever in introducing a parameter also in relation to fin efficiency. This parameter is generally defined for heat exchangers [5] and known as "effectivity" and denoted by Φ_h. The value of Φ_k that we wish to introduce is distinct from the Φ_h defined for heat exchangers in that our T_0 stands for the temperature of the fin base and not for the other heat transfer medium. If the temperature of the fin base fairly approximates that of the medium flowing on the yonder side, the two values will coincide. This is the case, for instance, when the fin transfers heat to gas, while fin base picks up heat from condensing steam or a liquid having a good heat transfer coefficient.

With the denotations of Figure 2, it can be written as a definition that

$$\Phi_k = \frac{\Delta t_f}{\Delta T_0}. \tag{43}$$

Furthermore it follows from the definition of the efficiency that:

$$Q = \varepsilon a \; F_0 \, \Delta T_{\log}, \tag{44}$$

respectively, introducing the water equivalent of the medium:

$$Q = \Phi_K \cdot J \cdot \Delta T_0. \tag{45}$$

Applying (45) to (44):

$$\Phi_k = \varepsilon \frac{aF_0}{J} \frac{\Delta T_{\log}}{\Delta T_0}. \tag{46}$$

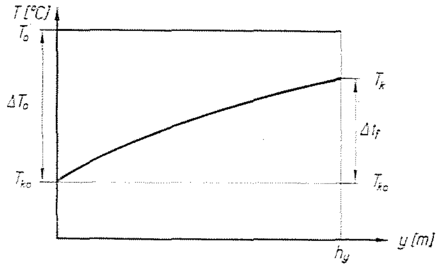

Fig. 2. Average temperatures of the medium and the fin base in the function of distance from the inlet

In view, however, of the interrelation between the mean temperature difference and the temperature difference at the inlet:

$$\frac{\Delta T_{\log}}{\Delta T_0} = \frac{1}{\Delta T_0} \frac{\Delta t_f}{\ln \dfrac{\Delta T_0}{\Delta T_0 - \Delta t_f}} = \frac{\Phi_k}{\ln \dfrac{1}{1-\Phi_k}}, \tag{47}$$

it may be written that

$$\frac{1}{C} = \frac{2ah_y}{C_k} = \frac{2ah_x h_y}{C_k h_x} = \frac{aF_0}{J} \tag{48}$$

and

$$\frac{\varepsilon}{C} = \ln \frac{1}{1-\Phi_k}. \tag{49}$$

What has gone before will prove that each Φ_k value has a straight in the $\varepsilon - C$ coordinate plane intersecting the origo. These straights, in the more illustrative $\ln \varepsilon - \ln C$ diagram, transform into parallelly shifted lines at 45°. The lines have not been plotted in our charts but have been made readily illustrable in a diagram with a scaled border.

Introducing the Φ_k value and scaled border in the charts, orientation became considerably easier, a given temperature pattern having been related to each point.

Let us finally briefly examine, how the value of ε_L is related to out fin efficiency chart.

The definition of ε_L is as follows:

$$Q = \varepsilon_L \ \alpha \ F_0 \Delta T_0. \tag{50}$$

Applying (50) to (45) we may write that:

$$\varepsilon_L \ \alpha F_0 = \Phi_k \cdot J. \tag{51}$$

Finally, taking into consideration also the (48) relationship, we may write that:

$$\varepsilon_L = \Phi_k \cdot C, \tag{52}$$

whereby the simple correlation of the Φ_k value of the fins, "fin effectivity" and the value of ε_L similar to fin efficiency, but in relation to the temperature, difference at the inlet, has been illustrated.

Definition of the fin efficiency and the heat transfer coefficient of fin-type heat exchangers for the evaluation of experiments

In evaluating the measurements carried out on fin-type heat exchangers, the determination of the fin efficiency and the heat transfer coefficient are fundamental requirements. In the approximate determination of the fin efficiency from equation (41) this is generally satisfied in such a way that the product of the heat transfer coefficient and the fin efficiency is calculated from the measured thermal output of the heat exchanger, then the values of the heat transfer coefficient and fin efficiency are determined through iteration, by using the (41) formula.

By appplying the enclosed charts and the accurately determined fin efficiency, not only the lengthly iteration process can be avoided but the actual value of the heat transfer coefficients can be determined. In the processes followed so far, namely, the error of the fin efficiency calculations was transposed into the value of the heat transfer coefficient, the measurement having given the product of these two.

To facilitate evaluation, let us introduce the following variable:

$$r = C\,(mh_x)^2 \frac{C_k}{\lambda_x} \frac{h_x^2}{v_0 h_y} = \frac{J}{F_0} \frac{2h_x^2}{\lambda_x v_0}\,. \tag{53}$$

Its curves have been indicated in the charts by dotted lines.

Now the characteristic values of the measured fins are easy to determine, since r may be computed from the known quantity of the heat transfer medium and the geometric characteristics of the fin design, while the value of Φ_k can be determined from the measured temperatures.

The so determined two values define one single point in our chart. This is the one characteristic of the fin design under the given operational conditions. Measuring the temperatures under various operational methods so as to calculate Φ_k, the points plotted in the chart will give the aforementioned characteristic curve of the fin design. In their possession both the accurate heat transfer coefficient and the accurate fin efficiency can be determined, from each point of the curve.

In spite of the fact that takes both of the heat conductance of the fin in flow direction and the warming up of the flow medium, into account the evaluation of the fin design according to this method is considerably easier than the usual approximate evaluation according to the (41) formula. This is due to the fact that the computation starts out from simply produced factors (r, Φ_k) derived in the direct way from measured values (temperatures, mass rates of flow) and yields the actual fin efficiency, respectively the actual heat transfer coefficient from the value of mh_x.

The error due to neglection of heat conductance in flow direction and the warming up of the medium

The significance and application sphere of the computation of the accurate fin efficiency can be determined by calculating the error of the value by means of the (41) formula as against the accurate fin efficiency as derived from the charts.

Taking the status of the fin to be $mh_x = 1.5$ as frequently encountered in heat engineering, we have illustrated in Figure 3, as the function of two specifically chosen parameters, the percentual error caused by the above neglection. The figure will clearly show that in several areas the magnitude of the error is considerable.

It is observable that the magnitude of the error increases with the fin depth. This means that by larger fin depth the value of the fin efficiency can substantially reduced in comparison to that calculable by means of the (41) formula.

Let us furthermore consider that the heat transfer coefficient tends to increases mostly according to the power of the mass rate of flow, having an exponent below 1 — which permits the conclusion to be drawn that the value of a/C_k generally increases with decreasing mass rates of flow. Thus, with a smaller mass rate of flow — as shown in Figure 3., the relative error will also increase.

All this is naturally quite obvious qualitatively, since the ε_R fin efficiency calculable with the (41) formula is derived just by means of an approximation, assuming the temperature of the flowing medium along the fin as being constant. This takes place with a zero depth of the heat exchanger, that is, with infinite water equivalent of the flow medium.

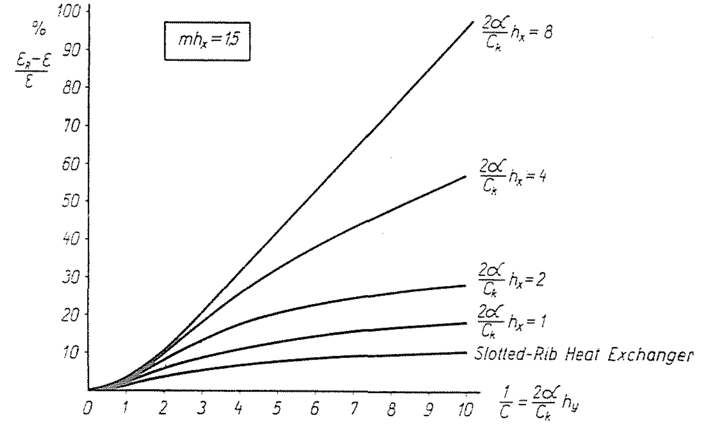

Fig. 3. Error due to neglection of the heat conductance in flow direction and the non-uniform changes in the temperature of the medium (at $mh_x = 1.5$)

The effect of heat conductance on fin efficiency in flow direction

The chart 1 shows the fin efficiencies (this case refers primarily to heat exchangers with slotted fins) with zero conductance in the direction of flow. It would be interesting to compare the curves of the chart 1. with those in the remaining five, in which heat conductance in the direction of flow and normal to it, coincide. The comparison shows that at given C and mh_x values the highest fin efficiency pertains to the value of heat conductance in flow direction being 0. This, in other words, means that the mere slotting of any kind of plate fins normal to the flow, even though it does not affect the heat transfer coefficient, does cut heat conductance in flow direction and increases the output of the heat exchanger.

This will throw light on the special advantage of slotted fins, over and above the amelioration of the heat transfer coefficient.

Summary

Authors point out that the non-uniform temperature changes of a medium flowing along plate fins (due to different temperature changes taking place adjacent to the fin base and at the fin tip) and the heat conductance of the fin in the flow direction should not be neglected in all cases when dimensioning the finned surface. It is shown that with small mass

rates of flow, with efficient fin design or with long fins in flow direction (as for instance with compact fins having very good heat transfer coefficient, tubes finned longitudinally in the axial direction, etc.) such neglections may lead to errors of up to 10 to 30 per cent magnitude.

Having evaluated by means of a computer the final results of their investigations [2] [3] [4] [6], authors present charts drawn up for dimensioning in consideration of the above effects.

Authors finally describe a process based on their own charts which helps simpler, faster and more accurate evaluation of the results of the measurements on finned surfaces than the usual fin efficiency formula which neglects the shove effects.

Nomenclature

h_x Fin dimension normal to flow

h_y Fin dimension in flow direction

Δh $\Delta h = \varepsilon_L - \varepsilon_{Lp}$

i The value of the "n" index at which the computer still calculates the S_n value

k The maximum value of the "n" index, still calculated in the computer

m $m = \sqrt{\dfrac{2a}{\lambda_x v_0}}$

n Index

r Value characteristic of the fin design:

$$r = \frac{J}{F_0} \frac{2h_x^2}{\lambda_x v_0}$$

v_0 Fin thickness

x, y Coordinates of place. x = normal to flow, y in flow direction

A_u ; A_v Dimensionless numbers

$$A_u = \frac{\lambda_x}{2a} \frac{v_0}{h_x^2}, \qquad A_v = \frac{\lambda_y}{2a} \frac{v_0}{h_y^2}$$

C Dimensionless number:

$$C = \frac{C_k}{2ah_y}$$

C_k Part of the water equivalent of the rate of mass flow related to unit fin length in x direction

F_0 $F_0 = 2h_x h_y$ the heat transfer surface of the fin

J $J = C_k h_x$ the water equivalent of the rate of mass flow along the fin

ΔH The error of the efficiency calculation; $\Delta H = |\varepsilon_k - \varepsilon|$.

ΔH_L A member in the error of the calculation of ε_L; $\Delta H_L = \varepsilon_{Lk} - \varepsilon_{Lp}$

S_n The denotation of equation (10)

T_0 Fin base temperature

T_{k0} Temperature of the medium at the inlet

T_k Temperature of the medium behind the fin

Δt_f $\Delta t_f = T_k - T_{k0}$ the warming up of the medium

ΔT_0 $\Delta T_0 = T_0 - T_{k0}$ the difference between the temperature of the medium at inlet and the fin temperature

$\Delta T_{\log}$ Logarithmic mean temperature difference between fin base and medium

Q Heat transferred to one fin during a unit of time

a Heat transfer coefficient between medium and fin

ε Fin efficiency

ε_k Calculated value of fin efficiency

ε_L Dimensionless number similar to fin efficiency

$$\varepsilon_L = \frac{Q}{2h_x h_y \Delta T_0 a}$$

ε_{Li} See equation (14)

ε_{Lp} See equation (12)

ε_{Lk} The calculated and corrected ε_L value

ε_R The approximate value of fin efficiency

$\varepsilon_{1,2,3}$ Roots of the equation (2)

$\varepsilon_{3;\,k+1}$ The ε_3 value pertaining to the k + 1 member

$\Delta\varepsilon'_L; \Delta\varepsilon''_L$ The limits of $\Delta\varepsilon_{Lp}$

$\Delta\varepsilon_{Lk}$ The error of the series calculated up to the i^{th} member

$\lambda_x; \lambda_y$ Conductance of fin in x respectively y direction

ω $\omega = \frac{2n-1}{2}\pi$

Φ_h Effectivity of the heat exchanger

Φ_k Dimensionless quantity characterising temperature changes in the medium flowing along the fin, fin effectivity

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Dr. László SZÜCS
Csaba TASNÁDI
Budapest, XI., Sztoczek u. 2–4. Hungary