EXPERIMENTAL INVESTIGATIONS OF SOME PROPERTIES OF CAVITATING FLOW

By

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Introduction

In the course of experimental studies carried out with circular cylinder models investigating the scale effect of cavitation erosion [1], [2], [3], a relation could be established between the extent of cavitation erosion and the frequencies of the eddies shedding periodically from the models. The frequencies of the wakes leaving the model could be characterized by a Strouhal number, concerning which there are only data measured in noncaviting flow [4], [5]. Earlier circumstances made it desirable to make further experimental investigations to determine some relations that appear to have a basic value in respect to cavitation erosion. These are the relations between the Strouhal number and the cavitation number, between the cavitation number and the length and width of the cavity, as well as the relation between the cavity length and the Strouhal number. The investigation of these relations is of significance first of all in the critical and supercritical range of the Reynolds number. Speaking of a critical range of Reynolds number, in the case of noncavitating flow, we mean the section extending from the beginning of a rapid decrease of the drag coefficient to that reaching the minimum value, i.e. the Reynolds number range between the two limits. Accordingly, the range of the supercritical Reynolds number will be given by the Reynolds numbers above the upper limit of the critical range of Reynolds numbers mentioned earlier.

It is well-known that the drag coefficient of a circular cylinder will suddenly decrease in the critical range of the Reynolds number. This decrease in the value of the drag coefficient begins at about $Re = 10^5$. In this respect the reference data appear to be unambiguous [6, p. 419]. There are, however, no such data existing, which mark the upper limits of the critical range of the Reynolds numbers, and thereby the beginning of the supercritical range: this is due to the influence of the surface roughness of the model and the turbulence of the flow. GOLDSTEIN [6, p. 423] puts the critical range to $10^5 < Re < 2.5 \cdot 10^5$. This is supported by Fig. 1, taken from KONSTANTINOV's paper [7]. Our measurements also prove that a sharp decrease of the drag coefficient begins with the value of $Re = 10^5$ and ceases at $Re < 2.5 \cdot 10^5$. In cavitating flow the upper limit of the critical range of the Reynolds number can be well determined, if the change in the drag coefficient is to be determined as a function of the cavitation number. This relation is shown in Fig. 2 plotted from measurements made in a test section of 48×200 sq mm with a circular cylinder model of $d = 24$ mm diameter. The smallest value of the drag coefficient will be reached at $Re \cong 2.3 \cdot 10^5$. With Reynolds numbers greater than this, the value of the drag coefficient is approximately constant. To the right of the sharply falling part of the curve the drag coefficient has a constant value even in the case of $2 \cdot 10^5 < Re < 2.1 \cdot 10^5$, but it is higher than

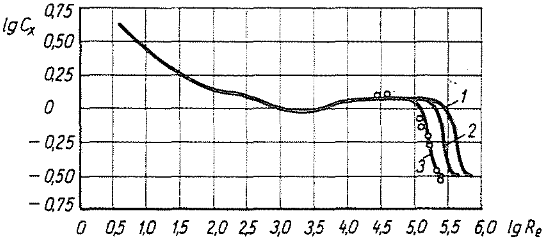

Fig. 1. The drag coefficient (C_x) as a function of Reynolds number (Re) following Konstantinov's paper. 1. Göttingen measurements (for air). 2. Eisner's measurements (for water). 3. NPL measurements (for air). Denoted points are the measurements of the Institute of Mechanics of the Academy of Sciences of the Soviet Union

the former value and corresponds to the values belonging to the rapidly decreasing section of the drag coefficient plotted as a function of the Reynolds number, as can be seen to the left of the figure. As an example, the figure shows pairs of points that belong to each other.

The experimental equipment

The experiments were carried out with circular cylinder models of $d = 12$, 24, 36 and 48 mm diameter (in some experiments also with models of $d = 60$ mm) placed in a water flow in a test section sized 48×200 sq. mm. The test section was horizontally arranged, in which the circular cylinders placed perpendicularly to the flow direction also lay in a horizontal plane. More particulars of the test equipment can be found in earlier papers [8], [9]. The experiments were carried out in the following ranges of the Reynolds number:

Diameter of circular cylinder model (mm)	Range of Re number
12	$0.86 \ldots 1.59 \cdot 10^5$
24	$2.12 \ldots 3.59 \cdot 10^5$
36	$2.69 \ldots 5.12 \cdot 10^5$
48	$3.28 \ldots 6.87 \cdot 10^5$

In view of the fact that the smallest value of the drag coefficient in experiments with a $d = 24$ mm diameter circular cylinder model appeared, in accordance with the earlier data of GOLDSTEIN at $Re \approx 2.3 \cdot 10^5$ (Fig. 2) and, with a circular cylinder model of $d = 48$ mm dia., this value was at $Re < 2.5 \cdot 10^5$, it can be stated that the experiments made with circular cylinder models of $d = 24$ mm and larger diameters occurred in the super-

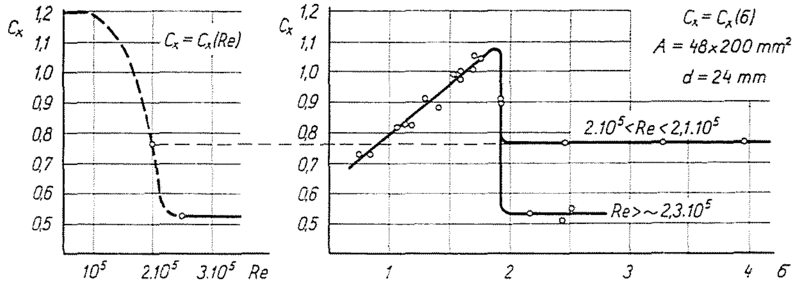

Fig. 2. The drag coefficient (C_x) as a function of Re number in non-cavitating flow (on the left) and the drag coefficient (C_x) as a function of the cavitation number (σ) (on the right) for a circular cylinder model of $d = 24$ mm diameter

critical range above the critical range of the Reynolds number. The measurements taken with the circular cylinder of $d = 12$ mm diameter fell in to the critical range of the Reynolds number.

Relation between the Strouhal number and the cavitation number

Following the determination made by several methods of the frequency of wakes shedding from circular cylinders, where the measurements were taken with circular cylinder models of $d = 12$, 24, 36, and 48 mm diameter, and with different cavitation numbers, the relation between the Strouhal number and the cavitation number could be established. More detailed particulars can be found in an earlier paper [10].

The values of the Strouhal number calculated with undisturbed flow velocity are shown in Fig. 3, as a function of the cavitation number also calculated with the undisturbed flow velocity. The experimental results obtained with circular cylinders of $d = 24$, 36, and 48 mm diameter are placed on a single curve thus plotting the relation $S = 0.2 \sqrt{\sigma}$. However, the results of measurements made with a model of $d = 12$ mm appear on another curve corresponding to the relation $S = 0.174 \sqrt{\sigma}$. The latter measurements were made within the critical range of Reynolds number, whereas the former ones occurred in the supercritical range of Reynolds number and, accordingly, the flow conditions were also different.

If the cavitation numbers are calculated with a uniform free downstream velocity of $\bar{v} = k \cdot v_\infty$, and the Strouhal number values calculated in the former way (with undisturbed flow velocity) are shown as a function of the

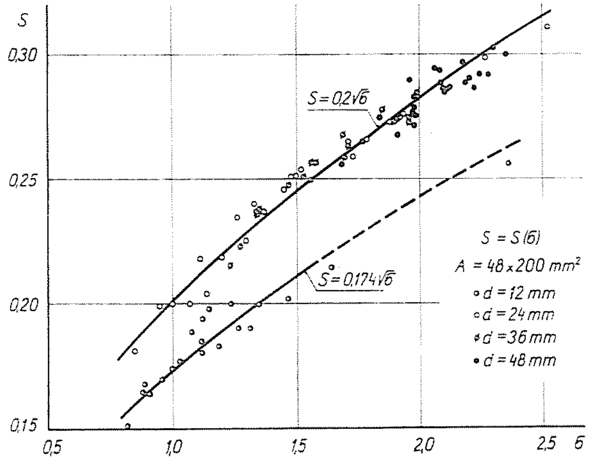

Fig. 3. The Strouhal number (S) as a function of the cavit. number (σ)

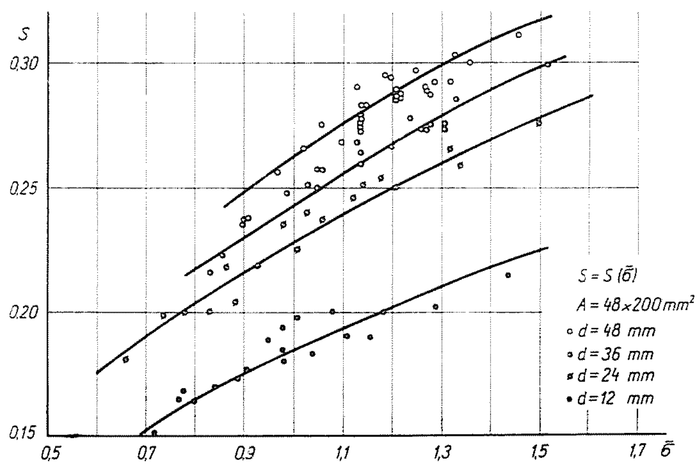

Fig. 4. The Strouhal number (S) as a function of the cavit. number ($\bar{\sigma}$) calculated with uniform free downstream velocity

cavitation number denoted as $\bar{\sigma}$, a separate curve will appear for each model diameter (Fig. 4). In this figure the effect of contraction appears, whereas Fig. 3 shows the wall effect, which does not seem to be significant.

If the Strouhal number is also calculated with uniform free downstream velocity, and the $\bar{S}$ Strouhal numbers thus obtained will be plotted against the $\bar{\sigma}$ cavitation number calculated with the uniform free downstream velocity, this will again result in the curves shown in Fig. 3. From the relations $\bar{\sigma} = \sigma/k^2$ and $S = 0.2\sqrt{\sigma}$ it will be easily seen that $\bar{S} = 0.2\sqrt{\bar{\sigma}}$.

Comparing the results shown in Fig. 3 with those of other authors (RELF and SIMMONS [4], ITAYA and YASUDA [5]), obtained in non-cavitating flow, it can be stated that the values of the Strouhal number are different in non-cavitating and cavitating flow. In the former case the Strouhal number depends only on the Reynolds number, whereas in cavitating flow it depends first and foremost on the cavitation number. In cavitating flow the influence

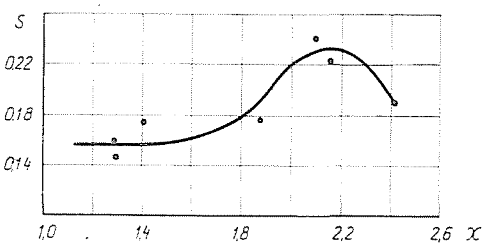

Fig. 5. The Strouhal number (S) as a function of the cavitation number ($\varkappa$) following Shalnev's paper

of Reynolds number appears in the fact that different Strouhal numbers will occur within the critical range of Reynolds number and in the supercritical range of Reynolds number.

In the literature only SHALNEV [11] made some reference to the relation between the Strouhal number and the cavitation number. In the diagram submitted by him (Fig. 5) there is only a small number of measurement points marked and thus, a clearcut relationship cannot be drawn from them. In this figure the curve presenting the changes of the Strouhal number has a peak value. From other publications [12] of the author it can be decided that the submitted diagram was drawn up from measurements made of $Re = 1{,}09 \cdot 10^5$ in the critical range of Reynolds number, with a circular cylinder model of $d = 12$ mm diameter. Converting the Strouhal number values in the diagram to the velocity of undisturbed flow we find that the majority shows good conformity with the values in Fig. 3 obtained with the cylinder model of $d = 12$ mm diameter. The last point of the curve at $\sigma = 2{,}4$ gives a Strouhal number value corresponding to non-cavitating flow. From these considerations it is highly probable that though the measured values are true, yet the measuring points require a curve of different character.

The relation between the cavitation number and cavity length

SILBERMAN and SONG [13] arrived at the conclusion that "Cavity length appears to be a unique function of cavitation number regardless of whether the cavity is natural or ventilated". This statement is supported by our experimental studies made with circular cylinder models having five different diameters ($d = 12 \ldots 60$ mm). The test points of cavity length measured from the centre line of the cylinder (l_z) as a function of the cavitation number, yield well determined curves. As an example, Fig. 6 presents such a curve. The dimensionless cavity length values ($\lambda_z = l_z/d$) determined with models of different diameters are shown as a function of the cavitation number in

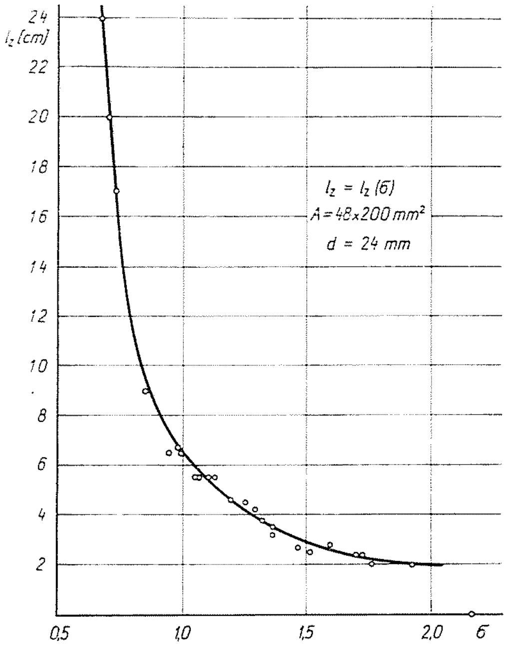

Fig. 6. Cavity length (l_z) as a function of the cavitation number (σ) for $d = 24$ mm cylinder diameter

Fig. 7. If the dimensionless zone-length values belonging to the four circular cylinders of different diameters shown in the figure are drawn up as a function of cavitation number $\bar{\sigma}$ calculated with uniform free downstream velocity (Fig. 8), then for all of the test points one single empirical curve can be laid and this can be expressed with the equation

$$\lambda_z \bar{\sigma}^n = \text{const} .$$

The curve shown in Fig. 8 is the result of experiments carried out in the supercritical range of Reynolds number, where the value of the exponent is $n = 2.5$, that of the constant is $C = 1.6$. The results of measurements made in the range of $Re = 0.6 \ldots 1.5 \cdot 10^5$ with a circular cylinder model of 12 mm diameter are given in Figure 9, where $n = 1.5$ and $C = 2.4$. Using the results of KONSTANTINOV's [7] experiments with circular cylinders of $d = 5$, 10, 20, 30, 40 and 50 mm diameter in a similar way, it is also to be found that the test points falling in the supercritical and critical ranges of Reynolds number are lying on two different curves. SONG's and SILBERMAN's [14] results obtained from their experiments with $d = 1/4''$ circular cylinder in a free jet of 6''

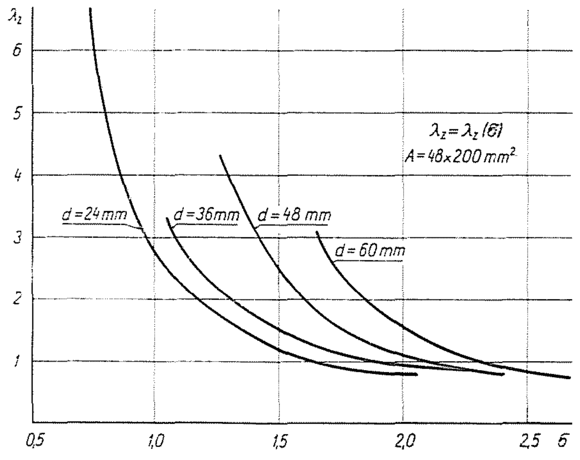

Fig. 7. Dimensionless cavity length (λ_z) as a function of the cavitation number (σ), with cylinder models of different dia

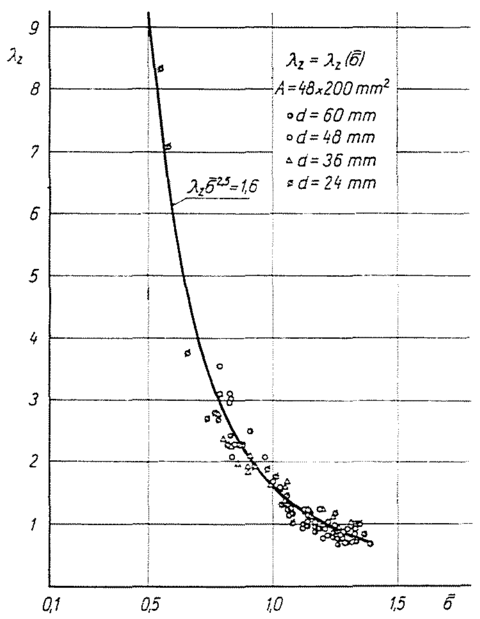

Fig. 8. Dimensionless cavity length (λ_z) as a function of $\bar{\sigma}$ cavitation number in the supercritical range of *Re* number

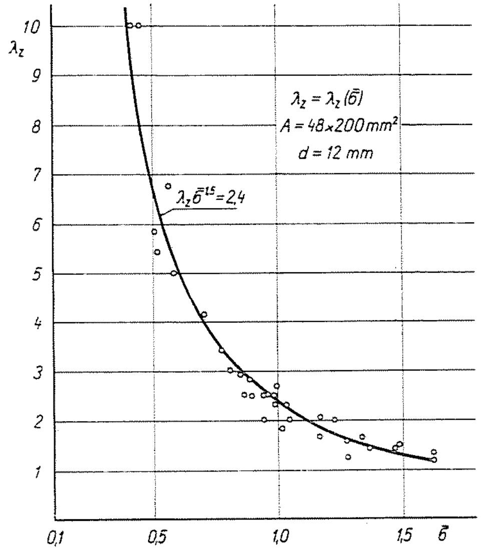

Fig. 9. Dimensionless cavity length (λ_z) as a function of $\bar{\sigma}$ cavitation number in the critical range of Reynolds number

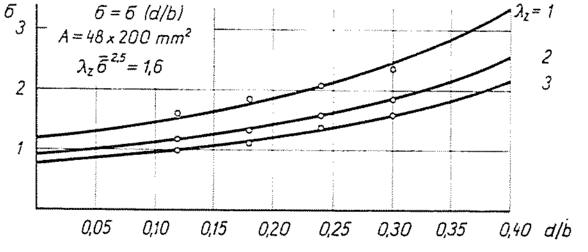

Fig. 10. Relation of the cavitation number (σ) to the quotient of the cylinder-diameter/test section height, (d/b), with different dimensionless cavity lengths (λ_z) in the supercritical range of Re number

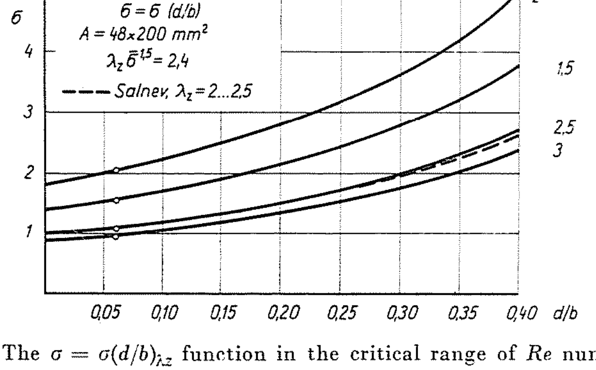

Fig. 11. The $\sigma = \sigma(d/b)_{\lambda z}$ function in the critical range of Re number

and 12″ diameter, and with 20...50 *ft/s* velocity in the $Re < 1.5 \cdot 10^5$ range, will complete the former relation with the values $n = 1.5$ and $C = 1.8$.

If the cavitation number values are drawn up as a function of the ratio d/b, that is the ratio of the cylinder diameter and the test section height, with different dimensionless zone lengths, the relation obtained for the supercritical range is shown in Fig. 10. The same relation for the critical Reynolds number is given in Fig. 11. A dotted line in the figure illustrates the curve corresponding to $\lambda_z = 2...2.5$ from SHALNEV's [12] diagram showing a similar relationship, and this conforms well with our measurements.

The relation between the Strouhal number and the cavity length

On the basis of the test results presented, the relations between the Strouhal number and the cavitation number, and between the dimensionless cavity length and cavitation numbers can be characterized by the equations

$$\bar{S} = 0.2\bar{\sigma}^{\frac{1}{2}}$$

and

$$\lambda_z \bar{\sigma}^{\frac{5}{2}} = 1.6$$

respectively. The two equations will yield the relation

$$\bar{S}\lambda_z^{\frac{1}{5}} = 0.22$$

between the Strouhal number and the dimensionless cavity length (Fig. 12., curve drawn in full line). In the critical range of Reynolds number an approximate relation expressed by the equation

$$\bar{S}\lambda_z^{\frac{1}{5}} = 0.233$$

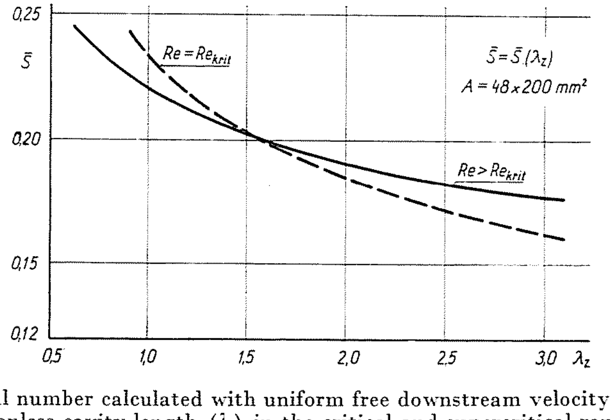

Fig. 12. Strouhal number calculated with uniform free downstream velocity ($\bar{S}$) as a function of the dimensionless cavity length (λ_z) in the critical and supercritical range of *Re* number

appeared (Fig. 12, curve drawn in dotted line). These empirical relations suggesting the connections between $\bar{S} - \bar{\sigma}$, $\lambda_z - \bar{\sigma}$ and $\bar{S} - \lambda_z$, can be interpreted as the projections of the space-curve

$$\bar{S} = \frac{\lambda_z \bar{\sigma}^3}{8},$$

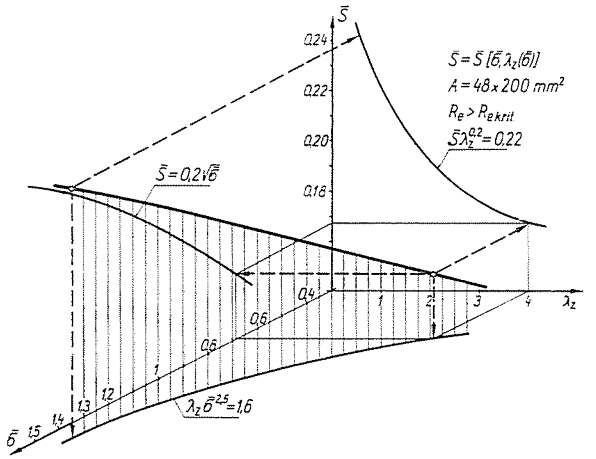

Fig. 13. The relation between the Strouhal number ($\bar{S}$), the cavitation number ($\bar{\sigma}$) and the dimensionless cavity length (λ_z)

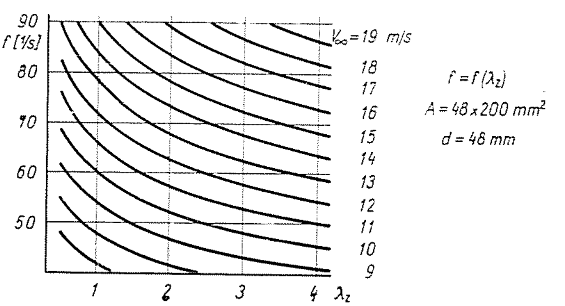

Fig. 14. The frequency (f) of the eddies shedding from a circular cylinder, as a function of the dimensionless cavity length (λ_z) in the case of a $d = 48$ mm diameter circular cylinder, with different flow velocities

coming from the previous relations onto the planes mentioned earlier (Fig. 13). The relations considered so far make it possible to draw up the relations between the frequency of shedding cavities and dimensionless cavity length as well (Fig. 14).

The relation between the cavity width and the cavitation number

The width of the cavitation zone is just as unique a function of the cavitation number as the cavity length, and suitable to characterize the state of cavitation. Measuring the cavitation zone width in an experimental way

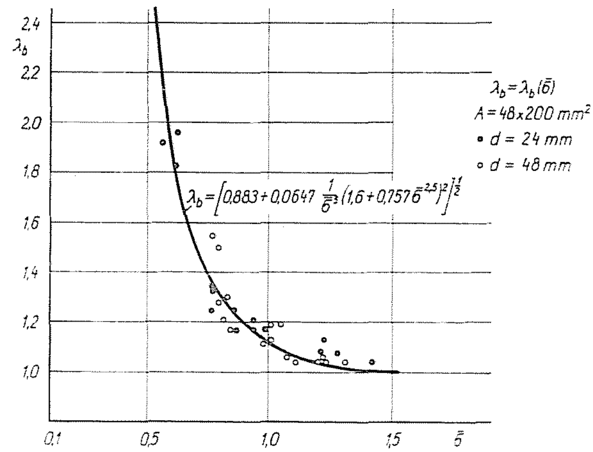

Fig. 15. Dimensionless width (λ_b) of the cavity, as a function of the cavitation number ($\bar{\sigma}$)

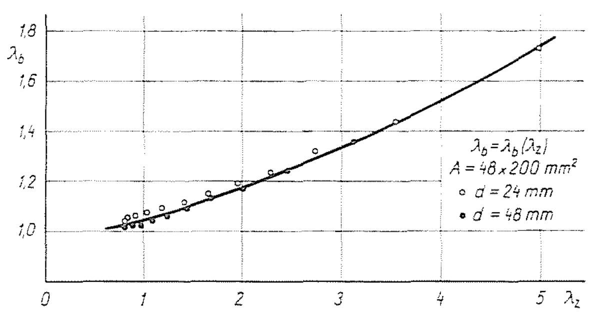

Fig. 16. Functional relation of the dimensionless cavity width (λ_b) and cavity length (λ_z)

gives less exact results. The changes of the dimensionless cavity width (λ_b) as a function of cavitation number $\bar{\sigma}$ with circular cylinder models of $d = 24$, and $d = 48$ mm diameter is shown in Fig. 15. The changes of λ_b as a function of λ_z for the former models, in the supercritical range is given in Fig. 16. The curve in the diagram can be described by the equation

$$\lambda_b = \left[0.883 + 0.0647\,(\lambda_z + 0.757)^2\right]^{\frac{1}{2}},$$

where λ_b and λ_z form the major axis of an ellipse. The relation between λ_b and $\bar{\sigma}$ is similar to that shown earlier for the relation between λ_z and $\bar{\sigma}$.

Summary

The authors report on their experiments made with circular cylinder models placed in a closed-circuit channel; these tests were made to determine the relations between the Strouhal number and the cavitation number, the cavitation number and the cavity length and width, the cavity length and the Strouhal number, respectively. The tests were made in the critical and supercritical range of Reynolds number. As a result, approximate relations were obtained, leading to the only relation between the Strouhal number, dimensionless zone length and cavitation number; on the basis of these findings, the relation between the frequency of shedding cavitations and the dimensionless cavitation zone length can be drawn up for different velocities. The results thus obtained are significant for the investigations of cavitation erosion.

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