DESIGN OF MIXED FLOW IMPELLER

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1. Introduction

In designing the blading of rotors or stators of pumps or turbines, one must consider a three-dimensional flow pattern even when assuming ideal liquid flow. The problem was solved by C. H. Wu [1], whose method requires a computing work too extensive for practical cases and is only practicable with an electronic computer.

Consequently, there is a need for a simpler method of calculation, giving a good approximation. The blading and the approximate velocity distribution can be calculated by one of the techniques on the basis of starting a more accurate calculation of velocity distribution.

This paper deals with design of mixed flow impeller having thin blades, the thickness of which is not infinitely small. The investigations were made assuming incompressible and perfect liquid. In the calculations the possibilities given by the conformal transformation and the method of singularities were used. For reducing the calculation work some allowances and simplifications should be introduced which can be tolerated according to our experience and nevertheless give a good approximation. The stream surfaces whose intersection with any plane perpendicular to axis z forming concentrical circles were regarded as surfaces of revolution and, in analysing the flow on the surfaces of revolution, the source distribution making a variable layer-thickness was considered only according to the integrated velocity means. On the basis of related investigations for practical cases by Betz [2] this approximation gives good results. After transforming the surfaces of revolution and the blade sections on them into a straight plain cascade, we could determine the blade-curves (blades having infinitely small thickness) by the method of singularities. The source distribution responsible for the blade thickness is considered, not on the blade-curves carrying the singularities, but only distributed on the whole field, corresponding to the conventional contraction coefficient. Nevertheless, when computing the velocity distribution of the blade, the value of the velocity-jump is determined by taking the accurate thickness-distribution into consideration. In fact, the thinner the blades are, the better is the approximation of the velocity distribution calculated in this way.

To determine the blades an iteration process is proposed here. As a first step — as approximation of zeroth order — similarly to other processes [3] the blade surface of an impeller with infinite blade numbers are calculated. This is necessary, on the one hand, because of the possibility of calculating the stream surfaces which are regarded as surfaces of revolution depending on the "blade load", on the other hand, because of the starting basis of the iteration determining the final blade surfaces. After the conformal transformation of the surfaces of revolution into the straight plain-cascades, the blade-curve is to be determined by iteration and, after fixing the blade thickness, the blade surface and the velocity distribution can be determined.

The process makes the determination of velocity distribution in the interblade channels possible, before and beyond the impeller, resp.

The paper shows the design of a pump impeller, but the relations can evidently be used in the case of turbines or, by putting $\omega = 0$, in the case of stators, as well.

2. Symbols used

H = head
Q_e = fluid mass delivered by the impeller
z = number of blades
Γ = blade circulation
ω = angular velocity of impeller
η_h = hydraulic efficiency
r, φ, z = cylindrical coordinates on the impeller
ξ, η = coordinates in the ζ plane in the straight cascade
$\xi^* = \xi/\xi_2$ = dimensionless coordinate perpendicular to the straight cascade
l, b = curved coordinates in the meridional section
t = blade pitch in the straight cascade
$\varphi_t = 2\pi/z$
φ_ϑ = angle characterizing blade thickness
$\gamma_s(s)$ = vorticity distribution along the camber line, in straight cascade
$\gamma_\xi(\xi)$ = vorticity distribution along the coordinate axis ξ, in straight cascade
$\gamma_\xi^*(\xi^*)$ = dimensionless vorticity distribution
β = blade angle
ϑ_n = blade thickness on the pressure side
ϑ_s = blade thickness on the suction side
ϱ_ζ = radius of blade curve in straight cascade
c = absolute velocity in the impeller system
c_ζ = absolute velocity in straight cascade
c_m = component of velocity c in the meridional section
c_u = component of velocity c in direction u
c_ξ, c_η = components of velocity c_ζ
c_{mk} = velocity c_m in the case of infinite number of blades
$c_{m\xi}$ = the correspondent of the velocity c_{mk} in straight cascade
$c_{\zeta\infty}$ = basic flow velocity in straight cascade
$c_{i\zeta}$ = induced velocity in straight cascade
$c_{p\eta}$ = the correspondent of the velocity of rotation before the impeller in straight cascade
$c_{k\eta} = \Gamma/2t$
c_{il} = component of velocity $c_{i\zeta}$ along the streamline
c_{in} = component of velocity $c_{i\zeta}$ perpendicular to streamline
u = peripheral velocity in the impeller
u_η = the correspondent of u in straight cascade

w = relative velocity in impeller
w_ζ = the correspondent of the velocity w in straight cascade
w_n = relative velocity on the pressure side of blade
w_s = relative velocity on the suction side of blade
$w_{n\zeta}$ = the correspondent of the velocity w_n in straight cascade
$w_{s\zeta}$ = the correspondent of the velocity w_s in straight cascade
w_ζ' = the correspondent of the relative velocity on the pressure side of the blade of infinitely small thickness, in straight cascade
w_ζ'' = the correspondent of the relative velocity on the suction side of the blade of infinitely small thickness, in straight cascade

3. Fundamental relations. Determination of the impeller element

With the known meridional curve (A, A) and the known breadth db (l) in Fig. 1, let us investigate the fundamentel relations required for the design of blading of the impeller element characterized by these "dimensions".

The surface of revolution characterized by the meridional curve (A, A) is to be regarded as approximated flowsurface. Now the flowpattern on the surface of revolution can be transformed to a flow pattern on the plane ζ, where the blade row of impeller element with a number of blades z, will appear as an infinite cascade of blades, with pitch t [4]. Transformation will be made by the relations

$$\eta = \frac{zt}{2\pi} \varphi \tag{1}$$

and

$$\xi = \frac{zt}{2\pi} \int_{r_1}^{r} \frac{dr}{r \sin \delta} \tag{2}$$

In the impeller system the symbol c denotes the absolute velocity of the flow which is regarded as a potential one. Its conformal value on plane ζ can be calculated from the known relation

$$c_\zeta = \frac{2r\pi}{zt} c\,. \tag{3}$$

In the impeller system the continuity can be expressed as

$$\nabla \mathbf{c} = \frac{1}{r} \frac{\partial}{\partial l} (c_m r) + \frac{1}{r} \frac{\partial c_u}{\partial \varphi} = - c_m \frac{d}{dl} \ln db \tag{4}$$

where c_m and c_u denote components of velocity c in meridional and rotational directions, resp. On plane ζ the same is

$$\nabla \mathbf{c}_\zeta = \frac{\partial c_\xi}{\partial \xi} + \frac{\partial c_\eta}{\partial \eta} = - c_\xi \frac{d}{d\xi} \ln\, db\,, \tag{5}$$

where c_ξ and c_η denote components of velocity c_ζ in directions of ξ and η, resp. [4]

It is evident from differential equation (5) that in straight cascade one must consider a flow with sources varying in directions ξ as well as η, because of $c_\xi(\xi, \eta)$. Nevertheless, experiences show, that acceptable accuracy can be

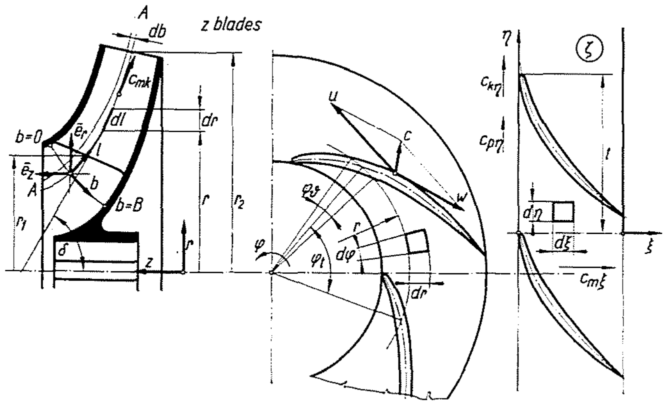

Fig. 1

achieved by setting into the differential equation (5) the mean integral values of

$$c_{\xi m}(\xi) = \frac{1}{t}\int_0^t c_\xi \, d\eta$$

$$c_{\eta m}(\xi) = \frac{1}{t}\int_0^t c_\eta \, d\eta$$

in lieu of $c_\xi(\xi, \eta)$ and $c_\eta(\xi, \eta)$.

According to [4]

$$c_{\xi m} = \frac{dQ_e}{ztdb} \tag{6}$$

$$c_{\eta m} = f(\xi) \tag{7}$$

where dQ_e is the fluid quantity delivered by the impeller of elementary breadth db.

This process holds for blading of impellers with "thin" blades, *i. e.* blades with small thickness. In this case we can achieve a good approximation, if the effect of the blade thickness will be taken into consideration by the only "contraction coefficient" from the viewpoint of continuity. Consequently, using the symbols in Fig. 1, instead of equation (6) we calculate with the relation

$$c_{\xi m} = \frac{2r\pi}{zt} c_{mk}, \tag{8}$$

where c_{mk} is the mean of meridional velocities of elementary impellers in the case of infinitely thin blades, $c_{mk} = dQ_e/2r\,\pi\,db(1 - \varphi_{\vartheta}/\varphi_t)$.

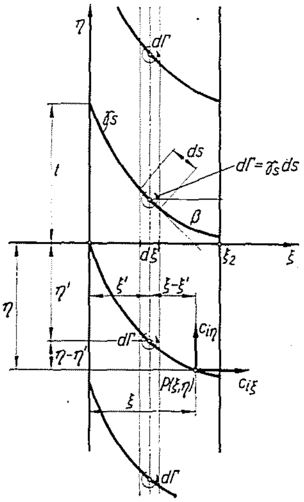

Fig. 2

In the straight cascade — in plane ζ — the flow will be generated in the conventional way, as sum of a basic flow and an induced one:

$$\mathbf{c}_{\zeta} = \mathbf{c}_{\zeta\infty} + \mathbf{c}_{i\zeta} \tag{9}$$

where $\mathbf{c}_{i\zeta}$ is the velocity induced by vorticity distribution placed on the streamline carrying the singularity. The velocity of the basic flow — as reported [5] — can be determined from the sum of

$$\mathbf{c}_{\zeta\infty} = \mathbf{c}_{\xi m} + \mathbf{c}_{p\eta} + \mathbf{c}_{k\eta} \tag{10}$$

where $c_{\xi m}$ can be calculated from equation (8) and

$$c_{p\eta} = \frac{\Gamma_p}{zt} \tag{11}$$

where $\Gamma_p = 2r_1\,\pi\,c_{1u}$ and c_{1u} is the component given by the prerotation at inlet, in the case of infinitely large numbers of blades. Finally

$$c_{k\eta} = \frac{\Gamma}{2t} \tag{12}$$

where

$$\Gamma = \frac{2\pi g H}{\omega \eta_h z} \tag{13}$$

denotes the blade circulation. Velocities of $c_{\xi m}$, $c_{p\eta}$ and c_{k_η} will be regarded as positive ones, if they show (see Fig. 1) in the positive directions of axes ξ and η resp., after the transformation relations (1) and (2) are used. The basic flow determined by Equ. (10) meets Equs. (7) and (8), resp.

The induced flow is generated by the vorticity distribution γ_s placed on the streamline carrying the singularity (see Fig. 2). Let it be

$$\gamma_s\, ds = \gamma_\xi\, d\xi = \frac{\Gamma}{\xi_2} \gamma_\xi^*\, d\xi = \Gamma \gamma_\xi^*\, d\xi^* \tag{14}$$

where $\xi^* = \xi/\xi_2$ and $\gamma_\xi^*(\xi^*)$ are dimensionless values. Using the symbols of Fig. 2 and remembering that $ds = \sqrt{1 + \tan^2\beta}\; d\xi$, so

$$\gamma_s = \frac{\Gamma}{\xi_2 \sqrt{1 + \tan^2\beta}}\, \gamma^*(\xi^*). \tag{15}$$

The element of $d\xi$ breadth of the straight cascade — it is in the fact an infinite vortex row of $d\Gamma$ intensity which lies parallel to axis η and is characterized by coordinates of ξ' and η' — induced velocity components of

$$dc_{i\xi} = \frac{d\Gamma}{t}\,\Phi$$

$$dc_{i_\eta} = \frac{d\Gamma}{t}\,\Psi$$

in point $P(\xi, \eta)$. In the above expression Φ and Ψ are influence functions expressed [6] as

$$\Phi = \frac{-e^{2\pi\frac{\xi-\xi'}{t}} \sin 2\pi \dfrac{\eta-\eta'}{t}}{1 - 2e^{2\pi\frac{\xi-\xi'}{t}} \cos 2\pi \dfrac{\eta-\eta'}{t} + e^{4\pi\frac{\xi-\xi'}{t}}} \tag{16}$$

and

$$\Psi = \frac{e^{4\pi\frac{\xi-\xi'}{t}} - 1}{2\left(1 - 2e^{2\pi\frac{\xi-\xi'}{t}} \cos 2\pi \frac{\eta - \eta'}{t} + e^{4\pi\frac{\xi-\xi'}{t}}\right)} \tag{17}$$

Taking relation (14) into consideration the components of induced velocity, parallel and perpendicular to the cascade, can be calculated in point $P(\xi, \eta)$ from the integrals

$$c_{i\xi} = \frac{\Gamma}{t} \int\limits_{\xi^{*\prime}=0}^{1} \gamma_{\xi}^{*}(\xi^{*\prime})\, \Phi\left(\frac{\xi - \xi'}{t}, \frac{\eta - \eta'}{t}\right) d\xi^{*\prime} \tag{18}$$

and

$$c_{i\eta} = \frac{\Gamma}{t} \int\limits_{\xi^{*\prime}=0}^{1} \gamma_{\xi}^{*}(\xi^{*\prime})\, \Psi\left(\frac{\xi - \xi'}{t}, \frac{\eta - \eta'}{t}\right) d\xi^{*\prime} \tag{19}$$

An iteration process can be suggested for determining the streamline carrying singularity. To set up the process we wish to take two items of the process proposed by CZIBERE [7]:

1. Approximation of order $i + 1$ can be obtained from that of order i in such a way, that the streamline of $i + 1$ will be computed as if the singularities were placed on curve i, then the streamline of $i + 2$ will be computed with singularities transmitted to the curve of $i + 1$, and so on.

2. We prescribe relations of

$$(c_{it})_i\, ds_i = (c_{it})_{i+1}\, ds_{i+1}$$

and

$$(c_{in})_i\, ds_i = (c_{in})_{i+1}\, ds_{i+1}$$

between the components $(c_{it})_i$ and $(c_{in})_i$ (relating to the arc element d_{si} as tangential and normal components) of velocity induced by singularities placed on the curve of i on one of the points of that curve, and, the correlate components of the induced velocity relating to the arc element ds_{i+1} of the approximation of order $i + 1$.

CZIBERE prescribed the second relation only for the 'eigeneffect' — for velocities induced by their own vortices of the investigated blade. However, no essential error can be made by extending the mentioned process to complete induced velocity for the sake of reducing the computation work. Consequently, paying attention to the relation of $ds = \sqrt{1 + \tan^2 \beta}\, d\xi$ we can write

$$(c_{it})_{i+1} = (c_{it})_i \sqrt{\frac{1 + \tan^2 \beta_i}{1 + \tan^2 \beta_{i+1}}} \tag{20}$$

$$(c_{in})_{i+1} = (c_{in})_i \sqrt{\frac{1 + \tan^2 \beta_i}{1 + \tan^2 \beta_{i+1}}} \tag{21}$$

where β_i and β_{i+1} are, respectively, tangent angles of approximations of order i, and $i + 1$ with axis ξ.

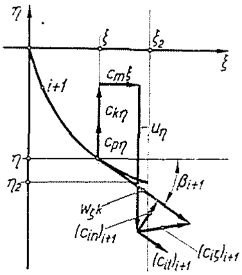

Fig. 3

Denoting $u_\eta = u\, 2r\pi/zt$ and taking into consideration, that $\beta_{i+1} < 0$ the streamline equation will be (see Fig. 3)

$$c_{p\eta} \cos \beta_{i+1} + c_{k\eta} \cos \beta_{i+1} - c_{m\xi} \sin \beta_{i+1} - u_\eta \cos \beta_{i+1} + (c_{in})_{i+1} = 0$$

or by taking Equ. (21) into consideration

$$\tan \beta_{i+1} = \frac{c_{p\eta} + c_{k\eta} - u_\eta + (c_{in})_i \sqrt{1 + \tan^2 \beta_i}}{c_{m\xi}} \tag{22}$$

and since $(c_{in})_i = (c_{i\eta})_i \cos \beta_i - (c_{i\xi})_i \sin \beta_i$ therefore

$$\tan \beta_{i+1} = \frac{c_{p\eta} + c_{k\eta} - u_\eta + (c_{i\eta})_i - (c_{i\xi})_i \tan \beta_i}{c_{m\xi}} \tag{23}$$

where $(c_{i\xi})_i$ and $(c_{i\eta})_i$ are values computable from the integrals (18) and (19) resp., at the point pertaining to the same value ξ of the streamline obtained as a result of the preceding iteration step. The streamline of order $i + 1$ will be given from the integral

$$\eta_{i+1} = \int_0^{\xi} \tan \beta_{i+1}\, d\xi = \xi_2 \int_0^{\xi^*} \tan \beta_{i+1}\, d\xi^* \tag{24}$$

As an approximation of zeroth order we accept the streamline relating to the blading with infinite number of blades. In this case $(c_{i\xi})_{i=0}$ and

$$(c_{i\eta})_{i=0}(\xi^*) = \frac{\Gamma}{t} \int_0 \gamma_\xi^* \, d\xi^* - c_{k\eta} \tag{25}$$

form can be used, or

$$\tan \beta_{i=0} = \frac{c_{p\eta} + c_{k\eta} + (c_{i\eta})_{i=0} - u_\eta}{c_{m\xi}} \tag{26}$$

and finally

$$\eta_{i=0} = \xi_2 \int_0^{\xi^*} \tan \beta_{i=0} \, d\xi^* . \tag{27}$$

Using relations (25), (26) and (27) the streamline resulting in the zeroth approximation can be calculated. As soon as the singularities (the γ_s distribution) will be placed along the mentioned curve, the approximation of first order can also be determined with the aid of expression (18), (19), (22) or (23) and (24). When transmitting the singularities to the curve obtained as the approximation of first order the iteration can be continued as long as the difference in approximation values of order j and $j + 1$ are negligible.

With the aid of the above shown iteration process the streamline carrying the singularities — the blade curve *i.e.* the blade of infinitely small thickness — can be determined along the surface of revolution (A, A) (see Fig. 1), that is along that approximate flow surface. The blade profile element can also be determined in the knowledge of function $\varphi_\partial/\varphi_t = f(l)$ taken from relation (8), only the angle of $\varphi_\partial/2$ is to be plotted on both sides of the blade curve.

But the blade element determined by the above shown method was related with a given — predetermined (!) — surface of revolution and a given velocity distribution $c_{mk}(l)$. On the basis of practical experiences one can categorically assert, that either the meridional curve (A, A) (see Fig. 1), or the velocity distribution $c_{mk}(l)$ should not be taken by a "set of trajectories" assuming the potentional flow pattern. It is conceivable, that stream surfaces could make surfaces of revolution only when the blading is an infinitely dense one (or in a bladeless space). However, in this case the distribution of singularities cannot be discrete and surfacial, but continuous and spatial one. Inasmuch as the said distribution has the feature containing vortex vectors with components perpendicular to the meridional plane, which is usual, so the surfaces of revolution (as approximate flow surfaces) are to be determined by considering these components. Consequently, as an approximation of zeroth order the impeller with infinite number of blades must be determined.

4. Approximation of zeroth order. Impeller with an infinite number of blades

As a starting point of the iteration process (as an approximation of zeroth order) an infinite number of blading will be used. In conformity with the task (Q, H, n) and the assumed vorticity distribution γ, and further, by allowing the determination of stream surfaces regarded as approximate surfaces of revolution, those as functions of the blade loading.

Principal dimensions for the impeller can be taken in the usual way [8], [9], [10] for the accumulated experiences of engineering practice, which

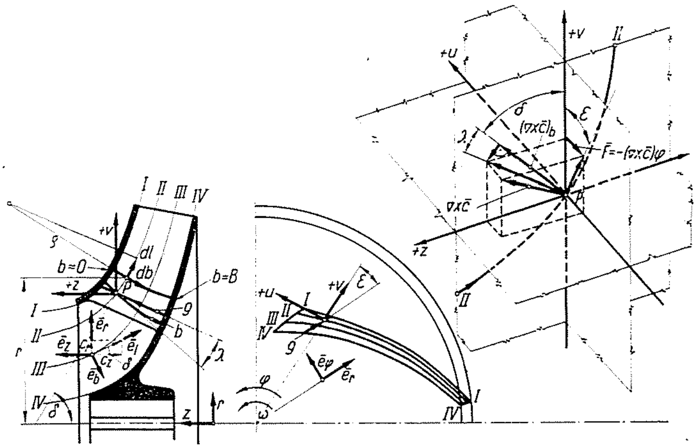

Fig. 4

allows the determination of these dimensions even by the conventional methods.

With known principal dimensions we will determine the impeller with infinite number of blades, which gives the approximation of zeroth order. In the course of the processing we shall determine the vortex components perpendicular to the meridional section of the impeller with an infinite number of blades, on account of the meridional curves (surfaces of revolution obtained from the step of order i and of the distributions $c_{mk}(l)$ and further, from the obtained values we will compute the meridional curves of order $i+1$ and the distributions $c_{mk}(l)$, resp. This iteration is to be continued as long as a negligible tolerance is reached.

At first the impeller will be disintegrated to partial channels (see Fig. 4) by meridional curves taken as is usual [11], in the meridional section of the impeller. At the same time the velocity distributions $c_{mk}(l)$ will also be determined. With known meridional curves (I, II, III, IV) defining the surfaces of revolution, the transformations can be made by relations of (1) and (2) and, after fixing the functions of $\varphi_\vartheta/\varphi_t = f(l)$ and $\gamma_\xi^*(\xi^*)$, the "blade curves" can be computed by relations of (25), (26) and (27). If the points of blade curves were computed for the same ξ^* values in the case of each surface of revolution and, if the computation were made with the same functions of γ_ξ^* (ξ^*), then the points pertaining to the same ξ^* value lie on the same vortex line. In Fig. 4 one of these vortex lines is drawn (g). From this diagram one can see, that the component perpendicular to the meridional section can be computed from relation

$$F(l, b) = \frac{\cos(\delta + \lambda)}{\cos \lambda} \tan \varepsilon \, |(\nabla \times \mathbf{c})_b| \tag{28}$$

where the symbols are those of the diagram. Here the vortex vector $\nabla \times \mathbf{c}$ lying on the blade surface has a component of $(\nabla \times \mathbf{c})_b$ placed in the meridional section, and directed perpendicular to the meridional streamline (II). The component can be determined at any point of the meridional section between the leading and trailing edges. This is the value of vorticity distribution, which affects the meridional flow pattern. In Equ. (28) the value $|(\nabla \times \mathbf{c})_b|$ is that of the continuous vorticity distribution on one of the surface of revolution, which was prescribed by us when taking the function of $\gamma_\xi^*(\xi^*)$. According to this

$$|(\nabla \times \mathbf{c})_b| = \frac{\Gamma}{\xi_2} \, \frac{z^2 t}{4\pi^2} \, \frac{\gamma_\xi^*}{r^2} \tag{29}$$

where γ_ξ^* is the function of ξ, consequently that of r on one of the meridional curves. With the aid of relations (28) and (29) the values of the function $F(l, b)$ at the various points of the meridional section can be determined as soon as the distribution γ_ξ^* is known.

In the coordinates r, φ, z (see Fig. 4)

$$(\nabla \times \mathbf{c})_\varphi = \mathbf{e}_\varphi \left(\frac{\partial c_r}{\partial z} - \frac{\partial c_z}{\partial r} \right).$$

The

$$\frac{\partial c_r}{\partial z} = - \frac{\partial c_r}{\partial l} \cos \delta - \frac{\partial c_r}{\partial b} \sin \delta$$

and

$$\frac{\partial c_z}{\partial r} = \frac{\partial c_z}{\partial l} \sin \delta - \frac{\partial c_z}{\partial b} \cos \delta$$

or by putting $c_r = c \sin \delta$ and $c_z = - c \cos \delta$

$$(\nabla \times \mathbf{c})_\varphi = - \mathbf{e}_\varphi \left(\frac{\partial c}{\partial b} + c \frac{\partial \delta}{\partial l} \right)$$

But from Fig. 4 it is $\mathbf{F}(r, z) = - (\nabla \times \mathbf{c})_\varphi$, and so the vorticity of the meridional flow can be described by the differential equation

$$\frac{\partial c}{\partial b} + c \frac{\partial \delta}{\partial l} = F(r, z) \tag{30}$$

(see Fig. 4), or, along a preferred orthogonal trajectory b

$$\frac{dc_{mk}}{db} + c_{mk} H(b) = F(b) \tag{31}$$

where $H(b) = (\partial \delta / \partial l)(b)$. Iteration process seems to be reasonable for determining the meridional streamlines and velocity distribution $c_{mk}(l)$. To determine the function $F(r, z)$ we have to start out from the flow before and after the impeller, the set of meridional streamlines and the distribution $c_{mk}(l)$ taken from conventional methods. Regarding this as an approximation of zeroth order of the meridional curves pertaining to the function $F(r, z)$. Now, along the approximate orthogonal trajectories the approximation of zeroth other $H_0(b)$ of the function $H(b)$ can be considered as known, and we must solve but the differential equation

$$\frac{dc_{mk}}{db} + c_{mk} H_0(b) = F(b) \tag{32}$$

The solution is

$$c_{mk} = e^{-\int\limits_0^b H d\lambda} \int\limits_0^b e^{\int\limits_0^\lambda H d\psi} F d\lambda + c_{mk_0} e^{-\int\limits_0^b H d\lambda} \tag{33}$$

or, because of

$$\frac{Q_e}{2\pi} = \int\limits_0^B r \left(1 - \frac{\varphi_v}{\varphi_t}\right) c_{mk}\, db = \int\limits_0^B \left\{ r \left(1 - \frac{\varphi_v}{\varphi_t}\right) e^{-\int\limits_0^b H d\lambda} \int\limits_0^b e^{\int\limits_0^\lambda H d\psi} F d\lambda \right\} db +$$

$$+ c_{mk_0} \int\limits_0^B r \left(1 - \frac{\varphi_v}{\varphi_t}\right) e^{-\int\limits_0^b H d\lambda} db$$

on the external streamline

$$c_{mk_0} = \frac{\frac{Q_e}{2\pi} - \int\limits_0^B r(b)\, A_1(b)\, A_2(b)\, A_3(b)\, db}{\int\limits_0^B r(b)\, A_1(b)\, A_3(b)\, db} \tag{34}$$

where

$$A_1(b) = e^{-\int\limits_0^b H(\lambda) d\lambda} \tag{35}$$

and

$$A_2(b) = \int\limits_0^b \frac{F(\lambda)}{A_1(\lambda)}\, d\lambda\, ; \;\; A_3(b) = 1 - \frac{\varphi_i}{\varphi_t}\, . \tag{36}$$

Along the orthogonal trajectory — or trajectories — the velocity distribution c_{mk} can be determined by Equ. (33), (34), (35) and (36). Now the liquid mass distribution is given by the integral

$$\psi(b) = \int\limits_0^b r(b)\, A_1(b)\, A_2(b)\, A_3(b)\, db + c_{mk_0} \int\limits_0^b r(b)\, A_1(b)\, A_3(b)\, db\, . \tag{37}$$

Knowing this, in the case of partial channels of number k, the intersection points of the orthogonal trajectory and the stream-lines disintegrating the partial channels for delivering equal partial volumes $\Delta\psi = Q_e/2\pi k$, can easily be plotted. After carrying out the computation along several orthogonal trajectories, if we know the conditions of the potential flow before and after the impeller,* the meridional streamlines could be plotted and, after taking new orthogonal trajectories, the new functions $H(b)$ could be determined. The process is to be iterated as long as no essential difference in values of the initial and the computed distributions appears. Thereafter, we have to check, whether the values (28) of function $F(l/b)$ do not essentially vary, otherwise the whole computation work is to be repeated.

Arriving at the end of the iterations the meridional curves and along these the velocity distributions $c_{mk}(l)$ are known, consequently those surfaces of revolution are available, which can be regarded as approximate flow surfaces. In addition the approximation of zeroth order of the "blade curve" is also known on each surface or revolution — and on the transformed straight cascade. All data are available to determine the final blading.

* More general conditions of this kind, having the basis of measurements and experiences will be published in a future paper.

5. Determination of blading. Computing velocity distribution

After the approximation of zeroth order is available, the blading of the "impeller elements" pertaining to the surfaces of revolution can be determined. The course of computation was already mentioned in Chap. 3. In connection with this we have still to deal with the difficulties connected with the integrals (18) and (19). In both cases, at the point $\xi = \xi'$ *i.e.* $\xi^* = \xi^{*'}$ integration of a discontinuity function takes place, for the influence functions in the said place are discontinuity ones and of infinite value, and for $\gamma_\xi^* \neq 0$.

In some ξ^* point the integrals will be computed by disintegrating them to sections. Let it be

$$c_{i\xi} = c'_{i\xi} + c''_{i\xi} \tag{38}$$

and

$$c_{i\eta} = c'_{i\eta} + c''_{i\eta} \tag{39}$$

where

$$c'_{i\xi} = \frac{\Gamma}{t}\left[\int_0^{\xi^*-\frac{\Delta\xi^*}{2}} \gamma_\xi^* \Phi\, d\xi^{*'} + \int_{\xi^*+\frac{\Delta\xi^*}{2}}^{1} \gamma_\xi^* \Phi\, d\xi^{*'}\right] \tag{40}$$

and

$$c'_{i\eta} = \frac{\Gamma}{t}\left[\int_0^{\xi^*-\frac{\Delta\xi^*}{2}} \gamma_\xi^* \Psi\, d\xi^{*'} + \int_{\xi^*+\frac{\Delta\xi^*}{2}}^{1} \gamma_\xi^* \Psi\, d\xi^{*'}\right] \tag{41}$$

or

$$c''_{i\xi} = \frac{\Gamma}{t}\int_{\xi^*-\frac{\Delta\xi^*}{2}}^{\xi^*+\frac{\Delta\xi^*}{2}} \gamma_\xi^* \Phi\, d\xi^{*'} \tag{42}$$

and

$$c''_{i\eta} = \frac{\Gamma}{t}\int_{\xi^*-\frac{\Delta\xi^*}{2}}^{\xi^*+\frac{\Delta\xi^*}{2}} \gamma_\xi^* \Psi\, d\xi^{*'} . \tag{43}$$

Integrals (40) and (41) can be computed directly. When computing integrals (42) and (43) one must take into consideration, that if $2\pi(\xi - \xi')/t$

and $2\pi(\eta-\eta')/t$ were of small value, then

$$\Phi \approx \frac{-t}{2\pi} \frac{\eta-\eta'}{(\xi-\xi')^2+(\eta+\eta')^2} = \frac{-t}{2\pi} \frac{\tan\beta}{1+\tan^2\beta} \frac{1}{\xi-\xi'} \tag{44}$$

and

$$\Psi \approx \frac{t}{2\pi} \frac{\xi-\xi'}{(\xi-\xi')^2+(\eta-\eta')^2} = \frac{t}{2\pi} \frac{1}{1+\tan^2\beta} \frac{1}{\xi-\xi'} . \tag{45}$$

With this

$$c''_{i\xi} = \frac{-\tan\beta}{1+\tan^2\beta} \frac{\Gamma}{2\pi\xi_2} \int\limits_{\xi^*-\frac{\Delta\xi^*}{2}}^{\xi^*+\frac{\Delta\xi^*}{2}} \frac{\gamma^*_\xi \, d\xi^{*\prime}}{\xi^*-\xi^{*\prime}} \tag{46}$$

and

$$c''_{i\eta} = \frac{1}{1+\tan^2\beta} \frac{\Gamma}{2\pi\xi_2} \int\limits_{\xi^*-\frac{\Delta\xi^*}{2}}^{\xi^*+\frac{\Delta\xi^*}{2}} \frac{\gamma^*_\xi \, d\xi^{*\prime}}{\xi^*-\xi^{*\prime}} . \tag{47}$$

After substituting into Equ. (38), (39) and (46) or (47) Equ. (23), which gives the streamline, the form obtained is

$$\tan\beta_{i+1} = \frac{c_{p\eta}+c_{k\eta}-u_\eta+(c'_{i\eta})_i-(c'_{i\xi})_i\tan\beta_i+c_{i\Delta})}{c_{m\xi}} \tag{48}$$

where $c_{i\Delta}$ can be computed from the integral

$$c_{i\Delta} = \frac{\Gamma}{\xi_2}\,(c_{i\Delta})^* = \frac{\Gamma}{\xi_2} \frac{1}{2\pi} \int\limits_{\xi^*-\frac{\Delta\xi^*}{2}}^{\xi^*+\frac{\Delta\xi^*}{2}} \frac{\gamma^*_\xi(\xi^{*\prime})\, d\xi^{*\prime}}{\xi^*-\xi^{*\prime}} . \tag{49}$$

The computation of the latter can be made without any difficulty. Because

$$\gamma^*_\xi(\xi^{*\prime}) \approx \gamma^*_\xi(\xi^*) + \frac{d\gamma^*_\xi}{d\xi^*}(\xi^{*\prime}-\xi^*) + \ldots$$

and $\Delta\xi^*$ is small, it is sufficient to consider the first three terms, and so

$$c_{i\Delta} \approx \frac{-\Gamma}{2\pi\xi_2} \frac{d\gamma^*_\xi}{d\xi^*}(\xi^*)\,\Delta\xi^* .$$

The "blade sections" can be computed for any surfaces of revolution when starting out for the approximation of zeroth order. Knowing these, the very blade is also known and its vane pattern sections (*i.e* board and radial sections) can be determined without any difficulties in the usual way.

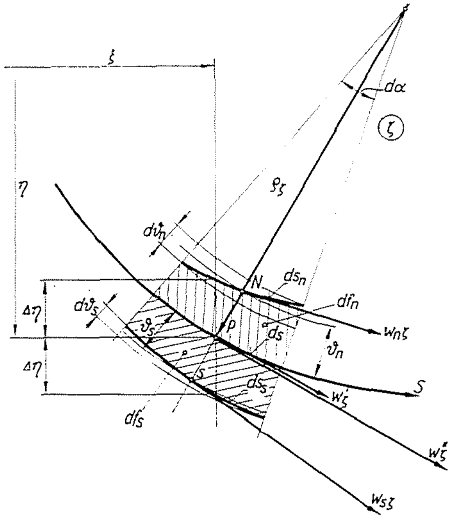

Fig. 5

In the following we shall deal with the method of determining velocity distribution.

As a first step, we have to determine the velocity distribution around the blade of infinitely small thickness. With the symbols used in Fig. 3 the projection in the plane ζ of the relative "mean" velocity (when $u_\eta = u2r\pi/zt$ and $\beta_{i+1} < 0$) can be expressed as

$$w_{\zeta k} = c_{p\eta} \sin \beta_{i+1} + c_{k\eta} \sin \beta_{i+1} + c_{m\xi} \cos \beta_{i+1} - u_\eta \sin \beta_{i+1} + (c_{i\xi})_{i+1}$$

or taking into consideration Equ. (20)

$$w_\zeta = \left[c_{m\xi} + (c_{i\xi})_i + (c_{i\eta})_i \tan \beta_i + (c_{p\eta} + c_{k\eta} - u_\eta) \tan \beta_{i+1} \pm \pm \frac{\Gamma}{2\xi_2} \gamma_\xi^* \right] \frac{1}{\sqrt{1 + \tan^2 \beta_{i+1}}} \tag{50}$$

where the positive sign refers to the suction side, and the negative one to the pressure side. In the following let us denote the first by w_ζ'' and the second by w_ζ'. Fig. 5 shows the element of a transformed blade section, which was sectioned out of the curvature centre of the curve carrying the singularities over an angle da. By determining the blade curve of infinitely small thickness on the plain ζ and plotting the measure of $\Delta\eta = zt\,\varphi_\vartheta/4\pi$ on both sides of the curve, the airfoil of the transformed blade section could be determined. There are known or there can also be determined $\vartheta_n(s)$, $\vartheta_s(s)$, $d\,\vartheta_n(ds, d\,\vartheta_s)\,ds$ and $\varrho_\zeta(s)$. Now, when these are known, the velocity distribution along the blade can already be determined with a good approximation.

Considering the absolute flow as being a potential one $\nabla\times\mathbf{w} = -2\boldsymbol{\omega}$ or the component of vortex vector perpendicular to the surface of revolution is $|(\nabla \times \mathbf{w})_b| = 2\omega\sin\delta$.

If we prescribed the transformation

$$w_\zeta = \frac{2r\pi}{zt}\,w$$

then

$$|\nabla\times\mathbf{w}_\zeta| = 2\omega\sin\delta\left(\frac{2r\pi}{zt}\right)^2 \tag{51}$$

also holds and it can be determined along the curve carrying the singularities as well as along the profile.

For the points S, P and N resp. in Fig. 5 let us introduce the symbols

$$|\nabla\times\mathbf{w}_\zeta|_S\,,\quad |\nabla\times\mathbf{w}_\zeta|_P \quad\text{and}\quad |\nabla\times\mathbf{w}_\zeta|_N$$

resp. With symbols in Fig. 5 equations

$$w_\zeta'\,ds - w_{n\zeta}\,ds_n = -\frac{1}{2}\left(|\nabla\times\mathbf{w}_\zeta|_P + |\nabla\times\mathbf{w}_\zeta|_N\right)df_n$$

and

$$w_{s\zeta}\,ds_s - w_\zeta''\,d_s = -\frac{1}{2}\left(|\nabla\times\mathbf{w}_\zeta|_s + |\nabla\times\mathbf{w}_\zeta|_P\right)df_s$$

can be set down. After putting the values $ds = \varrho_\zeta\,da$,

$$ds_n = \varrho_\zeta\,da\sqrt{(1-\vartheta_n/\varrho_\zeta)^2 + (d\vartheta_n/ds)^2}, \quad ds_s = \varrho_\zeta\,da\sqrt{(1+\vartheta_s/\varrho_\zeta)^2 + (d\vartheta_s/ds)^2},$$

$$df_n = \varrho_\zeta\,da\,(1-\vartheta_n/2\varrho_\zeta)\,\vartheta_n$$

and

$$df_s = \varrho_\zeta\,da\,(1+\vartheta_s/2\varrho_\zeta)\,\vartheta_s$$

we obtain, that

$$w_{n\zeta} = \frac{w_\zeta' + \dfrac{\vartheta_n}{2}\left(1-\dfrac{\vartheta_n}{2\varrho_\zeta}\right)\left(|\nabla\times\mathbf{w}_\zeta|_P + |\nabla\times\mathbf{w}_\zeta|_N\right)}{\sqrt{\left(1-\dfrac{\vartheta_n}{\varrho_\zeta}\right)^2 + \left(\dfrac{d\vartheta_n}{ds}\right)^2}} \tag{52}$$

and

$$w_{s\zeta} = \frac{w_\zeta'' - \frac{\vartheta_s}{2}\left(1 + \frac{\vartheta_s}{2\varrho_\zeta}\right)\left(|\nabla \times \mathbf{w}_\zeta|_S + |\nabla \times \mathbf{w}_\zeta|_P\right)}{\sqrt{\left(1 + \frac{\vartheta_s}{\varrho_\zeta}\right)^2 + \left(\frac{d\vartheta_s}{ds}\right)^2}} \tag{53}$$

On the surface of revolution, that is at the points correlate with point N and S (see Fig. 5), the velocity on the pressure side can be computed from equation

$$w_n = \frac{zt}{2r\pi} w_{n\zeta} \tag{54}$$

and the velocity on the suction side from

$$w_s = \frac{zt}{2r\pi} w_{s\zeta} \,. \tag{55}$$

The reported process affords easy way to determine velocities in whatever point of whichever surface of revolution: between the blades or before and downstream of the impeller.

Velocities referring to any point $P(\xi_P, \eta_P)$ can be determined by putting in the Equ. (16), (17), (18) and (19) of $\xi = \xi_P$ and $\eta = \eta_P$ and the components $c_{i\xi P}$ and $c_{i\eta P}$ of the induced velocity can be computed from Equ. (18) and (19) resp. Knowing these, can be determined without difficulties the components of absolute velocity, as

$$c_{uP} = \frac{zt}{2r\pi}\left(c_{p\eta} + c_{k\eta} + c_{i\eta P}\right) \tag{56}$$

and

$$c_{mP} = \frac{2t}{2r\pi}\left(c_{m\xi} + c_{i\xi P}\right) \tag{57}$$

as well as those of relative velocity, as

$$w_{nP} = c_{uP} - u_P \tag{58}$$

and

$$w_{mP} = c_{mP} \tag{59}$$

Summary

Paper presents an iteration process for designing mixed flow impeller or stator with thin blades, assuming the flow of an incompressible perfect fluid. Approximation is good and computation is relatively simple. As an approximation of zeroth order the blade surfaces of an impeller with infinitely numerous blades will be determined, then, preserving the flow surfaces of rotational form depending on the blade load, the blade sections will be computed using the singularities method after conformal transformation. After the blades had been determined, the velocity distributions along the blade, between the blades, or before and after the impeller could be calculated with good approximation.

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