EXAMINATION OF FRACTIONAL LOAD CONDITIONS IN GAS TURBINES BY MEANS OF LINEARIZATION

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Symbols

G = air intake

π_k = pressure ratio of compressor

δ_t = pressure ratio of turbine

T = temperature before the turbine and the compressor, respectively

η_k; η_t = compressor efficiency; turbine efficiency

σ = factor of pressure loss $\left(\sigma = \frac{p - \Delta p}{p}\right)$

AL_k; AL_t = compressor specific work; turbine specific work

index "o" = characteristics of the given starting point of process

$\overline{G} = \frac{G}{G_0}$ = ratio of characteristics for an arbitrary point of the process, but one sufficiently near to the starting point to the characteristics of the starting point

$\overline{\Delta G} = \frac{\Delta G}{G_0}$ = ratio of difference between the characteristics of an arbitrary point of the process but one sufficiently near to the starting point and those of the starting point of the process to characteristics of the starting point of the process. This definition also refers to the other characteristics of the gas turbine

c_{pl}; c_{pg} = specific heat of air and gas, respectively

$a = \frac{\varkappa_l - 1}{\varkappa_l}$; $b = \frac{\varkappa_g - 1}{\varkappa_g}$ = expression formed from the adiabatic exponents of air and gas, respectively

1. Specification of the problem

The accurate examination of the fractional loads in gas turbines necessitates a lengthy and difficult computation work. When designing gas turbines and determining their operations as well as their trends of development, it is often necessary to have an approximately realistic view, obtained by means of relatively simple calculation methods, of the fractional load conditions of gas turbines having different work cycles and structure — above all in order to be able to compare them with one another.

For the investigation of the technical-physical phenomena often a method is employed with good results when the mathematical expression describing a given process can be converted to a difference equation. The calculation for applying the difference equation assumes that the function between two points is sufficiently near to one another of the described function that the process can be substituted by a straight. This approximation is entirely sufficient when examining the fractional load conditions in gas turbines as in this case the exact and particular data of the gas turbines are not known, and the application of efficiencies and factors of the individual machine components is effected only on the basis of data drawn from similar machines.

The present study deals with the application of such a computation method for the above problems, as well as with the solution of a concrete problem.

2. Fundamental connections of the procedure

2.1 Difference equation of specific work consumption of the adiabatic compression

The adiabatic specific work consumption of the compressor at a point near to the initial (known) state is given by the formula:

$$AL_k = c_{pl}\, T(\pi_k^a - 1)\,. \tag{2.1}$$

The division of the expression (2.1) by the expression of the initial (known) state gives the formula

$$\overline{AL}_k = \frac{AL_k}{AL_{k0}} = \frac{c_{pl}}{c_{pl}} \cdot \frac{T}{T_0} \cdot \frac{\pi_k^a - 1}{\pi_{k0}^a - 1} = \frac{\pi_k^a - 1}{\pi_{k0}^a - 1} = \frac{\overline{\pi}_k^a \cdot \pi_{k0}^a - 1}{\pi_{k0}^a - 1} \tag{2.2}$$

$\frac{T_0}{T} = \overline{T} = 1$, since at a fractional load the possible change of the temperature of the induced air as well as that of the specific heat under compression is negligible.

As the value of $\overline{AL}_k$ is only function of $\overline{\pi}_k$, therefore, the change of the specific work of compression is expressed by the complete differential:

$$\begin{aligned} \mathrm{d}\overline{AL}_k &= \frac{\partial \overline{AL}_k}{\partial \overline{\pi}_k} \cdot \mathrm{d}\overline{\pi}_k = \frac{\partial}{\partial \overline{\pi}_k}\left(\frac{\overline{\pi}_k^a \cdot \pi_{k0}^a - 1}{\pi_{k0}^a - 1}\right) \cdot \mathrm{d}\overline{\pi}_k = \\ &= \frac{a\overline{\pi}_k^{a-1} \cdot \pi_{k0}^a}{\pi_k^a - 1} \cdot \mathrm{d}\overline{\pi}_k\,. \end{aligned} \tag{2.3}$$

At the computed point $\overline{\pi}_k^{a-1} = 1$, therefore

$$\frac{\partial \overline{AL}_k}{\partial \overline{\pi}_k} = \frac{a\pi_{k0}^a}{\pi_{k0}^a - 1}\,. \tag{2.4}$$

When resubstituting the expression (2.4) to that of (2.3)

$$\mathrm{d}\,\bar{AL}_k = \frac{a\cdot\pi_{k0}^a}{\pi_{k0}^a - 1}\cdot \mathrm{d}\,\bar{\pi}_k\,.$$

Substituting the differential equation by difference equation, the formula is as follows

$$\Delta\bar{AL}_k = \frac{a\,\pi_{k0}^a}{\pi_{k0}^a - 1}\cdot\Delta\bar{\pi}_k = k\cdot\Delta\bar{\pi}_k\,. \tag{2.5}$$

The value k can be computed from the initial characteristics of the process.

2.2 *Specific work of the adiabatic expansion*

Specific work of the turbine:

$$AL_t = c_{pg}\,T\left(1 - \frac{1}{\delta_t^b}\right) = c_{pg}\,T\left(\frac{\delta_t^b - 1}{\delta_t^b}\right). \tag{2.6}$$

On applying the procedure in the same way as in case of compression, but taking into account the possibility of the change of temperature before the turbine, the result is

$$\overline{AL_t} = \frac{AL_t}{AL_{t0}} = \frac{T}{T_0}\left(\frac{\delta_t^b - 1}{\delta_t^b}\right)\cdot\left(\frac{\delta_{t_0}^b}{\delta_0^b - 1}\right) = \overline{T}\left(\frac{\delta_t^b\cdot\delta_{t_0}^b - \delta_{t_0}^b}{\delta_t^b\cdot\delta_{t_0}^b - \delta_t^b}\right).$$

By dividing the preceding equation to the end with the value of $\delta_{t_0}^b$:

$$\overline{AL_t} = \overline{T}\cdot\frac{\bar{\delta}_t^b\cdot\delta_{t_0}^b - 1}{\bar{\delta}_t^b(\delta_{t_0}^b - 1)}\,. \tag{2.7}$$

After forming the total differential of the expression $\overline{AL_t}$ and taking into consideration that $\overline{AL_t}$ is a function both of $\overline{T}$ and $\bar{\delta}_t$

$$d\,\overline{AL_t} = \frac{\partial\,\overline{AL_t}}{\partial\,\overline{T}}\cdot \mathrm{dT} + \frac{\partial\,\overline{AL_t}}{\partial\,\bar{\delta}_t}\cdot d\,\bar{\delta}_t\,. \tag{2.8}$$

Having determined the differential of equation (2.8)

$$\frac{\partial\,\overline{AL_t}}{\partial\overline{T}} = \frac{\bar{\delta}_t^b\cdot\delta_{t_0}^b - 1}{\bar{\delta}_t^b(\delta_{t_0}^b - 1)}$$

$$\frac{\partial\,\overline{AL}_t}{\partial\,\bar{\delta}_t} = \overline{T}\cdot\frac{b}{\bar{\delta}_t^{b+1}\,(\delta_{t0}^b - 1)}\,.$$

Being at the computed point $\overline{T} = \bar{\delta}_t = 1$, consequently

$$\frac{\partial\,\overline{AL}_t}{\partial\,\overline{T}} = 1\,;\,\frac{\partial\,\overline{ALt}}{\partial\,\bar{\delta}_t} = \frac{b}{\delta_{t_0}^b - 1} = \mathrm{t}\,. \tag{2.9}$$

When substituting the expressions (2.9) into (2.8)

$$\mathrm{d}\,A\bar{L}_t = \mathrm{d}\overline{T} + t\cdot\mathrm{d}\,\bar{\delta}_t\,.$$

After having converted the difference equation

$$\Delta\,A\bar{L}_t = \Delta\overline{T} + t\Delta\,\bar{\delta}_t\,. \tag{2.10}$$

The value of the constant t can be determined from the initial (calculated) characteristics.

2.3 *Change of the quantity of medium passing through the gas turbine*

According to the elliptic equation valid for the multi-stage turbine

$$p_1 = \sqrt{\overline{G}^2\cdot\overline{T}(p_{10}^2 - p_{20}^2) + p_2^2}\,, \tag{2.11}$$

where

$p_{10};\ p_{20}$ = pressure before and after the turbine at the computed running state,

$p_1;\ p_2$ = pressure before and after the turbine in the operational state of fractional load.

Having divided the equation (2.11) to the end by the expression $\dfrac{p_{20}}{p_{10}\,p_2} = \dfrac{1}{\delta_{t0}^p\,p_2}$, the following, already dimensionless equation is given:

$$\bar{\delta}_t = \sqrt{\overline{G}^2\,\overline{T}\,\frac{1}{\bar{p}_2^2}\left(1 - \frac{1}{\bar{\delta}_{t0}^2}\right) + \frac{1}{\delta_{t0}^2}}\,, \tag{2.12}$$

where in accordance with the signs employed above

$$\overline{G} = G/G_0;\ \bar{\delta}_t = \frac{\delta_t}{\delta_{t0}}\,; \qquad \overline{T} = \frac{T}{T_0}\,; \quad \bar{p}_2 = \frac{p_2}{p_{20}}\,.$$

From equation (2.12) it can be seen that $\bar{\delta}_t$ can be expressed by the following function:

$$\bar{\delta}_t = f(\bar{G};\ \bar{T};\ \bar{p}_2)\,. \tag{2.13}$$

In order to determine the difference equation of the change of the quantity of medium passing through the turbine, let us form the complete differential of equation (2.13)

$$\mathrm{d}\bar{\delta}_t = \frac{\partial \bar{\delta}_t}{\partial \bar{G}} \cdot \mathrm{d}\bar{G} + \frac{\partial \bar{\delta}_t}{\partial \bar{T}} \cdot \mathrm{d}\bar{T} + \frac{\partial \bar{\delta}_t}{\partial \bar{p}_2}\, \mathrm{d}\bar{p}_2\,. \tag{2.14}$$

The partial differential quotients of equation (2.14) can be obtained by means of the differential quotients drawn from equation (2.12) according $\bar{G}$; $\bar{T}$ and $\bar{p}_2$

$$\frac{\partial \bar{\delta}_t}{\partial \bar{G}} = \frac{\bar{G}\,\bar{T}\,\frac{1}{\bar{p}_2^2}\,(1 - \frac{1}{\delta_0^2})}{\sqrt{\bar{G}^2\,\bar{T}\,\frac{1}{\bar{p}_2^2}\,(1 - \frac{1}{\delta_{t0}^2}) + \frac{1}{\delta_{t0}^2}}}\,. \tag{2.15}$$

At the computed and sufficiently approximate operational state of fractional load is $\bar{G} = \bar{T} = \bar{p}_2 = 1$. Taking this into account, equation (2.15) will be converted as follows:

$$\frac{\partial \bar{\delta}_t}{\partial \bar{G}} = 1 - \frac{1}{\delta_{t0}^2}\,. \tag{2.16}$$

In the same way the values of the other partial differential quotients can also be reckoned:

$$\frac{\partial \bar{\delta}_t}{\partial \bar{T}} = 0.5\left(1 - \frac{1}{\delta_{t0}^2}\right) \tag{2.17}$$

$$\frac{\partial \bar{\delta}_t}{\partial \bar{p}_2} = -\left(1 - \frac{1}{\delta_{t0}^2}\right). \tag{2.18}$$

After substituting the relations (2.16); (2.17); (2.18) into (2.14), the formula is as follows

$$\mathrm{d}\,\bar{\delta}_t = \left(1 - \frac{1}{\delta_{t_0}^2}\right) \cdot \left(\mathrm{d}\bar{G} + 0{,}5\,\mathrm{d}\bar{T} - \mathrm{d}\,\bar{p}_2\right).$$

When passing over to a change of operating state of finite value

$$\Delta\,\bar{\delta}_t = g\,(\Delta\,\bar{G} + 0{,}5 \cdot \Delta\,\bar{T} - \Delta\,\bar{p}^2)\,, \tag{2.19}$$

where $g = \left(1 - \frac{1}{\delta_0^2}\right)$ is the constant to be determined from the initial state.

The relation (2.19) represents the difference equation of the change of the quantity of working medium passing through the turbine.

2.4. *Equilibrium equation of coaxial turbine and compressor*

Let us establish the difference equation comprising the change of state of equilibrium of coaxial turbine and compressor by means of the difference equations concerning the variation of expansion and compression work.

In a state of equilibrium the performance of the turbine is equal to the assumed performance of the compressor

$$\frac{\overline{AL_k}}{\overline{\eta}_k} = \overline{AL_T}\,\overline{\eta}_T; \quad \overline{AL_k} = \overline{AL_T} \cdot \overline{\eta}_{\ddot{o}}\,, \tag{2.20}$$

where

$$\overline{\eta}_k\,\overline{\eta}_T = \overline{\eta}_{\ddot{o}}\,.$$

Having represented the complete differential of the expression $\overline{AL_k}$ (considering that $\overline{AL_k} = f/A\overline{T}_T : \overline{\eta}_{\ddot{o}})$

$$\mathrm{d}\,\overline{AL_k} = \frac{\partial\,\overline{AL_k}}{\partial\,\overline{AL_T}} \cdot \mathrm{d}\,\overline{AL_T} + \frac{\partial\,\overline{AL_k}}{\partial\,\overline{\eta}_{\ddot{o}}}\,\mathrm{d}\overline{\eta}_{\ddot{o}}\,. \tag{2.21}$$

The prescribed differential quotients should be determined, then

$$\frac{\partial\,\overline{AL_k}}{\partial\overline{AL_T}} = \overline{\eta}_{\ddot{o}}; \qquad \frac{\partial AL_k}{\partial\overline{\eta}_{\ddot{o}}} = \overline{AL_T}$$

at the computed point:

$$\frac{\partial\,\overline{AL_k}}{\partial\overline{AL_T}} = 1; \qquad \frac{\partial\overline{AL_k}}{\partial\overline{\eta}_{\ddot{o}}} = 1\,. \tag{2.22}$$

On substituting the relations (2.22) into (2.21)

$$\mathrm{d}\,\overline{AL_k} = \mathrm{d}\,\overline{AL_T} + \mathrm{d}\,\overline{\eta}_{\ddot{o}}\,. \tag{2.23}$$

When converting to difference equation ($\overline{\eta}_{\ddot{o}}$ being invariable):

$$\Delta\,\overline{AL_k} = \Delta\,\overline{AL_T}\,. \tag{2.24}$$

The equations (2.5) and (2.10) are to be substituted into (2.24):

$$k\,\Delta\bar{\pi}_k = \Delta\bar{T} + t\cdot\Delta\bar{\delta}_t$$

or

$$\Delta\bar{\pi}_k = \frac{1}{k}\,(\Delta\bar{T} + t\Delta\bar{\delta}_t)\,. \tag{2.25}$$

The equation (2.25) gives a relation between the three most important characteristics, $\Delta\bar{\pi}_k$, $\Delta\bar{T}$ and $\Delta\delta_t$ of the state of equilibrium of the coaxial compressor and turbine.

2.5. Behaviour of series-connected turbines under fractional load

When there are two turbines series-connected without intermediate firing, the relation of the degrees of heat of the incoming gas into the turbines is given by the formula:

$$T_2 = T_1\frac{1}{\delta_1^b}\,, \tag{2.26}$$

where

T_1; T_2 represents the temperature before the turbines (indexes according to the direction of stream)

δ_1 pressure ratio of the first (high-pressure) turbine.

When applying a dimensionless figure

$$\bar{T}_2 = \bar{T}_1\cdot\frac{1}{\bar{\delta}_1^b}\,.$$

Entirely equal to the method outlined until now $[\bar{T}_2 = f(\bar{T}_1;\ \bar{\delta}_1)]$:

$$\partial\bar{T}_2 = \frac{\partial\bar{T}_2}{\partial\bar{T}_1}\,\mathrm{d}\bar{T}_1 + \frac{\partial\bar{T}_2}{\partial\bar{\delta}_1}\,\mathrm{d}\bar{\delta}_1\,. \tag{2.27}$$

After having set up the differential quotients of equation (2.26), and taking the conditions ($\bar{\delta}_1 = 1$; $\bar{T}_1 = 1$) of the operating point into consideration:

$$\frac{\partial\bar{T}_2}{\partial\bar{T}_1} = 1; \qquad \frac{\partial\bar{T}_2}{\partial\bar{\delta}_1} = -\,b\,. \tag{2.28}$$

The substitution of the relations (2.28) into (2.27) gives the formula

$$\partial\bar{T}_2 = \mathrm{d}\bar{T}_1 - b\,\mathrm{d}\bar{\delta}_1\,.$$

Passing over to difference equation, this results in the following formula

$$\Delta \overline{T}_2 = \Delta \overline{T}_1 - b \Delta \bar{\delta}_1 . \tag{2.29}$$

Taking into consideration that the value $b = \frac{k_g - 1}{k_g} = 0.248$ is essentially a smaller number than the unit, this can be set up with a good approximation:

$$\Delta \overline{T}_2 = \Delta \overline{T}_1 .$$

2.6. Equilibrium equation of the complete gas turbine system

In the state of equilibrium ($\Delta\omega = 0$) of the gas turbine the pressure increase produced in the compressors will be consumed, on the one hand, in the turbines, on the other hand, it covers the pressure losses of the system:

$$\delta = \sigma \cdot \pi \tag{2.30}$$

where δ = complete pressure ration of turbines,
π = complete pressure ration of compressors,
σ = pressure loss factor of complete gas turbine system.

When applying a method identical with the above procedure

$$\Delta \bar{\pi} = \Delta \bar{\delta} + \Delta \bar{\sigma} .$$

In case of fractional load calculations $\bar{\sigma}$ = permissibly stands beside ($\Delta \bar{\sigma} = 0$)

$$\Delta \bar{\pi} = \Delta \bar{\delta} \tag{2.31}$$

2.7. Determination of the fractional load performance

The performance can be established as the product of the heat drop in the working turbine and of the gas quantity passing through it:

$$\overline{N} = \overline{AL}_T \cdot \bar{G} .$$

As a result of the method identical with the preceding steps:

$$\Delta \overline{N} = \overline{AL}_T + \Delta \bar{G} . \tag{2.32}$$

3. Application of the method

It is the fractional load operating state of a gas turbine having a given initial characteristic that should be determined by means of the method outlined above.

Thermic characteristics of the nominal operating state of gas turbine:

$\pi_{k0} = 6.25$; pressure ratio of gas turbine compressor
$\delta^I_{t0} = 3.32$; pressure ratio of turbine driving the compressor
$\delta^{II}_{t0} = 1.593$: pressure ratio of working turbine
$G_0 = 2.21$ kg/sec; air intake of gas turbine
$a = 0.286$
$b = 0.248$
$T_0 = 1073$ K^0; maximum temperature of gas turbine.

The switching diagram of the gas turbine is shown in Fig. 1. The gas turbine is performing a two-adiabatic work cycle without any heat exchanger, this being equipped with a separate working turbine; the working turbine runs at the low-pressure part of expansion.

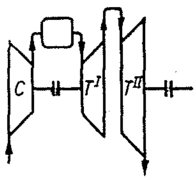

Fig. 1

To determine the fractional load operating state of the gas turbine, the following equations are to be set up:

The equilibrium equation of the quantity of medium passing through the turbine which drives the compressor can be established by means of the relation (2.19):

$$\Delta \bar{\delta}^I_t = g^I (\Delta \bar{G} + 0,5 \Delta \bar{T} - \Delta \bar{p}^I_2) .$$

As the pressure after the working turbine is invariable (atmospheric), therefore the change of the counter-pressure in the turbine driving the compressor determines the pressure ratio of the working turbine, thus:

$$\Delta \bar{\delta}^{II}_t = \Delta \bar{p}^I_2 .$$

The equilibrium equation of the quantity of medium passing through the working turbine is given by the formula:

$$\Delta \bar{\delta}^{II}_t = g^{II} \cdot (\Delta \bar{G} + 0.5 \Delta \bar{T}^{II}) .$$

It can be set up on the basis of equation (2.29) with good approximation, that:

$$\Delta \bar{T}^I = \Delta \bar{T}^{II} = \Delta \bar{T} .$$

Equilibrium equation of the compressor and of the turbine driving the compressor according to the relation (2.25)

$$\Delta \bar{\pi}_k = \frac{1}{k} (t^I \Delta \bar{\delta}_t^I + \Delta \bar{T}).$$

Equilibrium equation of the complete gas turbine system according to (2.31):

$$\Delta \bar{\pi}_k = \Delta \bar{\delta}_t^I + \Delta \delta_t^{II} .$$

Changing the values of the constants figuring in equations (g^I, g^{II}, k, t^I) is shown in Fig. 2 as a function of the characteristics determining the constants. Determination of the constants is to be achieved according to the following relations:

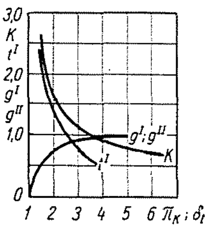

Fig. 2

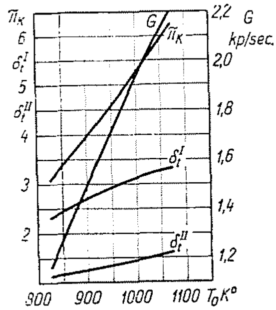

Fig. 3

$$g^I = 1 - \left(\frac{1}{\delta_t^I}\right)^2 ; \qquad g^{II} = 1 - \left(\frac{1}{\delta_t^{II}}\right)^2 ;$$

$$k = \frac{a \pi_k^a}{\pi_k^a - 1} ; \qquad t^I = \frac{b}{(\delta_t^I)^b = 1} .$$

After having performed the indicated operations, the function $\Delta\bar{G}/\Delta\bar{T}$ which determines the fractional load operating state of the gas turbine principally, will be constituted as follows:

$$\frac{\Delta \bar{G}}{\Delta \bar{T}} = \frac{\dfrac{0{,}5\, t^I g^I}{K} + \dfrac{0{,}5\, t^I g^I g^{II}}{K} + \dfrac{1}{K} - 0{,}5\, g^I - 0{,}5\, g^I g^{II} - 0{,}5\, g^{II}}{g^I - g^I g^{II} + g^{II} - \dfrac{t^I g^I}{K} + \dfrac{t^I g^I g^{II}}{K}} .$$

In this equation there only the constants that can be determined from the initial conditions occur. By means of the above relations, starting from the characteristics determining the nominal operating state and assuming the maximum heat drop as $\Delta T = 50°$ C, the fractional load thermic characteristics which belong to $T_0' = 1023°$ K can be determined (Fig. 3). Considering the values thus obtained as nominal ones as the characteristics of the fractional load operating state, but in this case those related to $T_0'' = 973°$ K can again be determined.

4. Accuracy of method

In order to show graphically the error made in the course of computations and the method, respectively, it is Fig. 4 which shows the change of the specific work of adiabatic expansion as a fuction of the pressure ratio both by means of equation (2.6) assuring an exact solution and the approximative difference equation (2.10). In the figure the fundamental conception of the method is clearly to be seen, that is, the original function will be approximated by a fraction line system made up of the tangents drawn at the given points of the function; in such a manner that the original function and the value of the ordinates of the fraction line system approximating that should coincide with each other at a given value of the independent variable.

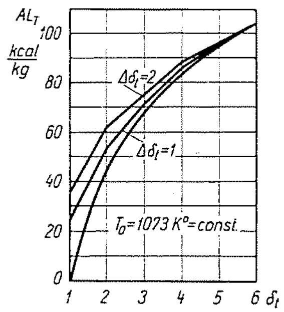

Fig. 4

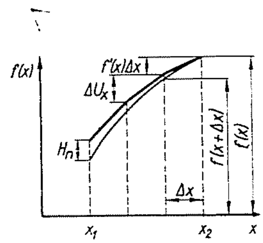

Fig. 5

From the system of approximation the results are that in the case a function has no point of inflexion in the given examined section which should be approximated, the made error will increase when moving away from the point of coincidence, however, on augmenting the number of the approximative line parts, the made error can be kept, in principle, within arbitrary limits. Obviously the problem is, how to divide the investigated interval, in order to establish a difference equation within the given limit of error, into partial intervals equal to each other.

Let us approximate (Fig. 5) the examined function $f(x)$ in the interval $(x_2 - x_1)$ with difference equation and check, into how many partial intervals

$\Delta x = \dfrac{(x_2 - x_1)}{n}$ the interval $(x_2 - x_1)$ should be divided while observing a given limit of error. Consequently, determining the gravity of the made error if the approximation $f(x + n \Delta x) \sim f(x) + f'(x) \Delta x + f'(x + \Delta x) \Delta x + \ldots$ $\ldots + f' [x + (n-1) \Delta x] \Delta x$ is applied.

The made error is:

$$H_n = f(x + n \Delta x) - f(x) - f'(x) \cdot \Delta x - \ldots f' [x + (n-1) \Delta x] \Delta x . \quad (4.1)$$

After a simple conversion of the right-hand side:

$$H_n = \sum_{i=0}^{n-1} \{f[x + (i+1) \Delta x] - f(x + i\Delta x) - f'(x + i\Delta x) \Delta x\} . \quad (4.2)$$

Having ranged the function $f(x)$ into Taylor's series and applied Lagrange's residual member, the formula will be

$$f(x + \Delta x) = f(x) + f'(x) \Delta x + \frac{f''(\varrho_i)}{2!} \Delta x^2$$

that is

$$f(x + \Delta x) - f(x) = f'(x) \Delta x + \frac{f''(\varrho_i)}{2!} \Delta x^2 , \quad (4.3)$$

where ϱ_i is an arbitrary point of the interval. When achieving the substitution

$$f(x + i\Delta x) - f[x + (i-1) \Delta x] = f'[x + (i-1)\Delta x] \Delta x + \frac{f''(\varrho_i)}{2!} \Delta x^2$$

at the right-hand side of the relation (4.2) on the basis of (4.3), then the value of H_n amounts to

$$H_n = \sum_{i=1}^{n} \frac{f''(\varrho_i)}{2!} \Delta x^2 < \frac{nM}{2!} \Delta x^2 = \frac{(x_2 - x_1)^2}{n} M , \quad (4.4)$$

where $M = f''(\varrho_i) = \max f''(x)$ in the examined interval, $x_2 - x_1$ represents the length of the interval and $\dfrac{x_2 - x_1}{n} = \Delta x$.

In expression (4.4) it can be seen that the value of the error decreases when increasing the number of the partial intervals, and it will become zero if $n = \infty$.

In practice, in most cases it is the relative value error but not the absolute one which is needed:

$$H_n\,\% = \frac{H_n}{f(x_1)}, \tag{4.5}$$

where $f(x_1)$ represents the function or its approximative value at the point x_1.

In applying the method, the exact course of the process is not known only its approximation, thus, the determination of the value of $f''(x)$ cannot be otherwise carried out, but with approximation in the following way:

When comparing Newton's interpolation formula with the figure ranged in Taylor's series of the same function, relations can be gained by means of which the differential quotients are to be determined from the differences:

$$f'(x) = \frac{1}{\Delta x}\left(\Delta U_x - \frac{\Delta^2 U_x}{2} + \frac{\Delta^3 U_x}{3} - \frac{\Delta^4 U_x}{4} + \dots\right) \tag{4.6}$$

$$f''(x) = \frac{1}{\Delta x^2}\left(\Delta^2 U_x - \Delta^3 U_x + \frac{11}{12}\Delta^4 U_x - \frac{5}{6}\Delta^5 U_x + \dots\right), \tag{4.7}$$

where $\Delta^n U_x$ (Fig. 5) represents the difference of the n-th order belonging to Δx at the given abscissa x. Among the values $f''(x)$ pertaining to different abscisse of the given complete interval $(x_2 - x_1)$ the maximum approximative value of $f''(x)$ can be selected directly.

The procedure for determining the error of the method outlined in this study is as follows:

Considering one of the characteristics determining the fractional load operating state of the gas turbine (in the present study ΔT) as an independent variable, the characteristic curves for fractional loads of the gas turbine can be determined as a function of ΔT. By means of forming the differences of characteristic curves, the max $f''(x)$ belonging to the complete interval can be determined, next the value of the error pertaining to ΔT can be assumed in advance. After having determined the value of the error and comparing with the permissible value, respectively, a possibility arises to correct the value of ΔT which was assumed in advance.

It is obvious that with a view to limiting error of the various dependent variables there are independent variables with different partial intervals which occur, and of these it is the independent variable with the least partial interval that has to form the basis of all approximative calculations.

Consequently, the correctness of the assumption of partial intervals for the independent variables of the process can be checked, and the value belonging to a given limit of error ΔT determined, by applying this method.

Summary

The study deals with a possible method for examining the fractional loads of gas turbines approximately. The main point of this method is that the fundamental equations to be applied for the fractional load calculations can be converted into differential, then into difference equations. Thus, the functions given in an exact way for fractional loads can be approximated by a fraction line system consisting of straight sections, then by establishing difference equations when a point of the operating state of the gas turbine (as a rule, the maximum operating state) is known, the characteristic curves for fractional loads of the gas turbine with a given working cycle can be determined. When applying the method exposed in this study and having approximative difference equations, the error made at the approximation can be specified. The more accurate the calculation is, the nearer the computed point of the rate operating state at fractional loads comes to the known starting data.

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