STUDY ON A PROBLEM OF PNEUMATIC CONTROL

ADIABATIC FLOW IN AN INFINITELY LONG TUBE BY ASSUMING PERIODIC BOUNDARY CONDITION

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The signal transmission in a pneumatic regulating system is often carried out to a long distance through a narrow piping; thus, the questions we have to put are: how the friction in the air filled piping acts as damping factor and how the form of the signal may be distorted.

In this paper this problem will be dealt with, for the case of the simplest initial signal having a sine form. Besides, frequency and amplitude are freely chosen within certain limits. Anyhow it is necessary to have a rather small amplitude compared with the unit.

The object of our consideration is a long (on principle infinite) air-filled tube; on one end there is a piston moving in periodic motion e.g. $A \sin \omega t$. Our task is to determine, on whichever point of the tube, the displacement of a gas particle from the point of equilibrium: $\Phi(x, t)$. Thus, $\Phi(x, t)$ denotes the displacement of the particle with the original position of x at the moment t. Now, in describing the motion, the Lagrange method will be applied; this choice is advantageous, because the displacements in question, in relation to the point of equilibrium, are of the smallest range. In this system, $\dot{\Phi}$ and $\ddot{\Phi}$ denote the velocity and the acceleration; both bound by more complicated forms of expression, if the Euler method is applied.

As is known, the differential equation relating to frictional motion, has the following form:

$$\frac{a_0^2}{(1+\Phi_x)^{\varkappa+1}} \Phi_{xx} - \Phi_{tt} - b_0(1+\Phi_x)\Phi_t = 0$$

According to the above described case, the boundary conditions are: at the front-end of the tube, we assume a harmonic oscillation, having a frequency ω and an amplitude A;

$$\varphi(0,t) = A \sin \omega t$$

and according to further suppositions, due to the influence of friction, at an infinite distance, the motion ceases i.e.

$$\lim_{x\to\infty} \Phi(x, t) = 0$$

Further, considering the process after a fairly long period of time, it disappears under the steady influence of the friction and the initial condition.

The physical para meters in the differential equation and the boundary condition are: a_0[m/sec] = 340, the sound velocity in the air; $\varkappa = 1.4$ (constant, dimensionless), pipe diameter 5 mm, and accordingly: $b_0 = 16$ [sec^{-1}], Φ [m], Φ_x (dimensionless), Φ_{xx} [m^{-1}], ω [sec^{-1}], $A = 0.05$ [m].

With a Φ_x having a value small enough, i.e. for waves having a rather flat form, the approximative form of the differential equation is:

$$[1-(\varkappa+1)\Phi_x]\,\Phi_{xx}-\frac{1}{a_0^2}\Phi_{tt}-\frac{b_0}{a_0^2}\Phi_t=0.$$

NB: It was GRUBER who drew attention to this problem.

The positive parameter λ shall be introduced as follows:

$$\lambda = A(\varkappa+1) \ll 1.$$

With this, the differential equation will be modified:

$$\left(1-\frac{\lambda}{A}\Phi_x\right)\Phi_{xx}-\frac{1}{a_0^2}\Phi_{tt}-\frac{b_0}{a_0^2}\Phi_t=0.$$

The function $\Phi(x,t)$ that satisfies both the differential equation and the boundary condition, can be written according to our supposition as a progressive series of powers of λ:

$$\Phi(x,t)=\sum_{n=0}^{\infty}\lambda^n\,\Phi_n(x,t).$$

With this, the differential equation can be written in this form:

$$\left(1-\frac{1}{A}\sum_{n=0}^{\infty}\lambda^{n+1}\,\Phi_{nx}\right)\sum_{n=0}^{\infty}\lambda^n\,\Phi_{nxx}-\frac{1}{a_0^2}\sum_{n=0}^{\infty}\lambda^n\,\Phi_{ntt}-\frac{b_0}{a_0^2}\sum_{n=0}^{\infty}\lambda^n\,\Phi_{nt}=0.$$

After rearrangement according to powers of λ, the functions Φ_n are described by the following series of differential equations:

$$\mathbf{D}[\Phi_0]=0 \tag{1}$$

$$\mathbf{D}[\Phi_n]=\frac{1}{A}\sum_{\mu=0}^{n-1}\Phi_{(n-1-\mu)x}\,\Phi_{\mu xx}; \qquad (n=1,2,\ldots) \tag{1a}$$

where

$$\mathbf{D}[\Phi_n]\equiv\Phi_{nxx}-\frac{1}{a_0^2}\Phi_{ntt}-\frac{b_0}{a_0^2}\Phi_{nt}; \qquad (n=0,1,2,\ldots)$$

Obviously, the boundary condition can be separated:

$$\Phi_0(0,t) = A \sin \omega t$$

$$\Phi_n(0,t) = 0, \qquad (n = 1, 2, \ldots)$$

$$\lim_{x \to \infty} \Phi_n(x,t) = 0, \qquad (n = 0, 1, 2, \ldots).$$

Let us try to establish the function Φ_0 in the following form:

$$\Phi_0(x, t) = u_0(x)\, e^{i\omega t}.$$

From this, we obtain for u_0 the differential equation of ordinary type:

$$u_0'' + \left(\frac{\omega^2}{a_0^2} - i\,\frac{\omega b_0}{a_0^2}\right) u_0 = 0\,.$$

Introducing the notation:

$$\frac{\omega^2}{a_0^2} - i\,\frac{\omega b_0}{a_0^2} = (\alpha + i\beta)^2$$

we obtain

$$\alpha^2 - \beta^2 = \frac{\omega^2}{a_0^2}, \qquad 2\alpha\beta = -\frac{\omega b_0}{a_0^2}\,.$$

The solved equations are:

$$\alpha = -\frac{1}{\sqrt{2}a_0}\sqrt{\omega(\sqrt{\omega^2 + b_0^2} + \omega)}, \qquad \lim_{\omega \to \infty} \alpha = -\frac{\omega}{a_0}$$

$$\beta = \frac{1}{\sqrt{2}a_0}\sqrt{\omega(\sqrt{\omega^2 + b_0^2} - \omega)}, \qquad \lim_{\omega \to \infty} \beta = \frac{b_0}{2a_0} = 0.023\,529\,411\ldots$$

For $u_0(x)$ we find the simple solution:

$$u_0(x) = C_{01} \sin(\alpha + i\,\beta)x + C_{02} \cos(\alpha + i\,\beta)\,x$$

and by considering the boundary condition we have:

$$C_{01} = A, \qquad C_{02} = -\,iA$$

i.e.

$$u_0(x) = -\,iAe^{(-\beta + i\alpha)x}\,.$$

Table I

ω	β	$-\alpha$	β_1	$-\alpha_1$	β_2	$-\alpha_2$	$\frac{\beta}{2}(\beta^2+\alpha^2)$	$-\frac{\beta}{2}(\beta^2-3\alpha^2)$
50	0.023241	0.148884	0.023455	0.295051	0.023478	0.442146	0.000264	0,000766
100	0.023455	0.295052	0.023510	0.588705	0.023521	0.882668	0.001027	0.003056
150	0.023496	0.411802	0.023521	0.882668	0.023526	1.323721	0.002300	0.006873
200	0.023511	0.588706	0.023525	1.176706	0.023527	1.764887	0.004081	0.012216
250	0.023517.	0.735670	0.023526	1.470776	0.023528	2.206015	0.006370	0.019085
300	0.023521	0.882665	0.023527	1.764872	0.023528	2.647218	0.009169	0.027481
350	0.023523	1.029684	0.023528	2.058982	0.023529	3.088289	0.012477	0.037404
400	0.023524.	1.176726	0.023528	2.353092	0.023529	3.529473	0.016293	0.048854
450	0.023525	1.323749	0.023528	2.647206	0.023529	3.970658	0.020618	0.061829
500	0.023526	1.470790	0.023528	2.941328	0.023529	4.411842	0.025453	0.076332

Accordingly, the real portion of the differential equation (1) is:

$$\Phi_0 = Ae^{-\beta x}\sin(\alpha\, x + \omega t).$$

This solution satisfies both the differential equation (1) and the boundary condition (2).

Based on the second differential equation (1a) we write

$$\mathbf{D}[\Phi_1] = \frac{1}{A}\,\Phi_{0x}\,\Phi_{0xx} \tag{3}$$

and from the boundary condition (2):

$$\Phi_1(0,t) = 0, \qquad \lim_{x\to\infty}\Phi_1(x,t) = 0. \tag{3a}$$

By complementing with the perturbation member, we obtain the second differential equation:

$$\mathbf{D}[\Phi_1] = \frac{A}{2}\,e^{-2\beta x}\big[-\beta(\alpha^2+\beta^2) + \beta(\beta^2-3\alpha^2)\cos 2(\alpha x+\omega t) +$$

$$+\,\alpha(3\beta^2-\alpha^2)\sin 2(\alpha x+\omega t)\big]\,.$$

For the range $\omega > 50$ the above constant between brackets

$$-\beta(\alpha^2+\beta^2)$$

$\frac{\alpha}{2}(3\beta^2-\alpha^2)$	$\frac{3\beta^2-\alpha^2}{\beta}$	$\frac{\beta^2-3\alpha^2}{\alpha}$	$-\frac{\alpha}{\beta}$	$-\frac{\omega}{\alpha}$	$\beta_1-\beta$	$\beta_2-\beta$	$\alpha_1-2\alpha$	$\alpha_2-3\alpha$
0.001529	0.884043	0.443024	6.4061	335.832	0.000214	0.000237	0.002717	0.004506
0.012599	3.641253	0.883291	12.5795	338.923	0.000056	0.000066	0.001399	0.002488
0.042751	8.236790	1.324155	18.8032	339.519	0.000025	0.000030	0.000935	0.001684
0.101527	14.670676	1.765179	25.0400	339.728	0.000014	0.000016	0.000706	0.001231
0.198465	22.942611	2.206257	31.2819	339.826	0.000009	0.000011	0.000563	0.000994
0.343108	33.052759	2.647367	37.5265	339.880	0.000006	0.000007	0.000458	0.000776
0.545005	45.001883	3.088513	43.7731	339.910	0.000004	0.000006	0.000385	0.000762
0.813720	58.791305	3.529708	50.0217	339.926	0.000004	0.000005	0.000360	0.000706
1.158712	74.415090	3.970830	56.2687	339.943	0.000003	0.000004	0.000292	0.000590
1.589602	91.878960	4.411994	62.5171	339.953	0.000002	0.000003	0.000252	0.000528

is negligible. This will easily be understood, after introducing the notation

$$-\frac{\alpha}{\beta} = k(\omega).$$

Accordingly we can write:

$$-\beta(\alpha^2+\beta^2) = -\beta^3(k^2+1) \approx -k^2\beta^3$$
$$\beta(\beta^2-3\alpha^2) = -\beta^3(3k^2-1) \approx -3k^2\beta^3$$
$$\alpha(3\beta^2-\alpha^2) = -k\beta^3(3-k^2) \approx k^3\beta^3$$

where $k(50) = 6{,}4$ and shows a monotone increase with ω increasing. (Table 1) Rewriting the perturbation member in exponential form:

$$R_1(x,t) = \frac{A}{4}\left(z_1 e^{\zeta_1 x + i2\omega t} + \bar{z}_1 e^{\bar{\zeta}_1 x - i2\omega t}\right)$$

where

$$z_1 = \beta(\beta^2-3\alpha^2) + i\,\alpha(\alpha^2-3\beta^2)$$

$$\zeta_1 = -2(\beta - i\alpha).$$

When looking for a solution of the differential equation (3), the presumed form remains the same. Here the perturbation member is composed of two parts, and thus the assumed solution will take the form:

$$\Phi_1(x,t) = u_{11}(x)e^{i2\omega t} + u_{12}(x)e^{-i2\omega t}.$$

So, the function u_{11} and u_{12} are described by the differential equations:

$$u''_{11} + \left(\frac{4\omega^2}{a_0^2} - i\,2\omega\,\frac{b_0}{a_0^2}\right) u_{11} = \frac{A}{4}\, z_1 e^{\zeta_1 x}$$

$$u'_{12} + \left(\frac{4\omega^2}{a_0^2} + i\,2\omega\,\frac{b_0}{a_0^2}\right) u_{12} = \frac{A}{4}\, \bar{z}_1 e^{\bar{\zeta}_1 x} \quad . \tag{4}$$

With the general solution of these differential equations we find the solution that will satisfy both the boundary condition (3a) and the conditions expressed by

$$u_{1j}(0) = \lim_{x\to\infty} u_{1j}(x) = 0, \qquad (j = 1, 2) \tag{4a}$$

Let us introduce the notations:

$$\frac{4\omega^2}{a_0^2} - i\,2\omega\,\frac{b_0}{a_0^2} = (\alpha_1 + i\,\beta_1)^2$$

and

$$\frac{4\omega^2}{a_0^2} + i\,2\omega\,\frac{b_0}{a_0^2} = (\alpha_1 - i\,\beta_1)^2$$

we obtain:

$$\alpha_1 = -\frac{\sqrt{2}}{a_0}\sqrt{\omega\left(\sqrt{\omega^2 + \frac{b_0^2}{4}} + \omega\right)}$$

$$\beta_1 = \frac{\sqrt{2}}{a_0}\sqrt{\omega\left(\sqrt{\omega^2 + \frac{b_0^2}{4}} - \omega\right)}.$$

By considering the second part of the boundary condition (3a) we obtain as solutions for the homogeneous differential equations to (4):

$$u_{11h} = C_{11} e^{-(\beta_1 - i\alpha_1)x} \qquad \text{resp.} \qquad u_{12h} = C_{12} e^{-(\beta_1 + i\alpha_1)x}\,.$$

The constants C_{11} and C_{12} must be determined according to the condition that the solution of (3) should satisfy the first part of the boundary condition (3a). This can be achieved, if we put, for the general solution of the inhomogeneous differential equations the following conditions:

$$u_{11}(0) = 0 \qquad \text{resp.} \qquad u_{12}(0) = 0.$$

As the particular solutions of the inhomogeneous differential equations (4) the following functions are to be found:

$$u_{11\,i} = B_1 e^{\zeta_1 x} \qquad \text{resp.} \qquad u_{12i} = \overline{B}_1 e^{\overline{\zeta}_1 x} = \overline{u}_{11i}$$

where

$$B_1 = \frac{A}{8}\left[\frac{1}{2\beta}(3\beta^2 - \alpha^2) + i\frac{1}{2\alpha}(\beta^2 - 3\alpha^2)\right].$$

Thus, solutions of (4) satisfying the boundary conditions (4a) are:

$$u_{11} = B_1(e^{(-2\beta+i2\alpha)x} - e^{(-\beta_1+i\alpha_1)x})$$
$$u_{12} = \overline{u}_{11}.$$

With these, the solution of the differential equation (3) is:

$$\Phi_1 = u_{11}(x)\, e^{i2\omega t} + \overline{u}_{11}(x)\, e^{-i2\omega t} =$$

$$= \frac{A}{8}\left\{e^{-2\beta x}\left[\frac{3\beta^2 - \alpha^2}{\beta}\cos 2(\alpha x + \omega t) - \frac{\beta^2 - 3\alpha^2}{\alpha}\sin 2(\alpha x + \omega t)\right] - \right.$$

$$\left. - e^{-\beta_1 x}\left[\frac{3\beta^2 - \alpha^2}{\beta}\cos(\alpha_1 x + 2\omega t) - \frac{\beta^2 - 3\alpha^2}{\alpha}\sin(\alpha_1 x + 2\omega t)\right]\right\}.$$

This solution satisfies both the differential equation (3) and the boundary condition (3a).

According to (1a), the differential equation to be solved is:

$$\mathbf{D}[\Phi_2] = \frac{1}{A}(\Phi_{0x}\Phi_{1xx} + \Phi_{1x}\Phi_{0xx}) \tag{5}$$

and the pertaining boundary condition:

$$\Phi_2(0, t) = 0, \qquad \lim_{x\to\infty} \Phi_2(x, t) = 0. \tag{5a}$$

If we adhere to the above applied form of the function the perturbation member

$$R_2(x, t) = \frac{1}{A}(\Phi_{0x}\Phi_{1xx} + \Phi_{1x}\Phi_{0xx})$$

in the differential equation, (5) could be established in a most complicated form only; therefore, it is advisable to find a simplification in the formula of the function Φ_1. This is possible, when the members containing the coefficient $\frac{\beta^2 - 3a^2}{a}$ are, in a range of ω that surpasses a determined limit (e.g. $\omega > 100$) neglected; this is the more justified, the higher value of ω is. Furthermore, the approximative equalities $\beta_1 \approx \beta$ and $a_1 \approx 2a$ will be, the more accurate as the value of ω becomes higher and higher (Table 1). Of course, the value of x must be kept within certain limits. By omitting the negligible members we have:

$$\Phi_1 \approx -\frac{A}{8}\frac{3\beta^2 - a^2}{\beta} e^{-\beta x}(1 - e^{-\beta x}) \cos 2(ax + \omega t) .$$

After having carried out the necessary calculating operations, the perturbation member of the differential equation (5) takes the form:

$$R_2(x, t) = \frac{A}{32}\frac{3\beta^2 - a^2}{\beta}(z_{21} e^{\zeta_{21}x + i3\omega t} + \bar{z}_{21} e^{\bar{\zeta}_{21}x - i3\omega t} + z_{22} e^{\zeta_{22}x + i\omega t} +$$

$$+ \bar{z}_{22} e^{\bar{\zeta}_{22}x - i\omega t} + z_{23} e^{\zeta_{23}x + i\omega t} + \bar{z}_{23} e^{\bar{\zeta}_{23}x - i\omega t} + z_{24} e^{\zeta_{24}x + i3\omega t} + \bar{z}_{24} e^{\bar{\zeta}_{24} x - i3\omega t})$$

where

$$\begin{aligned}
z_{21} &= 6[a(3\beta^2 - a^2) + i\,\beta(\beta^2 - 3a^2)] \\
z_{22} &= -2[a(a^2 + \beta^2) + i\,3\beta(a^2 + \beta^2)] \\
z_{23} &= 3a\beta^2 + i\,\beta(3a^2 + 2\,\beta^2) \\
z_{24} &= a(8a^2 - 13\beta^2) + i\,\beta(18a^2 - 3\beta^2)
\end{aligned}$$

$$\zeta_{21} = -3(\beta - i\,a), \qquad \zeta_{22} = -(3\beta - i\,a)$$

$$\zeta_{23} = -(2\beta - i\,a), \qquad \zeta_{24} = -(2\beta - i\,3a) .$$

The solution of the differential equation (5) can be formed as the sum of the solution of the homogeneous differential equation and that of the inhomogeneous differential equation:

$$\Phi_2 = \Phi_{2h} + \Phi_{2i} .$$

When looking for a solution for the homogeneous equation, the same form as before can be maintained:

$$\Phi_{2h} = u_{21}(x)\, e^{i3\omega t} + \bar{u}_{21}(x)\, e^{-i3\omega t} + u_{22}(x)\, e^{i\omega t} + \bar{u}_{22}(x)\, e^{-i\omega t}.$$

Consequently the functions u_{2j} and $\bar{u}_{2j}$ ($j = 1{,}2$) have to be determined. namely, supposing that Φ_2 satisfies the boundary condition (5a).

The solution of the homogeneous differential equation involves the following differential equation:

$$u''_{21} + \left(\frac{9\omega^2}{a_0^2} - i\, b_0 \frac{3\omega}{a_0^2}\right) u_{21} = 0$$

and its conjugated pair. By introducing the notation:

$$\frac{9\omega^2}{a_0^2} - ib_0 \frac{3\omega}{a_0^2} = (\alpha_2 + i\beta_2)^2$$

we obtain:

$$\alpha_2 = -\frac{3}{\sqrt{2}a_0} \sqrt{\omega\left(\sqrt{\omega^2 + \frac{b_0^2}{9}} + \omega\right)}$$

$$\beta_2 = \frac{3}{\sqrt{2}a_0} \sqrt{\omega\left(\sqrt{\omega^2 + \frac{b_0^2}{9}} - \omega\right)}.$$

Combining these expressions with the above calculated quantities α, β and α_1, β_1 the following can be easily verified:

$$\lim_{\omega \to \infty} \alpha = -\frac{\omega}{a_0}, \qquad \lim_{\omega \to \infty} \alpha_1 = -\frac{2\omega}{a_0}, \qquad \lim_{\omega \to \infty} \alpha_2 = -\frac{3\omega}{a_0}$$

and

$$\lim_{\omega \to \infty} \beta = \lim_{\omega \to \infty} \beta_1 = \lim_{\omega \to \infty} \beta_2 = \frac{b_0}{2a_0} = 0.023\,529\,411\ldots$$

For the range $\omega > 100$, the following approximations are valid:

$$\alpha_2 \approx 3\alpha$$

$$\alpha_1 \approx 2\alpha$$

$$\beta_2 \approx \beta_1 \approx \beta.$$

Further, it can easily be proved, that by applying the above accepted rule of formulation, we obtain:

$$\lim_{n \to \infty} \alpha_n = -\frac{n+1}{a_0}\,\omega$$

$$\text{and} \quad \lim_{n \to \infty} \beta_n = \frac{b_0}{2a_0}$$

whence

$$a_n \approx (n+1)\alpha \quad \text{and} \quad \beta_n \approx \beta, \ (n = 1, 2, \ldots)$$

in a certain range $\omega > \Omega$ (with the already assumed data $\Omega = 100$).
We find the particular solution of inhomogeneous differential equation (5):

$$\Phi_{2i} = \frac{A}{32}\frac{3\beta^2 - \alpha^2}{\beta}\left(B_{21} e^{\zeta_{21}x + i3\omega t} + \overline{B}_{21} e^{\overline{\zeta}_{21}x - i3\omega t} + B_{22} e^{\zeta_{22}x + i\omega t} + \overline{B}_{22} e^{\overline{\zeta}_{22}x - i\omega t} + \right.$$

$$\left. + B_{23} e^{\zeta_{23}x + i\omega t} + \overline{B}_{23} e^{\overline{\zeta}_{23}x - i\omega t} + B_{24} e^{\zeta_{24}x + i3\omega t} + \overline{B}_{24} e^{\overline{\zeta}_{24}x - i3\omega t}\right)$$

where

$$B_{21} = \frac{1}{2}\left(\frac{3\alpha^2 - \beta^2}{\alpha} + i\frac{3\beta^2 - \alpha^2}{\beta}\right) \approx \frac{1}{2}\left(3\alpha + i\frac{3\beta^2 - \alpha^2}{\beta}\right)$$

$$B_{22} = \frac{1}{2}\left(\frac{\alpha(\alpha^2 + \beta^2)}{\alpha^2 + 4\beta^2} - i\frac{\alpha^2 + \beta^2}{\beta}\frac{\alpha^2 + 6\beta^2}{\alpha^2 + 4\beta^2}\right) \approx \frac{1}{2}\left(\alpha - i\frac{\alpha^2}{\beta}\right)$$

$$B_{23} = \frac{\alpha(5\beta^2 - 6\alpha^2) + i\beta(15\alpha^2 + 6\beta^2)}{4\alpha^2 + 9\beta^2} \approx -\frac{3}{2}\alpha + i\frac{15}{4}\beta$$

$$B_{24} = \frac{\alpha\beta(83\beta^2 - 148\alpha^2) + i(48\alpha^4 - 168\alpha^2\beta^2 + 15\beta^4)}{\beta(36\alpha^2 + 25\beta^2)} \approx -3\alpha + i\frac{8}{3}\frac{\alpha^2}{\beta}.$$

All the approximative expressions are motivated for the range $\omega > 100$.

Taking all that has been stated above, the approximative solution of the differential equation (5) can be written as follows:

$$\Phi_2 \approx \frac{A\alpha}{32}\frac{3\beta^2 - \alpha^2}{\beta} e^{-\beta x}\left\{2\cos(\alpha x + \omega t) + 3\cos 3(\alpha x + \omega t) - \frac{\alpha}{\beta}\left[\sin(\alpha x + \omega t) - \right.\right.$$

$$\left. - \frac{5}{3}\sin 3(\alpha x + \omega t)\right] - e^{-\beta x}\left[3\cos(\alpha x + \omega t) + 6\cos 3(\alpha x + \omega t) + \right.$$

$$\left. + \frac{8}{3}\frac{\alpha}{\beta}\sin 3(\alpha x + \omega t)\right] + e^{-2\beta x}\left[\cos(\alpha x + \omega t) + 3\cos 3(\alpha x + \omega t) + \right.$$

$$\left.\left. + \frac{\alpha}{\beta}\left(\sin(\alpha x + \omega t) + \sin 3(\alpha x + \omega t)\right)\right]\right\}.$$

This solution satisfies both the differential equation (5) and the boundary condition (5a).

Consequently — on the condition that the value x remains below a certain limit — the function $\Phi(x, t)$ as an approximative solution will take the form:

$$\Phi \approx A\, e^{-\beta x} \left\{ \sin(ax + \omega t) - \frac{q}{8}\,(1 - e^{-\beta x}) \cos 2(ax + \omega t) + \right.$$

$$+ \frac{q^2}{32} \left[(1 - e^{-2\beta x}) \sin(ax + \omega t) - \left(\frac{5}{3} - \frac{8}{3}\, e^{-\beta x} + e^{-2\beta x} \right) \sin 3(ax + \omega t) - \right.$$

$$\left.\left. - \frac{\beta}{a}\,(2 - 3e^{-\beta x} + e^{-2\beta x}) \cos(ax + \omega t) - \frac{3\beta}{a}\,(1 - 2e^{-\beta x} + e^{-2\beta x}) \cos 3(ax + \omega t) \right] \right\}$$

where

$$q = \lambda\, \frac{3\beta^2 - a^2}{\beta} = A(\varkappa + 1)\, \frac{3\beta^2 - a^2}{\beta}\,.$$

When the value of ω increases (e.g. $\omega > 200$), the solution can be expressed by neglecting further members. In this way:

$$\Phi \approx A\, e^{-\beta x} \left\{ \sin(ax + \omega t) - \frac{q}{8}\,(1 - e^{-\beta x}) \cos 2(ax + \omega t) + \right.$$

$$\left. + \frac{q^2}{32} \left[(1 - e^{-2\beta x}) \sin(ax + \omega t) - \left(\frac{5}{3} - \frac{8}{3}\, e^{-\beta x} + e^{-2\beta x} \right) \sin 3(ax + \omega t) \right] \right\}.$$

Of course, when ω is increased, the value A has to be diminished, first in order to have a convergent series, and on the other hand, for the sake of good approximation with the first three members. Thus, the value of the product $A\, \dfrac{3\beta^2 - a^2}{\beta}$ has to be kept within a certain limit. It can be foreseen that the convergence is surely exists, if

$$\left| A\, \frac{3\beta^2 - a^2}{\beta} \right| < 0.4\,.$$

In considering the solution Φ the solution of the linear equation (1) proves satisfactory as a good approximation for the case of flat waves. This flatness of waves is only sufficient, if the value of $\left| A\, \dfrac{3\beta^2 - a^2}{\beta} \right|$ is small enough; consequently the approximation

$$(1 + \Phi_x)^{-(\varkappa+1)} \approx 1 - (\varkappa + 1)\, \Phi_x$$

as was put in the original differential equation, is motivated only on the same condition.

When for the sake of simplification, only the linear differential equation is to be taken as basis of calculation, we find, that with the solution Φ we have good information on the magnitude of the neglected members and also on the possible limits of the quantities A and ω.

Summary

In this paper, the approximative solution of a pneumatic telemechanical controlling problem is treated, that involves a quasi linear equation, when the physical parameters are assumed to be within certain limits. The solution of the differential equation is found by using the Calculus of Perturbation. This method serves as a good basis to estimate the range of the physical parameters for the sake of an acceptable approximative calculation of the linear differential equation.

References

1. COURANT—FRIEDRICHS: Supersonic flow and shock waves. Interscience Publishers, Inc., New York 1948.
2. STOKER, J. J.: Nonlinear vibrations. Interscience Publishers, Inc., New York 1950.
3. STOKER, J. J.: Water waves. Interscience Publishers, Inc., New York 1957.
4. SAUER, R.: Anfangswertprobleme bei partiellen Differenzialgleichungen. Springer-Verlag, Berlin—Göttingen—Heidelberg. 1958.
5. JEFFREY—TANIUTI: Nonlinear wave propagation. Academic Press Inc., New York—London. 1964.
6. FRIEDRICHS, K. O.: Nonlinear hyperbolic differential equations for functions of two independent variables. Am. J. Math. **70** 555—589. (1948).
7. LAX, P. D.: Nonlinear hyperbolic equations. Communs. Pure Appl. Math. **6** 231—258 (1953).

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