PLATE FIN EFFICIENCY. THE TEMPERATURE OF THE FIN BASE VARYING IN FLOW DIRECTION

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In two previous papers author [1], [2] had dealt with the calculation of plate fin efficiency at constant fin base temperature, partly considering plate conductance in either direction, and partly — in connection with Forgó's heat exchanger — similarly at constant fin base temperature, however neglecting fin conductance in the direction of flow.

In the present paper, also in connection with the Forgó-type slotted-rib heat exchanger, author will examine the case when the temperature of the fin base varies in the direction of flow. (Since the plates of the slotted-rib exchangers are densely intersected in flow direction, their thermal conductance in this direction is negligable.)

A characteristic feature of slotted-rib heat exchangers (and in general, of all heat exchanger types incorporating tubes and plate fins) is that the temperature distribution along the straight line $\overline{a\ a}$ (Fig.1) — which may be regarded as the fin base — undergoes changes which may, in the majority of cases, be expressed with a periodic function. This is due to the fact that the cooling resp. heating effect of the tubes taking place along the $\overline{a\ a}$ straight line is not uniform. This paper aims at carrying out a quantitative examination of this effect.

Let us start out from the differential equation (1) representing the temperature space of the plate fin:

$$\bar{\Phi}'' - \frac{1}{A_u}\frac{C \cdot s}{1 + C \cdot s}\bar{\Phi} = -\frac{1}{A_u}\frac{f_k}{s + 1/C},$$

where conductance in the direction of flow has been neglected ($A_v = 0$).

In the previous papers ([4] and [2]) it was proved that the generic solution of this equation is as follows:

$$\bar{\Phi} = \bar{\Phi}_0 + \bar{\Phi}_i,$$

where

$$\bar{\Phi}_0 = \bar{c}_1\, e^{+\sqrt{\frac{1}{A_u}\frac{C \cdot s}{1+C \cdot s}}\cdot u} + \bar{c}_2\, e^{-\sqrt{\frac{1}{A_u}\frac{C \cdot s}{1+C \cdot s}}\cdot u}$$

and

$$\bar{\Phi}_i = \sum_{n=1}^{\infty} \frac{2}{\omega} \frac{1}{1 + A_u \omega^2} \cdot \frac{1}{s + \frac{1}{C} \frac{A_u \omega^2}{1 + A_u \omega^2}} \cdot \sin \omega u \,,$$

provided that $f_k = \sum_{n=1}^{\infty} \frac{2}{\omega} \sin \omega u$, viz. the temperature distribution of the inlet air over the inlet cross section of the heat exchanger is homogeneous.

The solution must satisfy two boundary conditions by which it is possible to determine the $\bar{c}_1$ and $\bar{c}_2$ operators. According to the first boundary condition, if $u = 0$, then $\bar{\Phi} = \bar{f}_v$; where $f_v = \Phi\,(0, v)$ that is, the value of Φ as the function

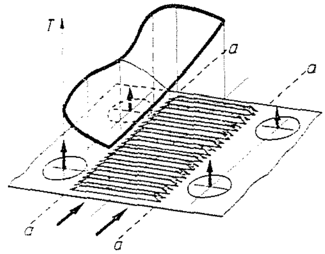

Fig. 1. Qualitative illustration of temperature distribution of a fin of the slotted-rib heat exchanger

of v, in the $u = 0$ position; while according to the second boundary condition if

$$u = 1, \qquad \text{then} \qquad \left.\frac{\partial \Phi}{\partial u}\right|_{u=1} = 0 \,.$$

The first boundary condition yields the following equation

$$\bar{f}_v = \bar{c}_1 + \bar{c}_2 \,,$$

while from the second the equation hereunder will result:

$$O = \sqrt{\frac{1}{A_u} \frac{C \cdot s}{1 + C \cdot s}} \left(\bar{c}_1 \, e^{+\sqrt{\frac{1}{A_u} \frac{C \cdot s}{1 + C \cdot s}}} - \bar{c}_2 \, e^{-\sqrt{\frac{1}{A_u} \frac{C \cdot s}{1 + C \cdot s}}} \right).$$

Provided that $\frac{1}{A_u} \frac{C \cdot s}{1 + C \cdot s} \neq 0$, it may be written that

$$O = \bar{c}_1 \, e^{+\sqrt{\frac{1}{A_u} \frac{C \cdot s}{1 + C \cdot s}}} - \bar{c}_2 \, e^{-\sqrt{\frac{1}{A_u} \frac{C \cdot s}{1 + C \cdot s}}} \,.$$

It follows from the two equations that the $\bar{c}_1$ and $\bar{c}_2$ operators are

$$\bar{c}_1 = \frac{\bar{f}_v}{1 + e^{2\sqrt{\frac{1}{A_u} + \frac{C \cdot s}{1 + C \cdot s}}}} \quad \text{and} \quad \bar{c}_2 = \frac{\bar{f}_v \cdot e^{2\sqrt{\frac{1}{A_u} \frac{C \cdot s}{1 + C \cdot s}}}}{1 + e^{2\sqrt{\frac{1}{A_u} + \frac{C \cdot s}{1 + C \cdot s}}}} \,,$$

whereby the generic solution presents itself in this form:

$$\bar{\Phi} = \frac{\bar{f}_v}{1 + e^{2\sqrt{\frac{1}{A_u}\frac{C\cdot s}{1+C\cdot s}}}} \left(e^{+\sqrt{\frac{1}{A_u}\frac{C\cdot s}{1+C\cdot s}}\cdot u} + e^{+\sqrt{\frac{1}{A_u}\frac{C\cdot s}{1+C\cdot s}}(2-u)} \right) + \Phi_i . \tag{1}$$

From the previous papers [1] we know that the ε_L, a factor similar to fin efficiency (its defining equation is $\varepsilon_L = \frac{Q}{a \cdot F \cdot \Delta T_0}\Big)$, may be derived from Φ in the following way:

$$\varepsilon_L = A_u \int_0^1 \frac{\partial \Phi}{\partial u}\bigg|_{u=0} dv . \tag{2}$$

Since both Φ and $\frac{\partial \Phi}{\partial u}$ consist of two members, it seems expedient to write the value of ε_L also as the sum of two members:

$$\varepsilon_L = \varepsilon_{L0} + \varepsilon_{Li} . \tag{3}$$

The second member is derived from $\bar{\Phi}_i$ and as its calculation was described in the quoted paper [1], in the present study we shall calculate only the ε_{L0} correction factor.

Abstaining from a detailed mathematical examination we shall restrict ourselves to indicating the ways and means which may help to obtain the best result.

Let us first compute the value of $A_u \frac{\partial \bar{\Phi}_0}{\partial v}\bigg|_{u=0}$:

$$A_u \frac{\partial \bar{\Phi}_0}{\partial u}\bigg|_{u=0} = A_u \bar{f}_v \frac{1 - e^{2\sqrt{\frac{1}{A_u}\frac{C\cdot s}{1+C\cdot s}}}}{1 + e^{2\sqrt{\frac{1}{A_u}\frac{C\cdot s}{1+C\cdot s}}}} \cdot \sqrt{\frac{1}{A_u}\frac{C\cdot s}{Cs+1}} =$$

$$= - A_u \bar{f}_v \sqrt{\frac{1}{A_u}\frac{C\cdot s}{1+C\cdot s}} \operatorname{th} \sqrt{\frac{1}{A_u}\frac{C\cdot s}{1+C\cdot s}} .$$

The validity of the following relationship can be proved[1]:

$$a \operatorname{th} a = \sum_{n=1}^{\infty} \frac{2\,a^2}{a^2 + \omega^2} ,$$

where a may even be an operator, and $\omega = \frac{2n-1}{2}\pi$.

[1] See e.g. PATTANTYUS' Manual (Műszaki Könyvkiadó Budapest 1961, Vol. I. p. 43.

Making use of the above series:

$$A_u \frac{\partial \bar{\Phi}_0}{\partial u}\bigg|_{u=0} = -A_u \bar{f}_v \sum_{n=1}^{\infty} \frac{\dfrac{2}{A_u}\dfrac{C\cdot s}{1+C\cdot s}}{\dfrac{1}{A_u}\dfrac{C\cdot s}{1+C\cdot s}+\omega^2} =$$

$$= -2A_u \bar{f}_v \sum_{n=1}^{\infty} \frac{1}{1+A_u\omega^2}\,\frac{s}{s+\dfrac{1}{C}\dfrac{A_u\omega^2}{1+A_u\omega^2}}.$$

With slight transformation:

$$A_u \frac{\partial \bar{\Phi}_0}{\partial u}\bigg|_{u=0} = -\sum_{n=1}^{\infty} \frac{2A_u}{1+A_u\omega^2}\cdot s\cdot \bar{f}_v \frac{1}{s+\dfrac{1}{C}\dfrac{A_u\omega^2}{1+A_u\omega^2}}. \tag{4}$$

Let us now establish the function pertaining to the $s\bar{f}_v$ operator. One of the basic relationships of the operational calculus is that

$$s\bar{f}_v = \left\{\frac{df_v}{dv}\right\} + f_v(0).$$

In our case, according to the definition:

$$f_v(0) = 0,$$

thus

$$s\cdot \bar{f}_v = \left\{\frac{df_v}{dv}\right\}.$$

Each member of the infinite series under examination is the convolution of two functions which consequently may be written in the following form:

$$s\bar{f}_v\cdot \frac{1}{s+\dfrac{1}{C}\dfrac{A_u\omega^2}{1+A_u\omega^2}} = \left\{\int_0^v \frac{df_v}{dv}\bigg|_{v=\tau} e^{-\frac{1}{C}\frac{A_u\omega^2}{1+A_u\omega^2}(v-\tau)}\,d\tau\right\}. \tag{5}$$

From what has gone before, ε_{L0} is readily expressed (see equations 3, 4, and 5) by:

$$\varepsilon_{L0} = A_u \int_0^1 \frac{d\Phi_0}{du}\bigg|_{u=0} dv = -\sum_{n=1}^{\infty} \frac{2A_u}{1+A_u\omega^2} \int_0^1\!\!\int_0^v \frac{df_v}{dv}\bigg|_{v=\tau} e^{-\frac{1}{C}\frac{A_u\omega^2}{1+A_u\omega^2}(v-\tau)}\,d\tau\,dv. \tag{6}$$

After integration and simplification, the following formula presents itself for the expression of ε_{L0}:

$$\varepsilon_{L0} = \sum_{n=1}^{\infty} \frac{2\,C}{\omega^2} \left[\int_0^1 \frac{df_v}{dv}\, e^{\frac{1}{C}\frac{A_u\omega^2}{1+A_u\omega^2}(v-1)} dv + \frac{\Delta T_t}{\Delta T_0} \right], \tag{7}$$

where ΔT_t denotes the total temperature increase of the fin base, in the direction of flow (see Fig. 2).

Since the member derived from the particular solution of the inhomogeneous equation has already been computed [2], for the sake of completeness, we shall write down its final result only:

$$\varepsilon_{Li} = \sum_{n=1}^{\infty} \frac{2\,C}{\omega^2} \left(1 - e^{-\frac{1}{C}\frac{A_u\omega^2}{1+A_u\omega^2}} \right). \tag{8}$$

With it, the solution — taking into consideration also equations (3) and (7):

$$\varepsilon_L = \sum_{n=1}^{\infty} \frac{2\,C}{\omega^2} \left[\left(1 + \frac{\Delta T_t}{\Delta T_0} \right) - e^{-\frac{1}{C}\frac{A_u\omega^2}{1+A_u\omega^2}} \left(1 - \int_0^1 \frac{df_v}{dv}\, e^{\frac{1}{C}\frac{A_u\omega^2}{1+A_u\omega^2} v} dv \right) \right]. \tag{9}$$

Whence, according to the well known correlation $\sum_{n=1}^{\infty} \frac{2}{\omega^2} = 1$, our equation may be put into the following final form:

$$\varepsilon_L = C \left(1 + \frac{\Delta T_t}{\Delta T_0} \right) - \sum_{n=1}^{\infty} \frac{2\,C}{\omega^2}\, e^{-\frac{1}{C}\frac{A_u\omega^2}{1+A_u\omega^2}} \left(1 - \int_0^1 \frac{df_v}{dv}\, e^{\frac{1}{C}\frac{A_u\omega^2}{1+A_u\omega^2} v} dv \right) \tag{10}$$

In the above the value of ε_L for arbitrarily varying fin base temperature has been computed in the direction of flow.

For practical considerations it seems expedient to calculate from the (10) relationship the value of ε_L for three special fin base temperature functions, viz. for linear, exponential and sine functions, since by their superposition all fin base functions encountered in practice may be produced and so ε_L always becomes calculable.

In cross-flow construction e.g. — which is frequently met in practice — the tubes, across the ever warmer gas flow passing through the heat exchanger and cooled by the ever warmer water, constitute the base of fins (counter-cross flow). The summation of the above-mentioned functions will obviously give a fair approximation of the fin base temperature even in this case (see Fig. 2).

First we shall present a solution for linearly varying fin base temperature.

Let us introduce for the designation of fin base temperature variations the following constant:

$$\varphi_v = -\frac{df_v}{dv}. \qquad 11)$$

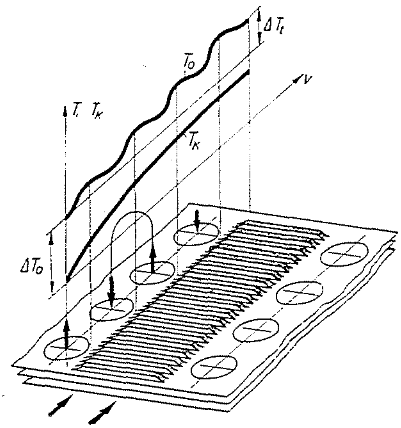

Fig. 2. Qualitative chart of the fin base temperature distribution in counter-cross flow

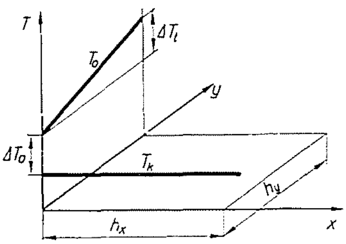

Fig. 3. Linearly varying fin base temperature

This definition, also using the defining equations of f_v and v, may be interpreted in the following manner (see Fig. 3):

$$\varphi_v = \frac{\Delta T_t}{\Delta T_0}, \qquad (12)$$

consequently φ_v denotes the total temperature increase of the fin base in flow direction, referred to the excess temperature of the fin base at the point of entry. With this denotation, after integration, the following relationship can be derived for the value of ε_L from equation (10):

$$\varepsilon_L = \sum_{n=1}^{\infty} \frac{2C}{\omega^2} \left[\left(1 - e^{-\frac{1}{C}\frac{A_u \omega^2}{1+A_u \omega^2}}\right) \left(1 - \varphi_v C \frac{1 + A_u \omega^2}{A_u \omega^2}\right) + \varphi_v \right], \qquad (13)$$

or, taking advantage of the $\sum_{n=1}^{\infty} \frac{2}{\omega^2} = 1$ summation:

$$\varepsilon_L = C(1+\varphi_v) - \sum_{n=1}^{\infty} \frac{2C}{\omega^2}\left[e^{-\frac{1}{C}\frac{A_u\omega^2}{1+A_u\omega^2}} + \varphi_v C \frac{1+A_u\omega^2}{A_u\omega^2}\left(1 - e^{-\frac{1}{C}\frac{A_u\omega^2}{1+A_u\omega^2}}\right)\right]. \tag{14}$$

Now substituting for φ_v the temperature differences (equation 12) we arrive at the following relationship:

$$\varepsilon_L = C\left(1+\frac{\Delta T_t}{\Delta T_0}\right) - C\sum_{n=1}^{\infty}\frac{2}{\omega^2} e^{-\frac{1}{C}\frac{A_u\omega^2}{1+A_u\omega^2}}\left[1 + \frac{\Delta T_t}{\Delta T_0}\frac{C(1+A_u\omega^2)}{A_u\omega^2}\left(e^{\frac{1}{C}\frac{A_u\omega^2}{1+A_u\omega^2}} - 1\right)\right]. \tag{15}$$

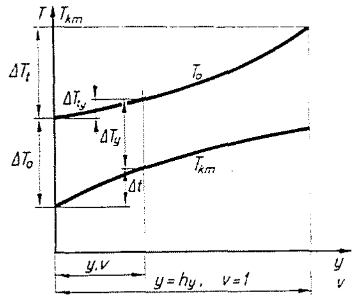

Fig. 4. Temperature changes taking place at the fin base along the $x = 0$ straight (Qualitative chart)

Thereby we have arrived at the ε_L factor for linearly changing fin base temperature.

Let us now examine heat transfer conditions at which temperature changes set in exponentially.

Since temperature increase taking place at the fin base is equal to ΔT_{ty} (see Fig. 4). and since according to the definition, in case $y = 0$, it is equal to nought, the temperature changes in the fin base may be written in the following form:

$$\Delta T_{ty} = a_e \Delta T_0 \left(e^{\frac{b_e}{h_y} y} - 1\right),$$

whence for the variables v and f_v:

$$f_v = a_e(1 - e^{b_e v}). \tag{16}$$

From the definition of f_v and v, and from the boundary conditions it follows that

$$a_e = \frac{\Delta T_t}{\Delta T_0} e^{\frac{1}{b_e - 1}} \quad \text{resp.} \quad b_e = \ln \frac{\Delta T_t + a_e \Delta T_0}{a_e \Delta T_0}. \tag{17}$$

Substituting this function of f_v into the (10) equation we arrive at the owing expression for ε_L:

$$\frac{\varepsilon_L}{C} = 1 + \frac{\Delta T_t}{\Delta T_0} - \sum_{n=1}^{\infty} \frac{2}{\omega^2} e^{-\frac{1}{C}\frac{A_u\omega^2}{1+A_u\omega^2}} \left[1 - \frac{a_e b_e}{\frac{1}{C}\frac{A_u\omega^2}{1+A_u\omega^2} + b_e} \left(1 - e^{\frac{1}{C}\frac{A_u\omega^2}{1+A_u\omega^2} + b_e} \right) \right] \tag{18}$$

Let us finally examine the value of ε_L provided that ΔT_{ty} is a trigonometric function of v. Let us assume that

$$f_v = -a_t \sin \omega_v v, \tag{19}$$

whereby the equations for a_t and ω_v will present themselves as

$$a_t \sin \omega_v = \frac{\Delta T_t}{\Delta T_0}. \tag{20}$$

With the constants so introduced the function of ε_L may be computed from the generic formula (equation 10):

$$\frac{\varepsilon_L}{C} = 1 + \frac{\Delta T_t}{\Delta T_0} -$$

$$- \sum_{n=1}^{\infty} \frac{2}{\omega^2} e^{-\psi} \left[1 + \frac{\omega_v}{\omega_v^2 + \psi^2} \left(e^{\psi} \frac{\omega_v \Delta T_t + \psi \sqrt{a_t^2 \Delta T_0^2 - \Delta T_t^2}}{\Delta T_0} - a_t \psi \right) \right], \tag{21}$$

where

$$\psi = \frac{1}{C} \frac{A_u \omega^2}{1 + A_u \omega^2}. \tag{22}$$

This yields the value of ε_L for the case when the fin base temperature is a trigonometric function.

As a conclusion we shall go into some details investigating the ε_L factor, as obtained by the calculations. It has already been stated that the defining equation of ε_L is as follows:

$$\varepsilon_L = \frac{Q}{2h_x h_y a \Delta T_0}, \tag{23}$$

thus ε_L — contrary to fin efficiency — refers to the inlet temperature difference of the heat exchanger. Accordingly, its value will not be equal to the unit even if fin length tends towards zero, because in this case it will take into account the effect of the temperature varying in the mass rate of flow, due to the heat exchange, which, ordinarily, may be considered with the logarithmic mean temperature difference.

Let us assume, as definition, the ε "fin efficiency" to be a factor which applied corrects the a film coefficient, permits fully to take into account the effect of the non-homogeneous temperature distribution of the rib causes to the heat transfer. In mathematical terms: ε is the factor which permits the computation of Q in the following form:

$$Q = \int_0^{h_y} \varepsilon a \, \Delta T_y \, 2h_x \, dy . \tag{24}$$

ΔT_y shall denote the difference between the mean temperature of the flowing medium and the fin base temperature at an y distance from the inlet:

$$\Delta T_y = T_0 - T_{km} , \tag{25}$$

where T_{km} is the mean temperature of the medium (see Fig. 4).

Denoting the heating up of the medium by Δt:

$$\Delta t = T_{km} - T_k(0, 0) ,$$

we may write down

$$Q = h_x C_K \, \Delta t . \tag{26}$$

On the other hand, it is evident from Fig. 4 that

$$\Delta T_y = \Delta T_0 + \Delta T_{ty} - \Delta t .$$

Reverting to our previous designations

$$\Delta T_y = \Delta T_0 (1 - f_v) - \Delta t$$

and

$$Q = \varepsilon a \, h_x h_y \, 2 \int_0^v \Delta T_y \, dv = h_x C_K \, \Delta t .$$

From the collation of the two equations:

$$\Delta T_y = \Delta T_0 (1 - f_v) - \frac{\varepsilon}{C} \int_0^v \Delta T_y \, dv .$$

The so obtained integral equation is readily solved e.g. by means of the operational calculus [3], the solution presents itself in the following form:

$$\Delta T_y = \Delta T_0 \left[e^{-\frac{\varepsilon}{C} v} - \int_0^v \frac{df_v}{dv} \bigg|_{v=\tau} e^{-\frac{\varepsilon}{C}(v-\tau)} \, d\tau \right] . \tag{27}$$

Let us define the mean temperature difference in the following way:

$$Q = \varepsilon\, \alpha\, 2h_x h_y\, \Delta T_m . \tag{28}$$

From the definition it follows that

$$\Delta T_m = \frac{1}{h_y}\int_0^{h_y} \Delta T_y\, dy = \int_0^1 \Delta T_y\, dv .$$

After integration, the appropriate rearrangement of the boundaries, and simplification, we arrive at

$$\Delta T_m = \Delta T_0 \frac{C}{\varepsilon}\left[1 - f_v(1) - e^{-\frac{\varepsilon}{C}}\left(1 - \int_0^1 \frac{df_v}{dv}\, e^{\frac{\varepsilon}{C} v}\, dv\right)\right]. \tag{29}$$

From the collation of the definitions of ε and ε_L it furthermore follows (see 28 and 23 equations) that there is a relationship between ε and ε_L:

$$\frac{\varepsilon_L}{\varepsilon} = \frac{\Delta T_m}{\Delta T_0} .$$

Substituting this into equation (29):

$$\frac{\varepsilon_L}{C} = 1 + \frac{\Delta T_t}{\Delta T_0} - e^{-\frac{\varepsilon}{C}}\left(1 - \int_0^1 \frac{df_v}{dv}\, e^{\frac{\varepsilon}{C} v}\, dv\right). \tag{30}$$

The so evolved relation for an arbitrary fin base temperature function will interrelate $\frac{\varepsilon_L}{C}$ with $\frac{\varepsilon}{C}$ and in this way make possible a comparison between the ε_L values as derived by the above detailed computation, and the ε value calculated according to the usual method.

Summary

In plate-fin heat exchangers, variations in the temperature of the fin base in the diection of flow are frequent. In compact high-efficiency heat exchangers the warming-up of rhe medium at different distances from the fin base also varies. These two effects make the talculation of fin efficiency rather difficult.

The paper presents a calculation method and a generic formula, in the form of an infinite series, directly suitable for the calculation with three functions of fin base temperature — linear, exponential and trigonometric. From the three types, the function of any arbitrary fin base temperature may be superposed. In this way the paper gives a clue for the computation of the fin efficiency of counter- or direct-cross-flow plate-fin heat exchangers (among them also for the efficiency of the Forgó-type).

The result of the calculations, instead of the usual fin efficiency, presents itself through an ε_L factor (already introduced in two previous papers of the author), referred to the inlet temperature difference.

The paper finally establishes a relationship between the ε_L factor and the fin efficiency for any arbitrary fin base temperature.

The series obtained as the final result rapidly converge and their divergence can readily be estimated.

Symbols

a_e, b_e	Constants, see equation (16)
a_t, ω_v	Constants, see equation (19)
c_1, $\bar{c}_2$	Arbitrary operators, constants
f_k	$f_k = \Phi_k$ (u, O)
f_v	$f_v = \Phi$ (0, v)
h_x	Fin length measured normal to flow
h_y	Fin length measured in the direction of flow
n	Index
s	Differential operator
Δt	$\Delta t = T_{km} - T_k(0, 0)$
u	Dimensionless coordinate in the direction of flow
v_0	Fin thickness
y	Coordinate in flow direction
x	Coordinate normal to flow
A_u	Dimensionless number $A_u = \lambda_x/2a \cdot v_0/h_x^2$
A_v	Dimensionless number $A_v = \lambda_y/2a \cdot v_0/h_y^2$
C	Dimensionless number $C = C_K/2ah_y$
C_k	Rate of water value for unit length of one fin in direction x. The full water rate value for each fin is: $\int_0^{h_x} C_k\, dx$
F	Total heat transfer surface
Q	Transferred heat
T	Fin temperature
T_0	Fin base temperature
T_k	Temperature of the medium
T_{km}	The mean temperature of the medium (calculated at a straight y = constant)
ΔT	$\Delta T = T - T_k$
ΔT_k	$\Delta T_k = T\ (0, 0) - T_k$
ΔT_0	$\Delta T_0 = T\ (0, 0) - T_k\ (0,0)$
ΔT_f	$\Delta T_f = T\ (0, 0) - T$
ΔT_m	The mean temperature difference prevailing between fin and medium
ΔT_t	The total temperature increase of the fin base in the direction of flow (see Fig. 2.)
ΔT_{ty}	The temperature increase of the fin base at place y (see Fig. 4.)
ΔT_y	$\Delta T_y = T_0 - T_{km}$
α	Heat transfer coefficient between fin and medium
ε_L	Dimensionless number, similar to fin efficiency
ε_{L0}, ε_{Li}'	See equation (3)
ε	Fin efficiency; see equation (24)
φ_v	Constant, see equation (11)
ω	$\omega = \dfrac{2n-1}{2}\pi$
ω_v, a_t	Constants, see equation (19)
ψ	See equation (22). $\psi = 1/C \cdot A_u\omega^2/(1 + A_u\omega^2)$

λ_x, λ_y	Fin conductivity in x, respectively in y direction
τ	Independent variable
Φ	Dimensionless variable, expressing temperature changes taking place in the plate $\Phi = \Delta T_f / \Delta T_0$
Φ_0, Φ_i	The homogeneous resp. inhomogeneous part of Φ. $\Phi = \Phi_0 + \Phi_i$
Φ_K	Dimensionless variable expressing temperature changes taking place in the medium $\Phi_K = \Delta T_K / \Delta T_0$
	$\bar{z}$ resp. $\{z\}$ denoting that z is an operator

Note: with respect to dimensions:

Physical equations have throughout been applied, consequently any consistent units may be employed.

References

1. Szücs, L.: Heat Transfer in Compact Plate-Fin Heat Exchangers. Periodica Polytechnica, 7, 21 (1963).
2. Szücs, L.: The Fin Efficiency of the Forgó-Type Slotted-Rib Heat Exchanger. Periodica Polytechnica, 7, 229 (1963).
3. Mikusiński, J. G.: Operational Calculus. Pergamon Press, 1959.
4. Szücs, L.: Thesis, 1962.

L. Szücs, Budapest XI., Stoczek u. 2—4. Hungary