REMARKS ON THE SOUND DETONATION PHENOMENON

SOUND CONDENSATION, SOUND RAREFACTION, SOUND PROLONGATION, ETC.

By

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The rise, propagation and perception of the sound are well-known, thoroughly investigated fields of physics. Nowadays, there is no problem in explaining the phenomena like reflexion, refraction, diffraction, echo of the sound or the effect of Doppler. However, there are some problems, connected with the arrival of the sound, to be cleared up. Essentially, they are of geometrical character and could be counted in the range of a still missing new branch of science. The sound detonation — discussed in a former paper by the authors[1] (quoted here as "S") — belongs here, but also do the problems we are now concerned with.

1. Sound detonation at two points

First we raise the problem: is it possible that a sound source (jet engine) travelling at constant speed v, gives rise (though not simultaneously) to sound detonations at two different points A, B of space? Thereby is understood that one part of the sound energy emitted by the jet plane on the same arc $\widehat{P_0P}$ of the path arrives at a moment t_1 at A and another part at a moment t_2 at B. (In general $t_1 \neq t_2$.)

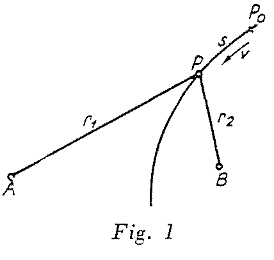

Fig. 1

According to the denotations used in Fig. 1, and to the last formula of "S" p. 288, the corresponding conditions are

[1] This Journal, vol. 1 (1957) pp. 288—295.

$$r_1 = r_{01} - \frac{c}{v} s, \qquad r_2 = r_{02} - \frac{c}{v} s;$$

$$(r_{01} = \overline{AP}_0,\ r_{02} = \overline{BP}_0)$$

whence

$$r_1 - r_2 = r_{01} - r_{02} = \text{const.} \tag{1}$$

This must hold all along the track. On the plane the condition (1) characterizes hyperbolae, the covering of those by a constant speed causes no sound detonation at all. Consequently, there are no such paths on the plane (let a path covered by the speed $v =$ const and causing sound detonation in a point A be called "s—d" path), but — on the contrary — there exist in space, since the condition (1) determines hyperboloids of two shells; the focus of these are in A and B. Really, there are "s—d" paths on these hyperboloids, moreover two of them traverse every point of the surface and they can be easily found (see "S" p. 291). All these paths produce sound detonation both in A and B (of course not simultaneously). In order to avoid misunderstandings, we once more emphasize the energy effecting sound detonation in A and B issues from the same part of the path.

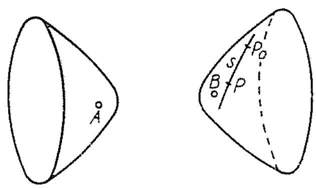

Fig. 2

2. Inversion of the sound detonation. Sound prolongation

Take an "s—d" path concerning point A and speed $v =$ const (either on the plane or in space) and let us assume that the jet plane covers this path moving from A away (see Fig. 3). The flyer steadily hears the sound emitted from A (e. g. like a cannon shot) provided that he heard it at all (at the initial

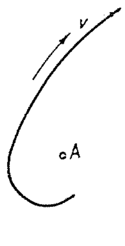

Fig. 3

moment). The explanation is evident. — We are faced with the simplest case, when the jet plane moves in a straight line with the speed of sound away and the cannon shot occurs immediately beside him, *viz.* when airplane and sound travel together. — This "sound prolongation" can be regarded as the inversion of sound detonation.

3. A somewhat similar — but still different — phenomenon, when a jet plane flying on a circular path with speed $v = \frac{\pi}{2} c$, the flyer continuously hears

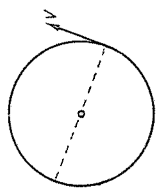

Fig. 4

his own engine's sound from the diametral opposite point, and only from this (during the time that the sound covers the diameter the airplane covers the half circle). In general, assuming $v = \lambda c$, his own engine's sound always arrives at a selfsame angle from the back or in front on the left side according to $\lambda < \frac{\pi}{2}$ or $\lambda > \frac{\pi}{2}$ (s. Fig. 4).

3. Sound condensation, sound rarefaction

This phenomenon occurs, when a sound emitted *e. g.* by a jet plane in a time interval of length t arrives at a point A in the time interval of — say — $\frac{t}{2}$. Does such a path exist? More precisely, the question is: What should be the path and the constant speed of a jet plane if there is a point in space to which

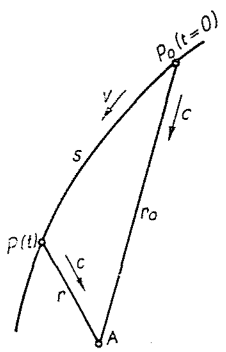

Fig. 5

the sound waves emitted by the jet engine in a time interval t along its course arrives in the time interval λt ($\lambda \lesseqgtr 1$)?

Suppose the sound phenomenon begins in A at a moment $t' = \frac{r_0}{c}$ and ends at $t'' = t + \frac{r}{c} = \frac{s}{v} + \frac{r}{c}$, hence its length is

$$t'' - t' = t + \frac{r}{c} - \frac{r_0}{c}. \tag{2}$$

Our requirement is that

$$t'' - t' = \lambda t$$

be satisfied. Then from (2)

$$t + \frac{r}{c} - \frac{r_0}{c} = \lambda t \qquad \text{or} \qquad r = r_0 - (1-\lambda) c t = r_0 - (1-\lambda)\frac{c}{v} s. \tag{3}$$

If we write this in form

$$r = r_0 - \frac{c}{\dfrac{v}{1-\lambda}} s$$

and compare with formula $r = r_0 - \frac{c}{v} s$ of the sound detonation, we see without further computation that these sound condensing paths ("s—c" paths) are the same as the "s—d" paths belonging to speed $v' = \frac{v}{1-\lambda}$, *i. e.* logarithmical spirals on the plane, helices on the cone, etc. For the angle formed by the tangent of these curves and the radius vector we have $\cos\alpha = (1-\lambda)\frac{c}{v}$. The lower limit of the necessary speed is now not c, but $c(1-\lambda)$, resp. — regarding the sound rarefaction too ($\lambda > 1$) — $c\,|1-\lambda|$. *E. g.* if $\lambda = \frac{1}{2}$ then $v \geqq \frac{c}{2}$. To a given λ and v satisfying this condition there always exist "s—c" paths.

There are two interesting boundary cases: $\lambda = 0$ involves the sound detonation (when duration of the sound shrinks to 0) and $\lambda = 1$ — the duration remains unchanged — involves $r = r_0$, the corresponding path is a circle of radius r_0 or an arbitrary curve on the surface of the sphere with radius r_0.

5. In the foregoing the two durations — the new and the original — were proportional to each other. However, their connection may otherwise be prescribed too (also by a different law). Let $f(u)$ be an arbitrary continuously differentiable function. Then, instead of the requirement $t'' - t' = \lambda t$, the following more general nonlinear "condition of condensation" can be assumed

$$t'' - t' = f(t).$$

Then (3) would be replaced by

$$t + \frac{r}{c} - \frac{r_0}{c} = f(t) \quad \text{or} \quad r = r_0 - c\,[t - f(t)] = r_0 - c\left[\frac{s}{v} - f\left(\frac{s}{v}\right)\right], \tag{3'}$$

whence — taking only the case of plane motion — the parametric equation of the path may easily be found. t plays the role of the parameter, therefore we also obtain the law governing the course of the path. *Viz.*, using polar co-ordinates

$$\frac{ds}{dt} = \sqrt{r^2\left(\frac{d\varphi}{dt}\right)^2 + \left(\frac{dr}{dt}\right)^2} = v,$$

but from (3′)

$$\frac{dr}{dt} = -c\,[1 - f'(t)].$$

Hence we have for $\frac{d\varphi}{dt}$ the value

$$\frac{d\varphi}{dt} = \frac{\sqrt{v^2 - c^2\,[1 - f'(t)]^2}}{r_0 - c\,[t - f(t)]}.$$

Thus the parametric equation of the curve is

$$r = r_0 - c\,[t - f(t)],$$

$$\varphi = \varphi_0 + \int_0^t \frac{\sqrt{v^2 - c^2\,[1 - f'(\tau)]^2}}{r_0 - c\,[\tau - f(\tau)]}\,d\tau.$$

The square root must be real and this implies a restriction on $f'(t)$

$$1 - \frac{v}{c} \leq f'(t) \leq 1 + \frac{v}{c}.$$

6. Interchanging the roles of t' and t'' in 4. (taking t'' as starting time and t' as end time) we have the relation

$$t' - t'' = -t - \frac{r}{c} + \frac{r_0}{c} = \lambda t, \qquad (\lambda \geqq 0)$$

or

$$r = r_0 - (1 + \lambda)\, ct = r_0 - (1 + \lambda) \frac{c}{v} s$$

and we are not faced with a mere sound condensation (or rarefaction), but the sound signs arrive at A in a reversed order. The necessary speed must exceed $c\,(1 + \lambda)$. *E.g.* the speed necessary for half its condensation and the reversion is at least $\frac{3}{2} c$. The non-negativity of λ need not be assumed in (3), only this involves the reversion of the sound signs.

7. The sound condensation phenomenon is also reversible. An airplane flying on an "s—c" path in a reversed direction and with the same speed, the sound emitted in A during a time interval t will be heard by the flyer during the time interval $\frac{t}{|\lambda|}$ and — provided $\lambda < 0$ — the order of the signs will be reversed.

Summary

This paper contains supplements and remarks on a former paper of the authors (this Journal, vol. 1 (1957) pp. 288—295) concerning the sound detonation phenomenon. Sound condensation, sound rarefaction, sound prolongation are treated here as new phenomena.

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