THE FIN EFFICIENCY OF THE FORGÓ-TYPE SLOTTED-RIB HEAT EXCHANGER

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The Forgó-type heat exchangers play an important role in Heller's air-cooled condensing system which is rapidly gaining ground all over the world. The Forgó heat exchangers essentially embody plate fins densely intersected in the direction of flow. Thanks to their design characteristics they ensure good heat transfer coefficients at relatively small pressure gradient, while their material — pure aluminium — makes for very good economy.

In view of the vast material requirements, the accurate dimensioning of the heat exchangers is of extraordinary significance (we mention for the sake of information that the frontal area of the heat exchanger elements incorporated in the power station built by the English Electric Company in Rugeley, of which fifty per cent was manufactured in Hungary, approximates 3600 sq. m.).

The Forgó heat exchanger may be termed a specially designed finned-tube water cooler.

From the technical literature it is known that due to the fact that the surface temperature of the fin extending from the tube into the air stream is not identical with the temperature of the tube, in computing finned tubes the air-side heat transfer coefficient is multiplied by ε-times the difference between the temperature of air and the temperature of the tube wall (where ε denotes the fin efficiency).

Accordingly, the calculation of fin efficiency calls for a study of fin temperature conditions.

The calculation of plate-fin heat exchangers with considerable dimensions in flow direction — to which Forgó's heat exchangers also belong — poses new problems, hitherto not dealt with, dissimilar to the calculation methods known from the relative literature. These calculations, namely, do not take into consideration that the air flowing adjacent to the fin base will warm up (or cool down) to a greater degree than the air streaming near midfin, on the one hand, and that in many cases heat conductance will take place in the fin not only normal to, but also parallel with the flow, on the other.

This problem had already been treated in its generic aspects [1], the present treatise — mainly on the basis of the results arrived at in the quoted paper — aims at drawing conclusions concerning the fin efficiency of the Forgó-type elements.[1]

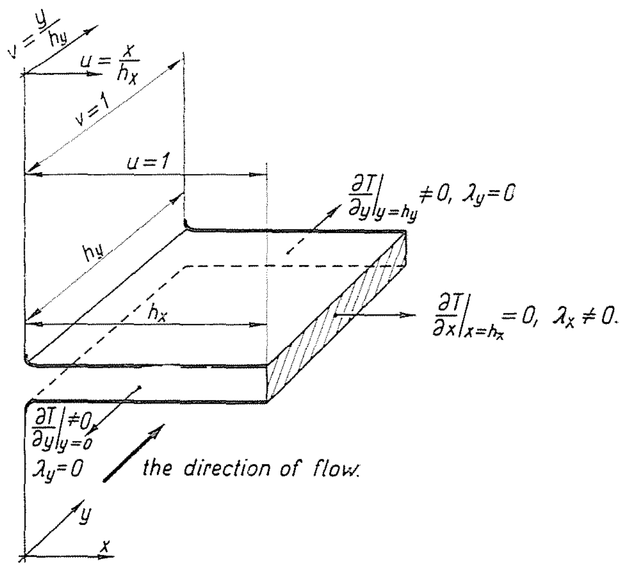

Fig. 1. Plate-fin dimensions

In the quoted paper [1] it was proved that the system of differential equations pertaining to the problem in hand[2] is as follows:

$$\Phi - \Phi_K = A_v \frac{\partial^2 \Phi}{\partial v^2} + A_u \frac{\partial^2 \Phi}{\partial u^2}$$

and (1)

$$\Phi - \Phi_K = C \frac{\partial \bar{\Phi}_K}{\partial v} .$$

Since the slotting of the fins practically checks heat conductance in the direction of flow, i.e. $A_v = 0$, in the case dealt with the equation will assume the following simplified form:

$$\Phi - \Phi_K = A_u \frac{\partial^2 \Phi}{\partial u^2}$$

[1] Our conclusions are naturally also applicable to other heat exchangers operating on similar principles.

[2] See SYMBOLS at the end of paper.

and (2)

$$\Phi - \Phi_K = C \frac{\partial \Phi_K}{\partial v} .$$

In our previous paper [1] the sequence followed inthe development of the (1) differential equation system, in its more generic form, was given. It will suffice to state here as much of it as is indispensable for the investigation of the more specific, (2) equation system.

In the previous paper [1] we also demonstrated that by means of the operational calculus — according to Mikusinsky's thesis[2], the (1) system of equations can be reduced to the following simple differential equation form:

$$\bar{\Phi}'' - \frac{A_v}{A_u}\left(\frac{1}{A_v}\,\frac{Cs}{1+Cs} - s^2\right) = \frac{A_v}{A_u} sf_u - \frac{1}{A_u}\,\frac{f_K}{s+1/C} . \qquad (3)$$

The evolution subsequently proceeded in the following manner: first it was proved that the pertinent homogeneous equation has no operator solution; then the particular solution to the inhomogeneous equation which suits the actual boundary conditions was sought for.

The boundary conditions were as follows:

A given constant fin temperature in the fin base ($x = 0$);

zero temperature gradient along the bounding straights of the fin ($x = 0$, $x = 1$, and $y = 1$); finally

a homogeneous temperature distribution of the entering air (along the $y = 0$ straight).

Proceeding according to the above outlined method, the solution was found to be an infinite series.

In the treated case — that is, if $A_v = 0$ — the following difficulties arise in the application of the solution derived:

As had already been stated in the first paper[1], the homogeneous equation pertaining to equation (3) has no solution unless $A_v \neq 0$; while in case $A_v = 0$, just on ground of the given evolution, it is self-evident that the differential equation does have a resultant operator. In this event the generic solution of the inhomogeneous equation will present itself as the sum of the solution of the particular and homogeneous equations, and not as the solution of the particular equation alone.

The second difficulty will be encountered in connection with boundary conditions. If, namely, fin conductance in the direction of v is 0, then the boundary condition stipulating that no heat flux is to take place in the outward direction along the fin edges may be fulfilled not only at 0, but also at optional temperature gradient in v direction. Therewith, two of our boundary conditions, however, will be forfeited.

It was stated that one of the specific features of equation (2) as against the more generic (1) form was, that the solution of the corresponding homogeneous differential equation was a function. Let us write down this function:

$$\bar{\Phi}_0 = \bar{c}_1\, e^{+\sqrt{\frac{1}{A_u}\frac{Cs}{1+Cs}}\cdot u} + \bar{c}_2\, e^{-\sqrt{\frac{1}{A_u}\frac{Cs}{1+Cs}}\cdot u}, \tag{4}$$

where $\bar{c}_1$ and $\bar{c}_2$ denote the arbitrary operators of the solution.

A particular solution may also be sought for, since the solution of the generic (1) equation under the said boundary conditions (given in the previous paper) with the substitution of $A_v = 0$ will obviously yield the particular solution of (2) equation:

$$\bar{\Phi}_i = \sum_{n=1}^{\infty} \frac{2}{\omega}\, \frac{1}{1 + A_u\, \omega^2}\, \frac{1}{s + \dfrac{1}{C} \cdot \dfrac{A_u\, \omega^2}{1 + A_u\, \omega^2}} \cdot \sin \omega u . \tag{5}$$

Accordingly, the generic solution can be derived in the following form:

$$\bar{\Phi} = \bar{\Phi}_0 + \bar{\Phi}_i . \tag{6}$$

Let us now examine the boundary conditions.

Studying the (1) differential equation and its solution, respectively, — here we refer once more to the previous paper (see literature [1]) respectively, to the Appendix — it will be apparent that its simplification due to A_v being equal to 0 will at the same time do away with the property of the solution, that the temperature gradient in flow direction, $\dfrac{\partial \bar{\Phi}}{\partial v}$, along the entering and exit edge of the plate ($v = 0$ and $v = 1$) is equal to zero.

On the other hand, the fin base temperature will remain constant, the temperature of the entering air will continue to be uniform and the temperature gradient $\dfrac{\partial \Phi}{\partial u}$ at the fin-top ($u = 1$) normal to the flow will remain zero.

This thesis has been verified in the quoted paper [1] or it may be proved by substitution into the correlation of $\bar{\Phi}_i$ (equation (5)).

From what has been said above it naturally follows that $\bar{\Phi}_i$ satisfies the said boundary conditions; accordingly, the particular solution adapted to our boundary conditions may be readily derived from the generic solution by selecting 0 for $\bar{c}_1$ and $\bar{c}_2$.

Summing up what has been stated so far:

If there is no thermal conductance along the fin edges and if temperature at the fin base is constant, further if the temperature distribution of the entering air is homogeneous, the operational form of the temperature distribution of the fin is represented by equation (5). From this equation, after producing the appropriate Φ function first, the ε_L factor may be produced with the help of the correlation obtained in our previous paper[1].

The defining equation of this factor is as follows:

$$Q = 2h_x h_y \varepsilon_L \alpha \Delta T_0, \quad \text{resp.} \quad \varepsilon_L = \frac{1}{\Delta T_0} \frac{Q}{\alpha F} = \frac{\Delta T_m}{\Delta T_0} \,. \tag{7}$$

ε_L, accordingly, differs from the fin efficiency insofar as it is referred to the entering (ΔT_0) and not to the mean temperature difference.

Performing the calculations (see also the Appendix) the following relationship presents itself for ε_L:

$$\varepsilon_L = \sum_{n=1}^{\infty} \frac{2C}{\omega^2} \left(1 - e^{-\frac{1}{C} \frac{\omega^2}{\omega^2 + (mh_z)^2}} \right), \tag{8}$$

or, with slight transformation:

$$\varepsilon_L = C \left(1 - \sum_{n=1}^{\infty} \frac{2}{\omega^2} e^{-\frac{1}{C} \frac{\omega^2}{\omega^2 + (mh_z)^2}} \right). \tag{9}$$

In the relationships $\omega = \frac{2n-1}{2}\pi$ is comprised the n index.

The series is rapidly converging and yields, even without a computer, an approximate result (the establishment of three to four members will almost always suffice and one or two members will be in many cases enough to obtain an approximate result).

It will be worthwhile to calculate the fin efficiency in a usual manner also for the logarithmic mean of the exit and inlet temperature differences, because it is only this method of fin efficiency calculation that permits a comparison to be made in the classical manner with the fin efficiency value, that is, a calculation which disregards the warming up of the air.

ε_L if referred to the entering temperature difference, namely, takes into consideration that due to the warming up of the air, the difference between the temperature of the heat exchanger surface and the mean air temperature will change along the heat exchanger, this being the condition generally included in the concept of logarithmic mean temperature difference. This can be proved in an interesting manner also by the fact that if mh_x together with the fin length tends to zero, the value of ε_L will tend to the mean temperature difference of the condenser, referred to the inlet temperature difference (to the mean temperature difference of the condenser, because wall temperature is constant both at the fin base and at one side of the condenser):

$$\lim_{mh_z \to 0} \varepsilon_L = C\left(1 - e^{-\frac{1}{C}}\right)$$

and with heat exchangers

$$\varepsilon_L = \frac{\Delta T_m}{\Delta T_0},$$

if condensers are concerned:

$$\varepsilon_L = C\left(1 - e^{-\frac{1}{C}}\right).$$

Let us now compute also the usual fin efficiency. In view of the fact that in our case the temperature difference between wall (fin base) and air will change from ΔT_0 to ΔT_v (mean), the fin efficiency referred to the logarithmic mean temperature difference can be computed without any difficulty. Denoting it with ε (see Appendix):

$$\varepsilon = C \ln \frac{1}{1 - \frac{\varepsilon_L}{C}} = C \ln \frac{C}{C - \varepsilon_L} = C \ln \frac{1}{\sum_{n=1}^{\infty} \frac{2}{\omega^2} e^{-\frac{1}{C} \frac{\omega^2}{\omega^2 + (mh_z)^2}}}. \tag{10}$$

It can be proved that this relationship already satisfies the requirement rightly set to "fin efficiency", that, once the length of the fin tends to zero, fin efficiency should tend to the unit:

$$\lim_{mh_z \to 0} \varepsilon = 1 .$$

In dimensioning, instead of the former method of calculating the fin efficiency, the above demonstrated calculation might be applied in all cases where the conditions outlined in this paper prevail.

This realization is significant not only for the appropriate dimensioning of plate-fin heat exchangers, but also for the evaluation of the data obtained in the measurement of such heat exchanger surfaces. The method followed so far in determining the heat transfer factor from the measured heat quantity had been to calculate fin efficiency in the traditional manner (valid for the case, if $C = \infty$) and subsequently to generalize through the theory of similarity.

The non-consideration of this effect must have had a share in the failure of attempts at generalization. It would be interesting to investigate and follow up the effects the accurate calculation of fin efficiency has on the relationships laid down for the heat transfer coefficient of plate fin heat exchangers.

Summary

On ground of a previously published paper [1] by the same author, the present treatise deals with the fin efficiency of plate fins having negligible heat conductance in flow direction, but non-negligible warming up of the flowing medium along the plate fins.

Constant temperature of the fin base has been assumed in flow direction.

These conditions have led to the result that fin efficiency depends, to a considerable degree, on the mass rate of flow.

The results were obtained with the use of dimensionless numbers.

This realization as well as the quantitative examination of the phenomenon are important, not only for dimensioning, but also for the evaluation of the measurement results.

APPENDIX

1. *The formation of the boundary conditions and the calculation of ε_L, if $A_v = 0$*

Transforming the (1) system of differential equations in the paper marked [1] in the literature into its operational form, the $\left.\frac{\partial \Phi}{\partial v}\right|_{v=0} = 0$ boundary condition was considered with its substitution into the $s^2 \overline{\Phi} = \left\{\frac{\partial^2 \Phi}{\partial v^2}\right\} + \left.\frac{\partial \Phi}{\partial v}\right|_{v=0} + s\Phi_{v=0}$ identity which permitted the clearing of $\frac{\partial^2 \Phi}{\partial v^2}$ from the differential equation.

Since, in the present case the coefficient of $\frac{\partial^2 \Phi}{\partial v^2}$ in the differential equation is $A_v=0$, the above identity falls away and it will be impossible to consider the $\left.\frac{\partial \Phi}{\partial v}\right|_{v=0} = 0$ boundary condition with it.

Since the condition $\left.\frac{\partial \Phi}{\partial v}\right|_{v=0} = 0$ is found at no other point of the calculation, should A_v be equal to 0, the solution will not necessarily comply with this requirement.

In the quoted paper the $\left.\frac{\partial \Phi}{\partial v}\right|_{v=1} = 0$ boundary condition has also been considered in a different manner. When the solution to satisfy the stipulations for entering air and wall temperature was produced, it appeared in the following form:

$$\overline{\Phi} = \sum_{n=1}^{\infty} \frac{A_v \left(b_n s^2 + \frac{b_n}{C} s\right) - \frac{2}{\omega}}{A_v \left(s^3 + \frac{1}{C} s^2\right) - (1 + A_u \omega^2) s - \frac{A_u \omega^2}{C}} \sin \omega_u.$$

Subsequently the coefficient of the trigonometric series was transcribed into simple fractional form (as the sum of three simple fractions). Still unknown in the expression of $\overline{\Phi}$ remained b_n which was then determined in such a way that from the above series the value of $\left.\frac{\partial \Phi}{\partial v}\right|_{v=1}$ was calculated and made equal to zero.

However, from the above expression of $\overline{\Phi}$ it will be clear, that if $A_v = 0$, this is impossible, since in this case b_n will drop out. In this event the simple fractional form will also be unnecessary because the solution can be derived directly, in the following way:

$$\overline{\Phi}_l = \sum_{n=1}^{\infty} \frac{2}{\omega (1 + A_u \omega^2)} \frac{1}{s + \frac{1}{C} \frac{A_u \omega^2}{1 + A_u \omega^2}} \sin \omega u.$$

Using the symbols as were used in [1] of the literature:

$$D_{1n} = \frac{2}{\omega(1 + A_u \omega^2)} \quad \text{and} \quad \varepsilon_{1n} = -\frac{1}{C} \frac{A_u \omega^2}{1 + A_u \omega^2},$$

while the members comprising the values of D_{2n} and D_{3n} will drop out.
Substituting these into the final result of the quoted paper:

$$\varepsilon_L = A_u \sum_{n=1}^{\infty} \omega \frac{2}{\omega(1 + A_u \omega^2)} \frac{e^{-\frac{1}{C}\frac{A_u \omega^2}{1+A_u\omega^2}} - 1}{-\frac{1}{C}\frac{A_u \omega^2}{1 + A_u \omega^2}} =$$

$$= \sum_{n=1}^{\infty} \frac{2C}{\omega^2} \left(1 - e^{-\frac{1}{C}\frac{A_u \omega^2}{1+A_u \omega^2}}\right).$$

Taking into consideration that

$$A_u = \frac{1}{(mh_x)^2},$$

we arrive at equation (8):

$$\varepsilon_L = \sum_{n=1}^{\infty} \frac{2C}{\omega^2} \left(1 - e^{-\frac{1}{C}\frac{\omega^2}{\omega^2+(mh_x)^2}}\right),$$

from which equation (9) will directly follow — on ground of the well known relationship whereby:

$$\sum_{n=1}^{\infty} \frac{2}{\omega^2} = 1.$$

2. *Calculation of the fin efficiency from the value of* ε_L

Let us assume a defining equation for the fin efficiency in our case, viz. when $\Delta T_m = \Delta T_{\log}$:

$$\varepsilon = \frac{Q}{\alpha F \Delta T_{\log}}.$$

Let us express the heat quantity absorbed by one fin, partly from the warming up of the medium, partly from the heat pick-up of the fin:

$$h_x C_k (\Delta T_0 - \Delta T_v) = 2 h_x h_y \varepsilon_L \alpha \Delta T_0.$$

From the definition of C, on the other hand, it follows that:

$$C = \frac{C_k}{2\alpha h_y} = \varepsilon_L \frac{\Delta T_0}{\Delta T_0 - \Delta T_v}. \tag{12}$$

Writing the logarithmic mean temperature difference by ΔT_0 and ΔT_v:

$$\Delta T_{\log} = \frac{\Delta T_0 - \Delta T_v}{\ln \frac{\Delta T_0}{\Delta T_v}}.$$

Collating the defining equation of ε_L (see equation (7) with the definition of ε (see equation (11) and taking equation (12) into consideration, we arrive at:

$$\varepsilon = \varepsilon_L \frac{\Delta T_0}{\Delta T_{\log}} = \varepsilon_L \frac{\Delta T_0}{\Delta T_0 - \Delta T_v} \ln \frac{\Delta T_0}{\Delta T_v} = C \ln \frac{\Delta T_0}{\Delta T_v}.$$

From equation (12):

$$\frac{\Delta T_0}{\Delta T_v} = \frac{1}{1 - \varepsilon_L/C},$$

whereby the relationship between the fin efficiency and ε_L can be written as:

$$\varepsilon = C \ln \frac{1}{1 - \varepsilon_L/C} = C \ln \frac{C}{C - \varepsilon_L}.$$

SYMBOLS

f_k — $f_k = \Phi_k(u, 0)$
f_u — $f_u = \Phi(u, 0)$
h_x — Fin length measured normal to flow
h_y — Fin length measured in flow direction
m — $m = \sqrt{\frac{2a}{\lambda_x v_0}}$
n — Index
s — Differential operator
u — Dimensionless coordinate normal to flow, $u = \frac{x}{h_x}$
v — Dimensionless coordinate in flow direction, $v = \frac{y}{h_y}$
v_0 — Fin thickness
y — Coordinate in flow direction
x — Coordinate normal to flow
A_u — Dimensionless number $A_u = \lambda_x/2a \cdot v_0/h_x^2$
A_v — Dimensionless number $A_v = \lambda_y/2a \cdot v_0/h_y^2$
C — Dimensionless number $C = C_k/2ah_y$
C_k — Rate of water value for unit length of one fin in direction x. The full water rate value for each fin is: $\int_0^{h_x} C_k \, dx$
D_{1n} — Coefficient, see item [1] in the literature, equation (23)
F — The total heat transfer surface
Q — Transferred heat
T — Fin temperature
T_0 — Temperature of the fin base
T_k — Temperature of the medium
ΔT — $\Delta T = T - T_k$
ΔT_k — $\Delta T_k = T(0,0) - T_k$
ΔT_0 — $\Delta T_0 = T(0,0) - T_k(0,0)$
ΔT_f — $\Delta T_f = T(0,0) - T$
$\Delta T_{\log}$ — The logarithmic mean temperature difference between fin base and medium
ΔT_m — The mean temperature difference prevailing between fin base and medium
ΔT_v — The difference between fin base and mean temperature of the leaving medium
α — Heat transfer coefficient between fin and medium
ε_L — Dimensionless number, similar to fin efficiency
ε — Fin efficiency; see literature [1], equation (23)
λ_x, λ_y — Fin conductivity in x, respectively, y direction
Φ — Dimensionless variable, expressing the temperature changes taking place in the plate, $\Phi = \frac{\Delta T_f}{\Delta T_0}$
Φ_K — Dimensionless variable expressing the temperature changes taking place in the medium, $\Phi_K = \frac{\Delta T_K}{\Delta T_0}$
$\bar{z}$ — resp. $\{z\}$ denoting that $\bar{z}$ is an operator

NOTE with respect to dimensions:

Since physical equations were applied throughout, any one consistent system of dimensions may be applied.

Literature

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