THE CALCULATION OF THE SHAPE FACTOR BETWEEN A CYLINDRICAL SURFACE AND A PLANE IN CONTACT THEREWITH

APPLICATION OF THE CALCULATION METHOD TO THE RADIANT SCREEN*

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I. Antecedents

Radiant heating, particularly where large halls are concerned, is ever more frequently used in heating engineering. The radiator applied in this heating method is mostly a radiant screen which is characterised by a heat loss of overwhelmingly larger ratio by radiation than by convection, due mainly to the elevated surface temperature and the free radiant surfaces.

The conventional design of the radiant screen is as follows.

Parallel-placed heating pipes are connected by a distributing and a collecting header. A metal plate between the heating pipes constitutes the major part of the heating surface. From the point of view of heat exchange, the contact existing between the heating pipe and the screen assumes considerable significance. In current practice this contact is ensured by the screen surrounding the pipe in an arc of up to 180 degrees, with some fixture — a yoke, stirrup, etc. — to press them together.

Fig. 1 shows a part of the radiator, in a sectional view normal to the heating pipe, however, without the clamping fixture.

The thermodynamical calculation of radiant screens may essentially be grouped into two main problems:

a) the conductive and convective heat exchange between the heating pipe and the screen, by contact between the pipe and the plate;

b) the heat exchange between the radiator and its surrounding, which determines the heat loss of the radiator.

In the following, one particular detail of the latter problem will be dealt with, viz. the heat exchange taking place between the screen and its surrounding.

Fig. 1 also shows an element of "dx" width and unit length of the screen. The thermal equilibrium of this element, in steady state, can be expressed

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by the following relationship:

$$q_x - q_{(x+dx)} = -\lambda i \left(\frac{d\vartheta}{dx}\right)_x + \lambda i \left(\frac{d\vartheta}{dx}\right)_{(x+dx)} =$$

$$= \{[1 - (\varphi_1 + \varphi_2)\, a_{is}] + a_{ic} + a_e\}\, \vartheta\, dx +$$

$$+ C \frac{\left(\frac{273 + \vartheta + t_i}{100}\right)^4 - \left(\frac{273 + \vartheta_f + t_i}{100}\right)^4}{\vartheta - \vartheta_f} (\vartheta - \vartheta_f)(\varphi_1 + \varphi_2)\, dx \quad (1)$$

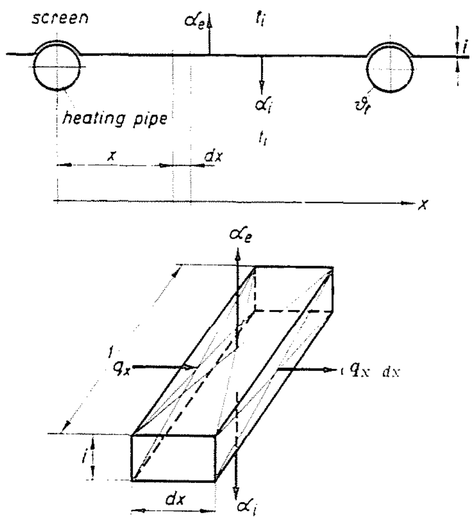

Fig. 1

where: q_x denotes the heat flow across the sectional area of the "i.1" screen at a distance of "x" in kcal/h

$q_{(x+dx)}$ the heat flow across the "i.1" surface at a distance of "$x + dx$", in kcal/h

λ the thermal conductivity of the screen material, in kcal/m h° C

i the thickness of the screen in m

ϑ the excess temperature of the screen in relation to "t_i" in centigrades

t_i the temperature surrounding the radiator, in centigrades

ϑ_f the excess temperature of the heating pipe surface in relation to "t_i", in centigrades

a_{ic} the average thermal convection coefficient from the lower surface of the screen downwards, in kcal/m²h°C

a_{is} the average thermal radiation coefficient from the lower surface of the screen to the surroundings, in kcal/m² h°C

a_e the average total heat transfer coefficient from the upper surface of the screen upwards, in kcal/m² h°C

C Boltzmann constant in kcal/m² h°K⁴

φ_1 the shape factor between the surface element of "dx" width and unit length, and one of the heating pipes

φ_2 the shape factor between the surface element and the other heating pipe.

The (1) relationship, as a differential equation, describes the following process:

The element cut out from the screen absorbs heat from the heating pipe side at the same time rejecting heat by conductance to the adjacent plate portion. Since the temperatures of the surroundings are mostly below those of the plate, heat is transmitted to the surroundings, viz. the quantity of heat arriving by conductance exceeds the quantity of heat extracted by same.

The heating pipes whence the element obtains heat by radiation, should also be regarded as the surroundings of the plate. This is expressed in the last term of the (1) relationship, whereby $\vartheta_f > \vartheta$.

The element transmits heat to the surroundings which lie beyond the heating pipes both by radiation and convection, downwards and upwards. However, since radiator tops are generally insulated, there is no need to go into details concerning the heat transmitted upwards.

The heat transmission of the element by radiation and convection is expressed in the first term on the right-hand side of the equation. The relationship presupposes constant temperature along the heating pipes, viz. that thermal conductance in the plate takes place only normal to the pipes. Accordingly, the temperature of the heating pipe is the arithmetical mean of the temperature prevailing at either end of the pipe.

With regard to the (1) relationship it should be noted that in the thermodynamical calculation of radiant screens the radiative heat exchange between the heating pipes and the screen has so far been neglected. Although this neglection is fully justified in some cases — pending on the particular structural design — whether this is so or not, should at all times be decided after thorough investigation.

The mathematical resolution of the (1) relationship as a differential equation creates considerable difficulties, as the "φ" shape factor is of the second degree and is still a trigonometric function of the "x" coordinate.

This difficulty can, however, be overcome by calculating with the average value of the shape factor — the latter arrived at as the shape factor between the screen surface and the surface of the heating pipe. Applying this method we avail ourselves of the assumption — completely justified in view of the very good approximation — that

$$\varphi \cong \Phi$$

The present paper deals with the calculation of the shape factor between a cylindrical and a plane surface in contact with each other, on the basis of what has been said above. We shall not go into any further examination of the (1) relationship, nevertheless, we wish to outline the fact that in the thermodynamical calculation of radiant screens, due to their structural design, we may come across instances when radiative heat exchange between the screen and the heating pipes is very considerable.

2. The derivation of the shape factor

It is known that in a relationship expressing radiative heat exchange between two optionally positioned surfaces, the average shape factor is interpreted by the following relation (shown on Fig. 2):

$$\Phi_{h-s} = \frac{1}{\pi F_h} \iint_{F_h F_s} \frac{\cos \beta_1 \cos \beta_2}{r^2} \, dF_h \, dF_s \tag{2}$$

where: Φ_{h-s} denotes the average shape factor between the cylindrical and the plane surface

F_h denotes the cylindrical surface

F_s denotes the plane surface,

all other notations are to be found in Figs. 2 and 3.

The problem to be solved can be seen from the (2) relationship. This must first be adapted to the problem in hand, then a double surface integration must be performed.

Avoiding all superfluous explanations and the eventual elucidation of simple mathematical processes, in what follows we shall present this deduction.

The determination by vectorial algebra of the

$$\frac{\cos \beta_1 \cos \beta_2}{r^2}$$

factor is as follows:

$$\bar{r}_1 = \{x;\ y;\ R \sin \varepsilon\}$$
$$\bar{r}_2 = \{\xi;\ R \cos t;\ R \sin t\}$$
$$\bar{r} = \{x - \xi;\ y - R \cos t;\ R \sin \varepsilon - R \sin t\}$$
$$r^2 = (x - \xi)^2 + (y - R \cos t)^2 + (R \sin \varepsilon - R \sin t)^2$$
$$\bar{n}_1 = \{0;\ 0;\ 1\}$$
$$\bar{n}_2 = \{0;\ R \cos t;\ R \sin t\} = R\{0;\ \cos t;\ \sin t\}$$

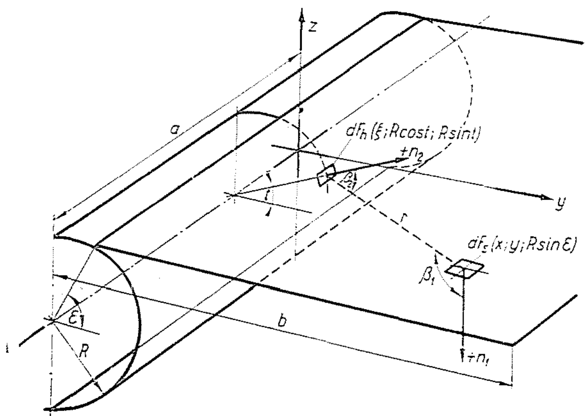

Fig. 2

$$\cos \beta_1 = \frac{\bar{n}_1 \cdot \bar{r}}{|\bar{n}_1|\,|\bar{r}|} = \frac{R \sin \varepsilon - R \sin t}{r} = \frac{R(\sin \varepsilon - \sin t)}{r}$$

$$\cos \beta_2 = \frac{\bar{n}_2 \cdot \bar{r}}{|\bar{n}_2|\,|\bar{r}|} = \frac{R \cos t\,(y - R \cos t) + R \sin t\,(R \sin \varepsilon - R \sin t)}{rR\sqrt{\cos^2 t + \sin^2 t}} =$$
$$= \frac{y \cos t + R \sin \varepsilon \sin t - R}{r}$$

$$\frac{\cos \beta_1 \cos \beta_2}{r^2} = \frac{R(\sin \varepsilon - \sin t)(y \cos t + R \sin \varepsilon \sin t - R)}{r^4} =$$

$$= \frac{R(\sin \varepsilon - \sin t)[(y - R \cos t + R \cos t) \cos t + R \sin \varepsilon \sin t - R]}{r^4} =$$

$$= \frac{(y - R \cos t)(\sin \varepsilon - \sin t) R \cos t + (R \sin \varepsilon - R \sin t)^2 \sin t}{[(x - \xi)^2 + (y - R \cos t)^2 + (R \sin \varepsilon - R \sin t)^2]^2}$$

A trigonometric determination will yield analogous result:

$$\cos\beta_1 = \frac{R\sin\varepsilon - R\sin t}{r} = \frac{R(\sin\varepsilon - \sin t)}{r}$$

$$\cos\beta_2 = \frac{\dfrac{(y - R\cos t)^2}{\cos^2 t} + (x-\xi)^2 + (y - R\cos t)^2 + (R\sin\varepsilon - R\sin t)^2}{2r\,\dfrac{y - R\cos t}{\cos t}} -$$

$$- \frac{(x-\xi)^2 - \dfrac{(y\sin t - R\cos t)^2}{\cos^2 t}}{2r\,\dfrac{y - R\cos t}{\cos t}} = \frac{y\cos t + R\sin\varepsilon\sin t - R}{r}$$

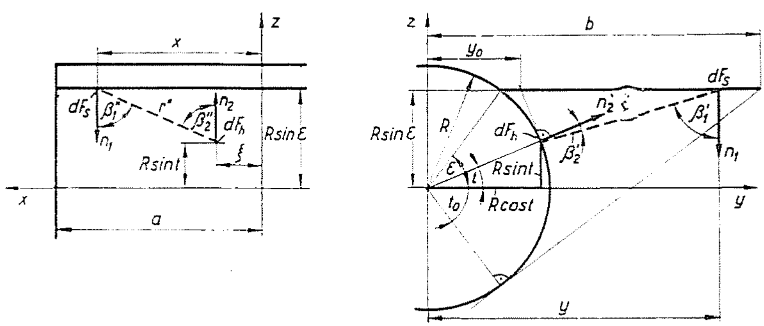

Fig. 3

Substituting the surface elements:

$$dF_s = dx\ dy$$

$$dF_h = R\,dt\ d\xi$$

Expanded, the (2) relationship assumes the following form:*

$$\Phi_{h-s} = \frac{R}{\pi F_h} \int\limits_x\int\limits_\xi\int\limits_y\int\limits_t \frac{(y - R\cos t)(\sin\varepsilon - \sin t)\,R\cos t + (R\sin\varepsilon - R\sin t)^2\sin t}{[(x-\xi)^2 + (y - R\cos t)^2 + (R\sin\varepsilon - R\sin t)^2]^2} \cdot$$

$$\cdot\ dx\ d\xi\ dy\ dt \qquad (3)$$

* In the setting up of this relationship, valuable help was given by Mr. Zs. Kokits, lecturer of the Mathematics Department of the Polytechnical University of Budapest.

Note: Lacking sufficient space, the multiplication has been written in two lines, with the multiplication point between the parts.

The integration limits may also be read from Fig. 3.:
according to x from $(-a)$ to $(+a)$;
,, to ξ from $(-a)$ to $(+a)$;
,, to y from y_0 to b;
,, to t from t_0 to ε; where

$$y_0 = R\cos t - R(\sin\varepsilon - \sin t)\frac{\sin t}{\cos t} = R\frac{1-\sin\varepsilon\sin t}{\cos t}$$

$$t_0 = \arcsin\frac{R^2\sin\varepsilon \pm b\sqrt{R^2(\sin^2\varepsilon - 1) + b^2}}{R^2\sin^2\varepsilon + b^2} \tag{4}$$

First integration, according to x :

$$A = (y - R\cos t)(\sin\varepsilon - \sin t)\,R\cos t + (R\sin\varepsilon - R\sin t)^2\sin t$$

$$B^2 = (y - R\cos t)^2 + (R\sin\varepsilon - R\sin t)^2$$

$$A\int_{x=-a}^{+a}\frac{1}{[(x-\xi)^2 + B^2]^2}\,dx = \frac{A}{B^4}\int\frac{1}{\left[\left(\frac{x-\xi}{B}\right)^2 + 1\right]^2}\,dx =$$

$$= \frac{A}{B^3}\int\frac{1}{(u^2+1)^2}\,du =$$

$$= \frac{A}{2B^3}\left[\operatorname{arc\,tg}\frac{x-\xi}{B} + \frac{\frac{x-\xi}{B}}{1+\left(\frac{x-\xi}{B}\right)^2}\right]_{-a}^{+a} =$$

$$= \frac{(y - R\cos t)(\sin\varepsilon - \sin t)\,R\cos t + (R\sin\varepsilon - R\sin t)^2\sin t}{2\,[(y-R\cos t)^2 + (R\sin\varepsilon - R\sin t)^2]^{3/2}}\,\cdot$$

$$\cdot\left[\operatorname{arc\,tg}\frac{a-\xi}{\sqrt{(y-R\cos t)^2 + (R\sin\varepsilon - R\sin t)^2}} + \right.$$

$$+ \operatorname{arc\,tg}\frac{a+\xi}{\sqrt{(y-R\cos t)^2 + (R\sin\varepsilon - R\sin t)^2}} +$$

$$+ \frac{(a-\xi)\sqrt{(y-R\cos t)^2 + (R\sin\varepsilon - R\sin t)^2}}{(a-\xi)^2 + (y-R\cos t)^2 + (R\sin\varepsilon - R\sin t)^2} +$$

$$\left. + \frac{(a+\xi)\sqrt{(y-R\cos t)^2 + (R\sin\varepsilon - R\sin t)^2}}{(a+\xi)^2 + (y-R\cos t)^2 + (R\sin\varepsilon - R\sin t)^2}\right].$$

Second integration, according to ξ :

$$\int\limits_{-a}^{+a} \operatorname{arc\,tg} \frac{a-\xi}{\sqrt{(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}}\,d\xi =$$

$$= -\sqrt{(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}\int \operatorname{arc\,tg} p\,dp =$$

$$= -\sqrt{(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}\,\cdot$$

$$\cdot\left\{\frac{a-\xi}{\sqrt{(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}}\,\cdot\right.$$

$$\operatorname{arc\,tg}\frac{a-\xi}{\sqrt{(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}}\;-$$

$$\left.-\frac{1}{2}\ln\left[1+\frac{(a-\xi)^2}{\left(\sqrt{(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}\right)^2}\right]\right\}_{-a}^{+a}=$$

$$= 2a\operatorname{arc\,tg}\frac{2\,a}{\sqrt{(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}}\;+$$

$$+\;\frac{\sqrt{(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}}{2}\;\cdot$$

$$\cdot\ln\frac{(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}{4\,a^2+(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}$$

$$\int\limits_{-a}^{+a} \operatorname{arc\,tg} \frac{a+\xi}{\sqrt{(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}}\,d\xi =$$

$$= \sqrt{(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}\int \operatorname{arc\,tg} p\,dp =$$

$$= \left[(a+\xi)\operatorname{arc\,tg}\frac{a+\xi}{\sqrt{(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}}\right]_{-a}^{+a}-$$

$$-\;\frac{\sqrt{(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}}{2}\;\cdot$$

$$\cdot\ln\left[1+\frac{(a-\xi)^2}{\left(\sqrt{(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}\right)^2}\right]_{-a}^{+a}=$$

$$= 2a\operatorname{arc\,tg}\frac{2\,a}{\sqrt{(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}}\;+$$

$$+\;\frac{\sqrt{(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}}{2}\;\cdot$$

$$\cdot\ln\frac{(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}{4a^2+(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}$$

$$\int_{-a}^{+a} \frac{a-\xi}{(a-\xi)^2+(y-R\cos t)^2+(R\sin\varepsilon - R\sin t)^2}\, d\xi =$$

$$= \frac{1}{2} \ln \frac{4a^2+(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}{(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}$$

$$\int_{-a}^{+a} \frac{a+\xi}{(a+\xi)^2+(y-R\cos t)^2+(R\sin\varepsilon - R\sin t)^2}\, d\xi =$$

$$= \frac{1}{2} \ln \frac{4a^2+(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}{(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}$$

"ξ" integration yields the following final result:

$$\frac{2a\cos t\,(y-R\cos t)\,(R\sin\varepsilon - R\sin t) + 2a\sin t\,(R\sin\varepsilon - R\sin t)^2}{[(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2]^{3/2}} \cdot$$

$$\cdot \operatorname{arc\,tg} \frac{2a}{\sqrt{(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}}$$

Third integration, according to y:

$$u = \operatorname{arc\,tg} \frac{2a}{\sqrt{(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}}$$

$$u' = -\frac{2a\,(y-R\cos t)}{[(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2+4a^2]\sqrt{(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}}$$

$$v' = \frac{2a\cos t\,(y-R\cos t)(R\sin\varepsilon-R\sin t)+2a\sin t\,(R\sin\varepsilon-R\sin t)^2}{[(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2]^{3/2}} =$$

$$= \frac{A\,(y-R\cos t)+B}{[(y-R\cos t)^2+C^2]^{3/2}} = \frac{Ap+B}{(p^2+C^2)^{3/2}}$$

$$v = \frac{2a\sin t\,(y-R\cos t) - 2a\cos t\,(R\sin\varepsilon-R\sin t)}{[(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2]^{1/2}}$$

$$\int_{y_0}^{b} \frac{2a\cos t\,(y-R\cos t)\,(R\sin\varepsilon-R\sin t)+2a\sin t\,(R\sin\varepsilon-R\sin t)^2}{[(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2]^{3/2}} \cdot$$

$$\cdot \operatorname{arc\,tg} \frac{2a}{\sqrt{(y-R\cos t)^2+(R\sin\varepsilon-R\sin t)^2}}\, dy = \int uv'\, dy =$$

$$= \left\{ \frac{2a \sin t\,(y - R\cos t) - 2a\cos t\,(R\sin\varepsilon - R\sin t)}{[(y - R\cos t)^2 + (R\sin\varepsilon - R\sin t)^2]^{1/2}} \cdot \right.$$

$$\left. \cdot \operatorname{arc\,tg} \frac{2a}{\sqrt{(y - R\cos t)^2 + (R\sin\varepsilon - R\sin t)^2}} \right\}_{y_0}^{b} +$$

$$+ \int_{y_0}^{b} 4a^2 \frac{\sin t\,(y - R\cos t)^2 - \cos t\,(y - R\cos t)(R\sin\varepsilon - \sin t)}{[(y - R\cos t)^2 + (R\sin\varepsilon - R\sin t)^2 + 4a^2][(y - R\cos t)^2 + (R\sin\varepsilon - R\sin t)^2]}\, dy$$

$$p = y - R\cos t; \quad p^2 = s$$

$$A^2 = (R\sin\varepsilon - R\sin t)^2 + 4a^2 = B^2 + 4a^2$$

$$\int_{y_0}^{b} 4a^2 \frac{\sin t\,(y - R\cos t)^2 - \cos t\,(R\sin\varepsilon - R\sin t)\,(y - R\cos t)}{[(y - R\cos t)^2 + (R\sin\varepsilon - R\sin t)^2 + 4a^2][(y - R\cos t)^2 + (R\sin\varepsilon - R\sin t)^2]}\, dy =$$

$$= 4a^2 \sin t \int \frac{p^2}{(p^2 + A^2)\,(p^2 + B^2)}\, dp -$$

$$- 2a^2 \cos t\,(R\sin\varepsilon - R\sin t) \int \frac{1}{(s + A^2)\,(s + B^2)}\, ds =$$

$$= 4a^2 \sin t \frac{A^2}{A^2 - B^2} \int \frac{1}{p^2 + A^2}\, dp + 4a^2 \sin t \frac{B^2}{B^2 - A^2} \int \frac{1}{p^2 + B^2}\, dp -$$

$$- 2a^2 \cos t\,(R\sin\varepsilon - R\sin t) \frac{1}{B^2 - A^2} \int \frac{1}{s + A^2}\, ds -$$

$$- 2a^2 \cos t\,(R\sin\varepsilon - R\sin t) \frac{1}{A^2 - B^2} \int \frac{1}{s + B^2}\, ds =$$

$$= \sin t \sqrt{(R\sin\varepsilon - R\sin t)^2 + 4a^2} \left[\operatorname{arc\,tg} \frac{y - R\cos t}{\sqrt{(R\sin\varepsilon - R\sin t)^2 + 4a^2}} \right]_{y_0}^{b} -$$

$$- \sin t\,(R\sin\varepsilon - R\sin t) \left[\operatorname{arc\,tg} \frac{y - R\cos t}{R\sin\varepsilon - R\sin t} \right]_{y_0}^{b} +$$

$$+ \frac{\cos t\,(R\sin\varepsilon - R\sin t)}{2} \left[\ln \frac{(y - R\cos t)^2 + (R\sin\varepsilon - R\sin t)^2 + 4a^2}{(y - R\cos t)^2 + (R\sin\varepsilon - R\sin t)^2} \right]_{y_0}^{b}$$

After the substitution of the limits, the final result of integration according to y will take the form:

$$\frac{2a\,[\sin t\,(b-R\cos t)-R\cos t\,(\sin\varepsilon-\sin t)]}{\sqrt{(b-R\cos t)^2+R^2(\sin\varepsilon-\sin t)^2}}\,\cdot$$

$$\cdot\ \operatorname{arc\,tg}\frac{2a}{\sqrt{(b-R\cos t)^2+R^2(\sin\varepsilon-\sin t)^2}}-$$

$$-\frac{2a\left[\sin t\left(\frac{1-\sin\varepsilon\sin t}{\cos t}-\cos t\right)-\cos t\,(\sin\varepsilon-\sin t)\right]}{\sqrt{\left(\frac{1-\sin\varepsilon\sin t}{\cos t}-\cos t\right)^2+(\sin\varepsilon-\sin t)^2}}\,\cdot$$

$$\cdot\ \operatorname{arc\,tg}\frac{2a}{R\sqrt{\left(\frac{1-\sin\varepsilon\sin t}{\cos t}-\cos t\right)^2+(\sin\varepsilon-\sin t)^2}}+$$

$$+\sin t\,\sqrt{R^2(\sin\varepsilon-\sin t)^2+4a^2}\,\cdot$$

$$\cdot\left[\operatorname{arc\,tg}\frac{b-R\cos t}{\sqrt{R^2(\sin\varepsilon-\sin t)^2+4a^2}}-\right.$$

$$\left.-\operatorname{arc\,tg}\frac{R\left(\frac{1-\sin\varepsilon\sin t}{\cos t}-\cos t\right)}{\sqrt{R^2(\sin\varepsilon-\sin t)^2+4a^2}}\right]-$$

$$-R\sin t\,(\sin\varepsilon-\sin t)\left[\operatorname{arc\,tg}\frac{b-R\cos t}{R\,(\sin\varepsilon-\sin t)}-\right.$$

$$\left.-\operatorname{arc\,tg}\frac{\left(\frac{1-\sin\varepsilon\sin t}{\cos t}-\cos t\right)}{\sin\varepsilon-\sin t}\right]+$$

$$+\frac{R\cos t\,(\sin\varepsilon-\sin t)}{2}\times$$

$$\times\left[\ln\frac{(b-R\cos t)^2+R^2(\sin\varepsilon-\sin t)^2+4a^2}{(b-R\cos t)^2+R^2(\sin\varepsilon-\sin t)^2}-\right.$$

$$\left.-\ln\frac{\left(\frac{1-\sin\varepsilon\sin t}{\cos t}-\cos t\right)^2+(\sin\varepsilon-\sin t)^2+4a^2}{\left(\frac{1-\sin\varepsilon\sin t}{\cos t}-\cos t\right)^2+(\sin\varepsilon-\sin t)^2}\right].$$

Substituting the result of the third integration into the (2) relationship, we arrive at:

$$\pi F_h \Phi_{h-s} =$$

$$= \int_{t_0}^{\varepsilon} \frac{2aR(b\sin t - R\sin\varepsilon\cos t)}{\sqrt{(b-R\cos t)^2 + R^2(\sin\varepsilon - \sin t)^2}} \cdot$$

$$\cdot \operatorname{arc\,tg} \frac{2a}{\sqrt{(b-R\cos t)^2 + R^2(\sin\varepsilon - \sin t)^2}}\, dt -$$

$$- \int_{t_0}^{\varepsilon} 2aR \operatorname{arc\,tg} \frac{2a\cos t}{R(\sin t - \sin\varepsilon)}\, dt +$$

$$+ \int_{t_0}^{\varepsilon} R\sin t \sqrt{R^2(\sin\varepsilon - \sin t)^2 + 4a^2} \cdot$$

$$\cdot \left[\operatorname{arc\,tg} \frac{b - R\cos t}{\sqrt{R^2(\sin\varepsilon - \sin t)^2 + 4a^2}} - \right.$$

$$\left. - \operatorname{arc\,tg} \frac{R\dfrac{\sin^2 t - \sin\varepsilon\sin t}{\cos t}}{\sqrt{R^2(\sin\varepsilon - \sin t)^2 + 4a^2}} \right] dt -$$

$$- \int_{t_0}^{\varepsilon} R^2(\sin\varepsilon\sin t - \sin^2 t)\left[\operatorname{arc\,tg} \frac{b - R\cos t}{R(\sin\varepsilon - \sin t)} + t \right] dt +$$

$$+ \int_{t_0}^{\varepsilon} \frac{R^2\cos t(\sin\varepsilon - \sin t)}{2} \cdot$$

$$\cdot \left[\ln \frac{(b-R\cos t)^2 + R^2(\sin\varepsilon - \sin t)^2 + 4a^2}{(b-R\cos t)^2 + R^2(\sin\varepsilon - \sin t)^2} - \right.$$

$$\left. - \ln \frac{R^2\dfrac{(\sin\varepsilon - \sin t)^2}{\cos^2 t} + 4a^2}{R^2\dfrac{(\sin\varepsilon - \sin t)^2}{\cos^2 t}} \right] dt \qquad (5)$$

where:

$$F_h = 2aR\, 0.01745 \left[\varepsilon + \arcsin \frac{R^2\sin\varepsilon \pm b\sqrt{R^2(\sin^2\varepsilon - 1) + b^2}}{R^2\sin^2\varepsilon + b^2} \right]. \qquad (6)$$

Although a fourth integration according to t, should be carried out, a careful scrutiny of the (5) relationship will prove that no result can be arrived at by purely analytical means.

Graphic integration, viz. the calculation of the value of the (5) relationship at various angular values between "t_0" and "ε" and their representation — whereby the area beneath the curve yields the result sought for — is a fairly good and adequately accurate, but highly painstaking solution.

The correctness of the result depends on the following conditions:

a) the density of the "t" angular values between the limits of integration,

b) the precision observed in the partial processes,

c) the accuracy of planimetry.

The rigorous accuracy of the final result up to its fifth decimal is an absolute criterion. This degree of accuracy can be readily achieved.

Summing up, we may state that *the shape factor between a cylindrical and a plane surface which is in contact, may be determined by the graphic integration of the (5) relationship.*

3. The value of the shape factors in connection with radiant screens

It becomes apparent from the result arrived at in the aforementioned that the determination of the shape factor between the heating pipe and the screen is a laboursome procedure, and thus unsuitable for general practice.

Table I

		$d = ½''$				$d = 1''$			
		b [m]				b [m]			
		0,12	0,16	0,24	0,48	0,12	0,16	0,24	0,48
$\varepsilon = 90°$	F_h	0.12572	0.12757	0.12944	0.13133	0.19154	0.19595	0.20053	0.20516
	Φ_{h-s}	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000
$\varepsilon = 60°$	F_h	0.10405	0.10573	0.10750	0.10925	0.15743	0.16185	0.16619	0.17054
	Φ_{h-s}	0.39751	0.39448	0.39444	0.40097	0.39577	0.39010	0.38992	0.39946
$\varepsilon = 30°$	F_h	0.08331	0.08459	0.08597	0.08738	0.12593	0.12941	0.13277	0.13638
	Φ_{h-s}	0.28898	0.28717	0.28757	0.29278	0.28306	0.27973	0.28100	0.28984
$\varepsilon = 0°$	F_h	0.06285	0.06387	0.06474	0.06564	0.09558	0.09795	0.10025	0.10258
	Φ_{h-s}	0.17861	0.17793	0.17901	0.18208	0.17583	0.17418	0.17528	0.18125
$\varepsilon = -30°$	F_h	0.04245	0.04299	0.04341	0.04390	0.06537	0.06644	0.06761	0.06877
	Φ_{h-s}	0.08343	0.08335	0.08447	0.08700	0.08325	0.08334	0.08354	0.08536

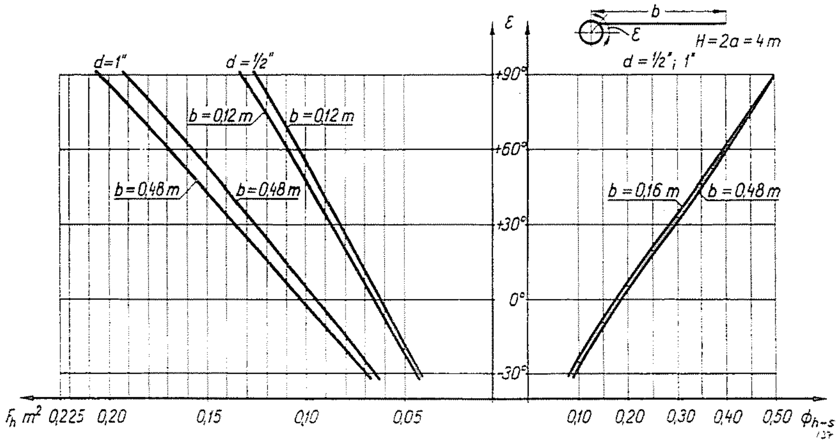

Fig. 4

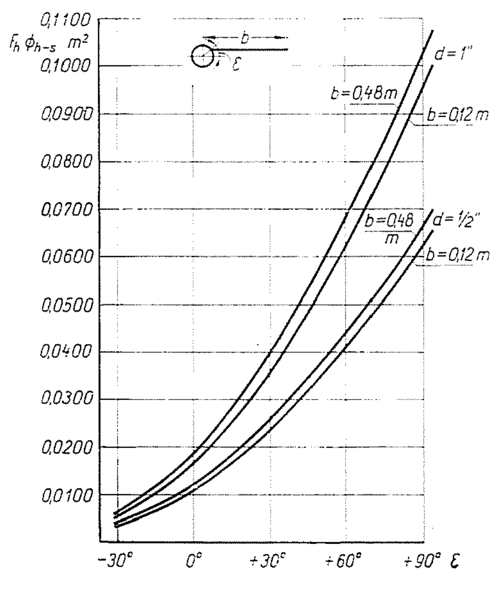

Fig. 5

To meet the case, it became necessary to calculate, once and for all and to graphically determine, the shape factors for the most frequent radiant screen designs.

The calculation was performed under the conditions given below — which at the same time represent the most frequent structural layouts of radiant screens.

a) Heating pipe diameter, $d = \frac{1}{2}''$ and $1''$.

b) Spacing of the heating pipes, $b = 0.12$ m; 0.16 m; 0.24 m and 0,48 m.

c) Length of the heating pipes and of the screen, $H = 2a = 4$ m.

Due to the small "d" values, the changes in the value of "a" above 0.5 m will cause no appreciable difference.

d) The factor having the greatest influence on the value of the shape factor is "ε", its value being $+90°$, $+60°$, $+30°$, $0°$ and $-30°$.

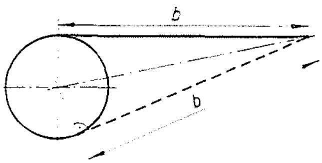

Fig. 6

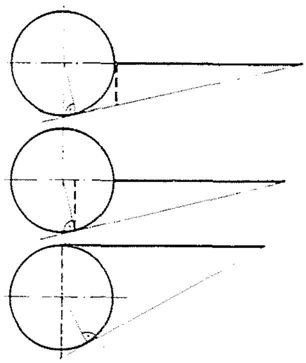

Fig. 7

For greater accuracy

a) the "t" value has been changed by 10 degrees;

b) the calculations were performed by means of a computer, at an accuracy of at least five decimals;

c) planimetry was carried out on coordinate paper.

Owing to its length, we shall not give the calculation in detail here. The results yielded have been compiled in Table I. and plotted in a chart.

Fig. 4 shows values of "Φ_{h-s}" as the function of "d", "b" and "ε" while Fig. 5 indicates the value of the product of "$F_h \; \Phi_{h-s}$" as the function of "ε", with the parameters "d" and "b".

Neither must we omit to mention in what manner the results and the correctness of the derivation of the relationship can be checked. Checking can be performed by simple means, on the following considerations:

At an arbitrary "d" and "b" but considerable "a" value, let us assume the case where $\varepsilon = +90°$ viz. the plane touches on the cylinder surface, although its elongation does not intersect it. A plane surface of the same dimension may be placed symmetrically on the other side of the cylinder in such a way that the two planes touch along a straight line. This arrangement, in a section normal to the cylinder, is illustrated in Fig. 6.

Since the shape factor indicates which part of the total radiation from one surface into the hemispheral space reaches the other, we may safely say in the case in question, that from the cylinder surface symmetrically located between the two planes, the bulk of the radiated heat acts on the planes and only a negligably small portion of it leaves through the gap at the pipe ends.

This means that the value of the shape factor between the cylinder surface and the two planes is approximately equal to unit. The two planes being symmetrical, the shape factor between one of the planes and the cylinder surface is equal to about half the unit. The correctness of this result has been accurately verified also by the above described calculations process.

Finally, reverting to the application of the (1) relationship, we wish to add that the average radiation value between the screen and the heating pipe, being in possession of the shape factor between them, can be readily calculated by means of the following relationship:

$$F_h \Phi_{h-s} = F_s \Phi_{s-h} \tag{7}$$

Since our original purpose was to demonstrate a calculation method to establish the shape factor between a cylindrical surface and a plane in contact with it — for which there are no data available in the technical literature — we shall not go into any more details in discussing the (1) relationship.

4. Approximations in the calculation of the shape factor

The idea might obviously arise that the neglections permissible with the (5) relationship considerably simplifies the calculation and substantially saves in the computfair accuracy on the basis of relatively simple relations and little work, these kinds were examined. The conditions of three approximations have been illustrated in Fig. 7: with the cylindrical surface substituted by planes normal to each other, or easily calculable as to location and size.

The calculations of all three cases have without a doubt shown that the assumed normal plane is not equivalent to the cylindrical surface it is destined to substitute, because the values of the two shape factors differ substantially. This fact has verified — and this follows unambiguously from the (2) basic relationship referring to the shape factor — that in radiative heat exchange no spatial surface can be substituted by one of its planar projections chosen at random, but only by a particular one of them. However, the determination of the location of this particular surface — as was also in the discussed case — is lengthy and creates considerable difficulties, consequently, from the practical point of wiev it is still cumbersome.

Summary

To determine the radiative heat exchange taking place between the heating pipes and the screen plate of a radiant screen, the shape factor must be known. The shape factor in technical literature can be found for specific cases only, these having no relations to radiant screens. The paper presents a relationship for the calculation of the general layout of the heating pipe and the screen, and gives the numerical value of the shape factor for the most frequently applied structural dimensions.

Literature

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