CONSIDERATION OF FLOW CONDITIONS AT ROTOR INLET IN BLADING DESIGN

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The present paper complement of a former one by the same author [1] which deals with the blading design of mixed flow impeller, assuming a potential flow pattern before the impeller. In practical cases this assumption does not hold, and therefore an additional generalization of the computation method would be needed, which does not involve such severe restrictions. In our treatment we prescribe only the axial symmetry of the flow towards the impeller (in case of ideal liquids) and merely deal with the differences obtained in comparison with (1).

Let us begin with a given flow pattern before the impeller, i. e. in the section 0 (see Fig. 1). (The flow data can be determined e.g. by measurements.) For the blading design the iteration process proposed by us (1) can be used, and the only difference will present itself in the calculation of an impeller with infinite blade numbers giving an approximation of zeroth order. The differential equation for the determination of flow surfaces — rotational surfaces with infinite blade numbers — separating the impeller to impeller-elements delivering equal fluid quantities has the form of

$$\frac{\partial c}{\partial b} + c\frac{\partial \delta}{\partial l} = F(r, z)$$

when the flow towards the impeller has a potential character [1]. The purpose of the present paper is to generalize this equation for the case of a distribution of axial symmetry with any value of $c_0(b_0)$, $c_{0u}(b_0)$, $p_0(b_0)$.

As will be seen, even in a generalized case the differential equation having the same form will be obtained, and this, in comparison with other methods published (2), represents an effective simplification.

The symbols used correspond with those of [1].

Symbols used

z	=	blade number of the real impeller
Γ	=	blade circulation
ω	=	angular velocity of impeller

r, φ, z	= cylindrical co-ordinates on the impeller
ξ, η	= co-ordinates in plane ζ in the straight cascade
$\xi^* = \xi/\xi_2$	= dimensionless co-ordinate perpendicular to the straight cascade
l, b	= curved co-ordinates in the meridional section
t	= pitch in the straight cascade
δ	= angle between the tangent of meridional streamline and the axis of rotation
φ_t	= $2\,\pi/z$
φ_ϑ	= angle characterizing blade thickness
γ	= vorticity distribution along blade sections of the impeller
$\gamma_\xi^*\ (\xi^*)$	= dimensionless vorticity distribution in the straight cascade
β	= blade angle
c	= absolute velocity in the impeller system
c_m	= component of velocity c in the meridional section
c_u	= component of velocity c in direction u
c_ζ	= absolute velocity in straight cascade
$c_\xi,\ c_\eta$	= components of velocity c_ζ
c_∞	= basic flow velocity in the impeller
$c_{\zeta\infty}$	= basic flow velocity in straight cascade
c_i	= induced velocity in the impeller
$c_{i\zeta}$	= induced velocity in straight cascade
$c_{p\eta}$	= the correspondent of the velocity of rotation before the impeller in straight cascade
$c_{k\eta}$	= $\dfrac{\Gamma}{2\,t}$
u	= peripheral velocity in the impeller
w	= relative velocity in the impeller
w_m	= c_m
w_u	= $c_u - u$
w_ζ	= correspondent of the relative velocity in straight cascade
$\mathbf{e}$	= unit vector
ϱ	= density of the medium delivered
I	= $\dfrac{u^2 - w^2}{2} - \dfrac{p}{\varrho}$

Meridional flow in impeller of infinitely numerous blades

In putting down the motion equation the influence of the blading regarded as infinitely numerous manifests itself as a field intensity $\mathbf{g}_n$ of an unconservative type:

$$-\nabla I + (\nabla \times \mathbf{c}) \times \mathbf{w} - \mathbf{g}_n = \mathbf{0} \tag{1}$$

where

$$I = \frac{u^2 - w^2}{2} - \frac{p}{\varrho} \tag{2}$$

and, due to this fact we do not calculate with shear stresses $\mathbf{w} \cdot \mathbf{g}_n = 0$. In addition, it will immediately be seen, that

$$\mathbf{w} \cdot \nabla I = 0 \tag{3}$$

so ∇I is perpendicular to $\mathbf{w}$.

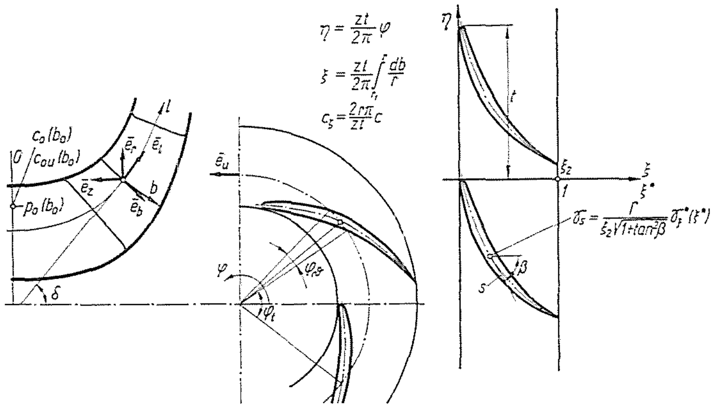

Fig. 1

Before the blading the flow is assumed to be symmetric to the wheel axis, consequently its symmetry will hold true in all parts of the impeller. Therefore, the flow surfaces (rotational surfaces) desintegrating the impeller to elementary impellers are at the same time surfaces of $I =$ const, for ∇I lies perpendicular to these surfaces everywhere.

With symbols $\mathbf{c} = \mathbf{c}_m + \mathbf{c}_u$ and $\mathbf{w} = \mathbf{w}_m + \mathbf{w}_u$ respectively, equation (1) multiplied with the unity vector $\mathbf{e}_b$ of Fig. 1 and arranged properly, takes the following form

$$(\nabla \times \mathbf{c}_m)\mathbf{e}_n = \frac{1}{w_m} \mathbf{e}_b \nabla I + \frac{w_u}{w_m} (\nabla \times \mathbf{c})\mathbf{e}_l + \frac{1}{w_m} \mathbf{e}_b \mathbf{g}_n \,. \tag{4}$$

On the right side of the equation suitable transformations can be made.

Because of the constanst of I on each of the flow surfaces of rotation form with symbols used in Fig. 2 and, with the use of the relation $2\, r_0\, \pi\, w_{m0}\, \mathrm{d}\, b_0 = 2\, r\, \pi\, w_m\, (1 - \varphi_\vartheta/\varphi_t)\, \mathrm{db}$ the following relation will hold as well

$$\frac{1}{w_m} \mathbf{e}_b \nabla I = \frac{1}{w_m} \frac{\partial I}{\partial b} = \frac{r}{r_0\, w_{m0}} \left(1 - \frac{\varphi_\vartheta}{\varphi_t}\right) \left(\frac{\partial I}{\partial_b}\right)_0$$

and, for $\mathbf{w}^2 = \mathbf{c}^2 + \mathbf{u}^2 - 2\,\mathbf{cu}$

$$\left(\frac{\partial I}{\partial b}\right)_0 = \left[\frac{\partial}{\partial b}\left(\frac{u^2 - w^2}{2} - \frac{p}{\varrho}\right)\right]_0 =$$

$$= \omega \left[\frac{\partial}{\partial b}(r\,c_u)\right]_0 - \left[\frac{\partial}{\partial b}\left(\frac{c^2}{2} + \frac{p}{\varrho}\right)\right]_0$$

or else, taking into consideration $w_{m0} = c_{m0}$

$$\frac{1}{w_m}\mathbf{e}_b \nabla I = \frac{r}{r_0\,c_{m0}}\left(1 - \frac{\varphi_\vartheta}{\varphi_i}\right)\left\{\omega\left[\frac{\partial}{\partial b}(r\,c_u)\right]_0 - \left[\frac{\partial}{\partial b}\left(\frac{c^2}{2} + \frac{p}{\varrho}\right)\right]_0\right\}. \quad (5)$$

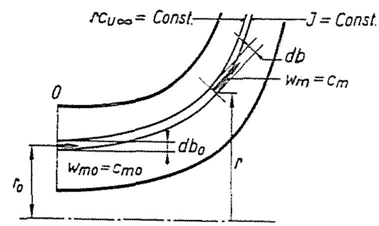

Fig. 2

With a right-hand system $\mathbf{e}_r$, $\mathbf{e}_\varphi$, $\mathbf{e}_z$ (see Fig. 3)

$$\nabla \times \mathbf{c} = -\frac{\partial c_u}{\partial z}\mathbf{e}_r + \left(\frac{\partial c_r}{\partial z} - \frac{\partial c_z}{\partial r}\right)\mathbf{e}_\varphi + \frac{1}{r}\frac{\partial}{\partial r}(rc_u)\mathbf{e}_z$$

and so in direction of $\mathbf{e}_l$ we can substitute respectively, $-\mathbf{e}_r\mathbf{e}_l = \mathbf{e}_z\mathbf{e}_b$ and $\mathbf{e}_z\mathbf{e}_l = \mathbf{e}_r\mathbf{e}_b$, after which

$$(\nabla \times \mathbf{c})\,\mathbf{e}_l = \frac{1}{r}\left[\frac{\partial}{\partial z}(rc_u)\,\mathbf{e}_z\,\mathbf{e}_b + \frac{\partial}{\partial r}(rc_u)\,\mathbf{e}_r\,\mathbf{e}_b\right] = \frac{1}{r}\frac{\partial}{\partial b}(rc_u).$$

According to our method [1] the blade sections along the rotational surfaces will be calculated in the straight cascade after the conformal transformation characterized by the transformation relations of

$$\eta = \frac{zt}{2\pi}\varphi$$

and

$$\xi = \frac{z \cdot t}{2\pi}\int_{r_1}^{r}\frac{dl}{r}$$

further, the velocity distribution $\mathbf{c}_\zeta$ in the straight cascade will be calculated in the conventional way — $\mathbf{c}_\zeta = \mathbf{c}_{\zeta\infty} + \mathbf{c}_{\zeta i}$ — by summing up the basic flow and the induced one ($\mathbf{c}_{\zeta\infty} = \mathbf{c}_{\xi\infty} + \mathbf{c}_{\eta\infty}$, $\mathbf{c}_{i\zeta} = \mathbf{c}_{i\xi} + \mathbf{c}_{i\eta}$) consequently

$$rc_u = \frac{z \cdot t}{2\pi} \left[c_{\eta\infty} + c_{i\eta}\right] = rc_{u\infty} + rc_{ui} .$$

So we obtain

$$(\nabla \times \mathbf{c})\, \mathbf{e}_l = \frac{1}{r} \frac{\partial}{\partial b} (rc_{u\infty}) + \frac{1}{r} \frac{\partial}{\partial b} (rc_{ui}) \tag{6}$$

or for the basic flow (see [1])

$$\frac{1}{r} \frac{\partial}{\partial b} (rc_{u\infty}) = \frac{1}{r} \frac{\partial}{\partial b} \left[\frac{zt}{2\pi} (c_{p\eta} + c_{k\eta})\right] = \frac{1}{r} \frac{\partial}{\partial b} \frac{\Gamma_b}{2\pi} = \frac{1}{r} \frac{\partial}{\partial b} (r_0\, c_{0u}),$$

assuming the same Γ, consequently the same $c_k = \Gamma/2\,t$ on each of the flow surfaces which form rotational surfaces. With symbols used in Fig. 2 and, considering the respective relations of $c_m = w_m$ and $r_0 w_{m0}\, db_0 = r\, w_m\, db\, (1 - \varphi_\vartheta/\varphi_t)$ we can put down, that

$$\frac{1}{r} \frac{\partial}{\partial b} (r_0\, c_{u\infty}) = \frac{w_m}{r_0\, c_{0m}} \left(1 + \frac{\varphi_\vartheta}{\varphi_t}\right) \left[\frac{\partial}{\partial b} (rc_u)\right]_0 \cdot \tag{7}$$

In the light of equations (6) and (7)

$$\frac{w_u}{w_m} (\nabla \times \mathbf{c}) \mathbf{e}_l = \frac{w_u}{r_0\, c_{m0}} \left(1 - \frac{\varphi_\vartheta}{\varphi_t}\right) \left[\frac{\partial}{\partial b} (rc_u)\right]_0 + \frac{w_u}{w_m} \frac{1}{r} \frac{\partial}{\partial b} (rc_{ui}) =$$

$$= \frac{w_u}{r_0\, c_{m0}} \left(1 - \frac{\varphi_\vartheta}{\varphi_t}\right) \left[\frac{\partial}{\partial b} (rc_u)\right]_0 + \frac{w_u}{w_m} (\nabla \times \mathbf{c}_i)\, \mathbf{e}_l . \tag{8}$$

As is known [1], if we took for an infinitely numerous blading a dimensionless vorticity distribution of $\gamma_\xi^* (\xi^*)$ in the straight cascade and, took into consideration,

$$c_{\eta\infty} = c_{p\eta} + c_{k\eta} = (2 r_0\, \pi c_{0u}/zt) + \Gamma/2t$$

we have

$$w_u = c_u - u = \frac{zt}{2r\pi} (c_{\eta\infty} + c_{\eta i}) - r\omega =$$

$$= \frac{1}{r} \left[(r_0\, c_{0u}) + \frac{z\Gamma}{2\pi} \int_0^{\xi^*} \gamma_\xi^*\, d\xi^* \right] - r\omega \tag{9}$$

according to

$$c_{i\eta} = \frac{\Gamma}{t} \int_0^{\xi^*} \gamma_\xi^*\, d_\xi^* - \frac{\Gamma}{2t} .$$

By summing up the equations of (4), (5), (8) and (9) we have

$$(\nabla \times \mathbf{c}_m)\mathbf{e}_u = \left(1 - \frac{\varphi_v}{\varphi_t}\right)\left\{-\frac{r}{r_0\, c_{m0}}\left[\frac{\partial}{\partial b}\left(\frac{c^2}{2} + \frac{p}{\varrho}\right)\right]_0 + \right.$$

$$+ \frac{1}{r}\left[\frac{\partial}{\partial b}(r\, c_u)\right]_0 \cdot \left(\frac{c_{u0}}{c_{m0}} + \frac{2\Gamma}{2\pi\, r_0\, c_{m0}} \int_0^{\xi^*} \gamma_\xi^*\, d\xi^*\right)\bigg\} +$$

$$+ \frac{w_u}{w_m}(\nabla \times \mathbf{c}_i)\,\mathbf{e}_l + \frac{1}{w_m}\,\mathbf{g}_n\,\mathbf{e}_b\,. \qquad (10)$$

In the following let us consider the last two members on the right side of equation (10).

In the case of a finite number of blades a differential pressure of

$$\Delta\, p = \varrho\, w\, \gamma$$

arises at the elementary blade surface of $d\mathbf{F} = dF\mathbf{e}_n$ (where $\mathbf{e}_n$ is a unity vector perpendicular to the blade, and $\mathbf{e}_n\mathbf{e}_u > 0$) consequently, in the case of a blading with infinite blade numbers the field acting against the unity mass will be

$$\mathbf{g}_n = \frac{z\,\Delta p\; d\mathbf{F}}{2r\,\pi\varrho\; d\mathbf{F}\;\mathbf{e}_u} = \frac{w\,\gamma z\;\mathbf{e}_u}{2r\,\pi\;\mathbf{e}_n\;\mathbf{e}_u}$$

and (having interpreted the sign before γ) substituting [1]

$$\gamma = -\frac{2r\,\pi}{z}\sin\beta\,(\nabla \times \mathbf{c}_i)\,\mathbf{e}_b$$

and taking into consideration $w_m/w = \sin\beta$ we obtain

$$\frac{1}{w_m}\mathbf{g}_n\,\mathbf{e}_b = -\,[(\nabla \times \mathbf{c}_i)\,\mathbf{e}_b]\,\frac{\mathbf{e}_n\,\mathbf{e}_b}{\mathbf{e}_n\,\mathbf{e}_u}.$$

We can agree that due to $\mathbf{w}\mathbf{e}_n = 0$

$$\frac{w_u}{w_m} = -\frac{\mathbf{e}_n\,\mathbf{e}_l}{\mathbf{e}_n\,\mathbf{e}_u}$$

and, with this the last two members on the right side of equation (10) will be:

$$\frac{w_u}{w_m}(\nabla \times \mathbf{c}_i)\,\mathbf{e}_l + \frac{1}{w_m}\,\mathbf{g}_n\,\mathbf{e}_b = -\left\{[(\nabla \times \mathbf{c}_i)e_l]\,\frac{\mathbf{e}_n\,\mathbf{e}_l}{\mathbf{e}_n\,\mathbf{e}_u} + \right.$$

$$\left. + \,[(\nabla \times \mathbf{c}_i)\,\mathbf{e}_b]\,\frac{\mathbf{e}_n\,\mathbf{e}_b}{\mathbf{e}_n\,\mathbf{e}_u}\right\} = (\nabla \times \mathbf{c}_i)\,\mathbf{e}_u \qquad (11)$$

because of $(\nabla \times \mathbf{c}_i)\mathbf{e}_n = 0$.

The reduction of equations (10) and (11) gives us:

$$(\nabla \times \mathbf{c}_m)\,\mathbf{e}_u = -\left(\frac{\varphi_\vartheta}{\varphi_t} - 1\right)\left\{-\frac{r}{r_0\,c_{m0}}\left[\frac{\partial}{\partial b}\left(\frac{c^2}{2} + \frac{p}{\varrho}\right)\right]_0 + \right.$$

$$+ \frac{1}{r}\left[\frac{\partial}{\partial b}(rc_u)\right]_0 \cdot$$

$$\cdot\left.\left(\frac{c_{u0}}{c_{m0}} + \frac{z\,\Gamma}{2\pi\,r_0\,c_{m0}}\int\limits_0^{\xi^*}\gamma_\xi^*\,d\xi^*\right)\right\} + (\nabla \times \mathbf{c}_i)\,\mathbf{e}_u \qquad (12)$$

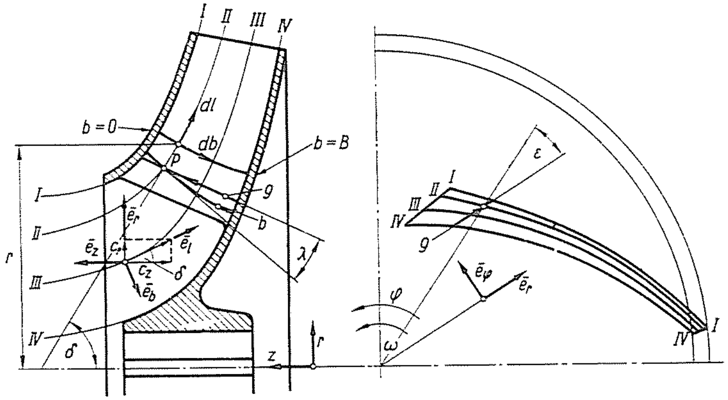

Fig. 3

and, in addition, putting the differential formula of

$$(\nabla \times \mathbf{c}_m)\,\mathbf{e}_u = -\frac{\partial c_m}{\partial b} - c_m\frac{\partial \delta}{\partial l}$$

into the above equation (see [1]), and likewise taking the following equation from [1]

$$(\nabla \times \mathbf{c}_i)\,\mathbf{e}_u = -\operatorname{tg}\varepsilon\frac{\cos(\lambda + \delta)}{\cos\lambda}\;\frac{\Gamma}{\xi_2}\;\frac{tz^2}{4\pi^2}\;\frac{\gamma_\xi^*}{r^2} \qquad (13)$$

(see also Fig. 3), after substitutions

$$\frac{\partial c_m}{\partial b} + c_m H = F(r,z) \qquad (14)$$

where [1] $H = \partial\delta/\partial l$ and

$$F(r,z) = C_1\,G_1(r,z) + C_2\,G_2(r,z) + C_3\,G_3(r,z) + C_4\,G_4(r,z) \qquad (15)$$

or else, the functions on the right side

$$G_1 = r\left(1 - \frac{\varphi_\vartheta}{\varphi_t}\right) \tag{16a}$$

$$G_2 = \frac{1}{r}\left(\frac{\varphi_\vartheta}{\varphi_t} - 1\right) \tag{16b}$$

$$G_3 = G_2 \int_0^{\xi^*} \gamma_\xi^* \, d\xi^* \tag{16c}$$

$$G_4 = \frac{\gamma_\xi^*}{r^2 \xi_2} \operatorname{tg} \varepsilon \frac{\cos(\lambda + \delta)}{\cos \lambda} \tag{16d}$$

and

$$C_1 = \frac{1}{r_0 c_{m0}} \left[\frac{\partial}{\partial b}\left(\frac{c^2}{2} + \frac{p}{\varrho}\right)\right]_0 \tag{16e}$$

$$C_2 = \frac{c_{u0}}{c_{m0}} \left[\frac{\partial}{\partial b}(rc_u)\right]_0 \tag{16f}$$

$$C_3 = \frac{z\Gamma}{2\pi r_0 c_{m0}} \left[\frac{\partial}{\partial b}(r\, c_u)\right]_0 \tag{16g}$$

$$C_4 = \frac{\Gamma t\, z^2}{4\pi^2} \tag{16h}$$

are constants on each of the flow surfaces (revolutional surfaces).

When, before the impeller in section 0 the flow is potential, then, according to $C_1 = 0$, $C_2 = 0$ and $C_3 = 0$ we obtain

$$F(r, z) = C_4 G_4$$

already known from [1].

Even in the general case equation (14) can be solved by the iteration process already made known [1]. The only difference lies in fixing the function $F(r, z)$ (see first three members on the right side of equation (15)), however, this can also be made without difficulties.

Functions $G_1(r, z)$ and $G_2(r, z)$ can be taken in advance and fixed independent of the blading. Though the function $G_3(r, z)$ depends according to the integral on the blade form, but when we take the same γ_ξ^* (ξ^*) function for each of the blade sections, its distribution will show no appreciable change in the course of iterations. What depends to the greatest extend on the blade form, is function G_4 (r, z), but the member C_4 G_4 of the equation (15) has played a part as early as in the potential flow towards the impeller [1], and so generalization is of no consequence, it is independent of edge conditions in the section 0. Consequently, the solution of a more generalized problem can be reduced to the solution according to [1].

Summary

Paper presents a generalization of a formerly published one by the same author [1] and shows, that the formerly proposed iteration process [1] can be used even in the case of any flow towards the impeller being symmetrical to impeller axis, and only formal differences will be given. Paper presents the generalized form of function $F(r, z)$ giving an approximation of zeroth order and representing the inhomogenity in the linear inhomogenous partial differencial equation of first order characterizing a meridional flow of the infinitely numerous blading.

Literature

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