ECONOMIC HEAT EXCHANGER DESIGN

By

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The need of efficient heat exchangers is ever increasing in many fields of modern engineering. In the course of development new fields are opened up, whose progress is primarily dependent on the quality of available heat exchangers. In fact the problem of heat exchangers is one of the bottlenecks in modern engineering.

Let us quote, as an example, one of the most interesting and important technical fields, that of nuclear science.

The limiting factor of the existing nuclear reactors is the heat exchange taking place within the reactor on the one hand — (since the reactor core may be considered as a special heat exchanging equipment) — and the process of heat exchange of the activated medium in a conventional heat exchanger, on the other.

Another example: With the exhaust gases of diesel engines significant heat quantities, theoratically still to be utilized, escape into the atmosphere. The utilization of this heat is economical only if the process can be realized by appropriate compact heat exchangers, at small cost.

Heat exchangers are especially significant in connection with gas turbines. When the application of the Joule-cycle was first considered, the first problem to be solved was the construction of a compressor of adequate efficiency. Since poor compressor efficiency affects cycle efficiency to such extent, that below a certain level of the cycle would not yield any work whatever and so on the solution of this problem hinged the realization of the cycle.

When this problem was solved, the technical realization of the Joule-cycle became possible and at present the efficiency of both compressor and turbine have reached such a high level that a sudden or sensational improvement in this sphere can not be expected in the foreseeable future. Notable improvement is only that of the cycle-thermodynamics which promises good results.

Improvement may be expected from rendering compression and expansion isothermic, respectively, from their prerequisite, recuperation, that is, from recooling of the turbine exhaust gases by compressed gases. As is known, isothermic compression and expansion bring about improved efficiency only if recuperation, too, takes place. While in the absence of recuperation isothermic compression and expansion not only do not improve, but might adversely affect cycle efficiency, recuperation in itself, without simultaneous isothermic compression and expansion, greatly improves it.

Owing to the decisive importance of heat exchangers, the ever growing interest shown as to their economic design throughout the technical world is comprehensible.

Since the economy of a heat exchanger, for a given task, is determined by different data which depend on the purpose and sphere of application, there are many sides of the problem. The many points of view may ultimately be traced back to two basic principles which fundamentally affect heat exchanger economy.

One is the cubic capacity of the heat exchanger and the requisite cross sectional area of flow on each side. This will determine the space requirement of the heat exchanger and the difficulties which are to be expected with its installation.

The other factor — more closely related to economics — is the cost of the heat exchanger proper. Instead of this item, as a fairly good approximation, the weight and material of the heat exchangers may be substituted, the expression "material" naturally including eventual differences, caused by manufacturing processes. The consideration of these conditions will not create any difficulties.

It is, naturally, possible to express the space requirement and space limitations in terms of money but its prerequisite is to know what purposes the equipment is going to serve. The value of built-in space will be widely different in cases where each cubic metre has to be spared from useful storage space, as for instance in ships, planes, motorcoaches, as for in instances where sufficiant space is available. While the application of a huge cross sectional flow area in engine-borne heat exchangers will meet with insurmountable difficulties, the same will be an easy task to solve in the natural draught cooling towers of an air condensing equipment. It is just on account of the manyfold points of view arising in this field that both factors — the space problems on one hand and the costs on the other — are being discussed separately, as in this way they will lend themselves better as a basis for further economic examinations.

The two points of view as to the adequacy of a heat exchanger can ultimately be followed back to four basic data: the heat transfer area (F m^2), the requisite cross sectional area of flow (F_0 m^2), the type of the heat exchanger under consideration (smooth ribs, tubes, strip-finned ribs, etc.), and, finally, the material of the heat exchanger (which latter may be considered as material costs depending on the quality of the surface). Among these factors an interdependence is created by the work applied in forcing through the flowing media, by the heat transferred per unit of temperature difference, and by the volume of the flowing medium. Accordingly, the sole practical basis for investigations as to heat exchanger economics should be the solution of this complex system of functions.

The economy of heat exchangers depends on the quality and geometry of the surface, the factors which seriously affect heat transfer coefficient (as termed previously: it is dependent on the heat exchanger type). For this reason it seems expedient to divide the exchanger into two parts. Let us first follow the path of heat. One of the heat carriers transfers its heat across one of the surfaces, at an intermediary temperature, to a dividing plane or to an inter-

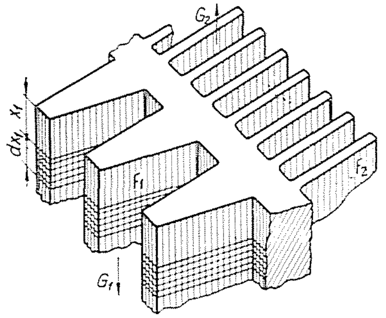

Fig. 1

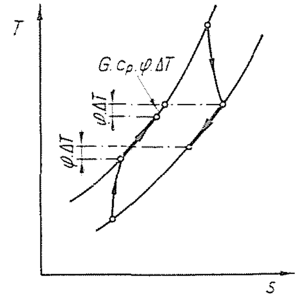

Fig. 2

mediary medium, and the dividing plane or medium will transfer the heat, through another surface, to another heat absorbing medium.

Let us examine the two surfaces separately (Fig. 1).

The entire temperature difference between the two heat transmitting media is $t_1 - t_2$. On the effect of the $t_1 - t_0$ temperature difference the heat through the F_1 surface reaches the t_0 temperature level, whereafter, owing to the $t_0 - t_2$ temperature difference, it warms the other medium up through the F_2 surface. The quality of heat is perfectly determined by the properties and speed of one of the flowing media, by the surface type (fin efficiency), and size of one of the surfaces, and by the temperature difference between the medium and the plane at t_0 temperature, which is considered as the theoretical boundary of the surface. Thus, assuming the above theoretical plane, surfaces F_1 and F_2 can be easily divided and the two heat transfers separately discussed. Since the pressure drop of one medium is entirely independent from that of the other, the same division can be carried out in connection with the examination of pressure drops.

This concept is of special interest in the examination of nuclear reactors. Owing to the fact that the heat producing medium — which may be considered as "the other side" — has no effect whatsoever on the type of the heat transmitting surface, they are easily separable.

During the thermodynamic examination of nuclear reactors the calculation will naturally be somewhat different but, along the principles laid down here, they can be carried out without any difficulty.

In the frequently occurring cases, when heat exchange between two gases of poor heat transfer coefficients is carried out by inserting a third medium of excellent heat transfer coefficient — may it be a liquid metal or another fluid — the division may be considered as actually effected. Under such circumstances in the limitation case, if heat transfer coefficient may be considered as being infinite, the construction of the two heat exchangers is entirely independent, in spite of the fact that these may be taken as the two sides of one single gas-to-gas heat exchanger.

This separation is justified — particularly in gas-to-gas heat exchangers — even in connection with the conventional types, because improved heat transfer as well as compactness has to be aimed at on both sides. Thus, in order to find the optimal solution, every combination of all heat exchanger types should be tested.

Such a combination, of course, is subject to conditions, the first being that both surfaces have to transfer the same given heat performance, while the sum of the temperature differences of both surfaces just equals the temperature difference, permissible for the whole heat exchanger. (Assuming the permissible temperature difference at $t_1 - t_2$, the F_1 area is to take over from the first medium, at a $t_1 - t_0$ temperature difference, the same heat quantity as F_2 area transfers to the second medium, at a temperature difference of $t_0 - t_2$.)

The second condition, valid only if both surfaces are united in the same space, is the existence of certain geometric conditions. If heat is transferred through an intermediary liquid, this condition is of secondary importance or may be eliminated (ignored) altogether.

At first our investigations will be restricted to one single surface. Let us assume a heat transfer area of F m², a cross sectional area for the flow of the medium (most frequently the narrowest) of F_0 m², a temperature difference of ΔT between the medium and the theoretical limiting plane of the area, and a so-called fin efficiency of ε. Restricting our investigations to a section of the heat exchanger of dx length, in the direction of flow (Fig. 1)

$$dQ = G \cdot c_p \cdot dt \tag{1}$$

respectively,

$$dQ = \varepsilon \cdot a \cdot \Delta T \cdot dF \tag{2}$$

Equations (1) and (2) denote the principle of conservation of energy for a section of dx length on one side of the heat exchanger, assuming implicitely that the heat exchanger is in a stationary state. dQ kcal/h is, namely, the quantity of heat transferred to G kg/h flowing medium, computed from the equation of heat transfer and the heat absorption in connection with the temperature rise of the medium. c_p kcal/kg C° denotes the specific heat measured at constant pressure, dt C° the change in the temperature of the medium, taking place along the length of dx meter, and finally a kcal/m² h C° the heat transfer coefficient between medium and surface.

In view of the fact that it is only the theory of models and the impulse theory that afford a relatively accurate calculation of the coefficients of heat transfer and friction — these factors being of decisive importance in the dimensioning of heat exchangers — thus, these theories seemed to be the best starting points for calculations. The application of the Stanton-number (which simultaneously considers heat transfer coefficient and velocity) and the Euler-number (considering pressure drop) seems to serve this purpose best.

In the form of definition:

$$N_{St} = \frac{a}{w \cdot c_p \cdot \gamma} . \tag{3}$$

The volume of medium flow may be measured by the cross sectional area of flow and the velocity of medium flowing in this area.

$$G = F_0 \cdot w \cdot \gamma \tag{4}$$

where w m/h flow velocity, and γ kg/m³ the specific weight of the medium.

In consideration of equations (1), (2), (3) and (4) it is at once apparent that

$$N_{St} = \frac{F_0}{\varepsilon \cdot \Delta T} \cdot \frac{dt}{dF} . \tag{5}$$

On the other hand, from the equations (3) and (4)

$$a = c_p \cdot \frac{G}{F_0} \cdot N_{st} . \tag{6}$$

The (5) equation may be written in the following form

$$F_0 = \varepsilon \cdot \Delta T \cdot N_{St} \cdot \frac{dF}{dt} . \tag{7}$$

Let us now introduce the Fanning friction factor, characteristic of the pressure drop. As a definition:

$$\frac{dN_{Eu}}{dx} = \frac{2f}{D} \tag{8}$$

where D is a length measured perpendicular, and characteristic to the flow (diameter in case of a flow along a tube, hydraulic diameter otherwise).

N_{Eu} is the Euler-number

f the Fanning factor

This is the definition of the Euler-number.

$$dN_{Eu} = \frac{g \cdot dP}{\gamma \cdot w^2} = g \cdot \gamma \cdot \frac{F_0^2}{G^2} \cdot dP \tag{9}$$

where g represents the constant of gravitation expressed in m/h^2, dP the pressure drop of the medium along the dx section expressed in kg/m^2.

In the second part of the (9) equation, the (4) equation had been taken into consideration.

Combining equations (1) and (8):

$$\frac{dQ}{dN_{Eu}} = \frac{G \cdot c_p \cdot D}{2f} \cdot \frac{dt}{dx}$$

respectively

$$\frac{dx}{dt} = \frac{G \cdot c_p \cdot D}{2f} \cdot \frac{dN_{Eu}}{dQ} . \tag{10}$$

Writing the (7) equation in the following form:

$$F_0 = \varepsilon \cdot \Delta T \cdot N_{St} \cdot \frac{dx}{dt} \cdot \frac{dQ}{dN_{Eu}} \cdot \frac{dN_{Eu}}{dP} \cdot \frac{dF}{dx} \cdot \frac{dP}{dQ} \tag{11}$$

and applying equations (9) and (10) respectively, we come to the following result:

$$F_0 = \varepsilon \cdot \Delta T \cdot N_{St} \cdot \frac{G \cdot c_p \cdot D}{2f} \cdot g \cdot \gamma \cdot \frac{F_0^2}{G^2} \cdot \frac{dF}{dx} \cdot \frac{dP}{dQ} . \tag{12}$$

Arranged:

$$F_0 = \sqrt{\frac{2}{g \cdot \gamma \cdot c_p}} \cdot \sqrt{F_0 \frac{dx}{dF}} \cdot \sqrt{\frac{f}{\varepsilon \cdot D \cdot N_{St}}} \cdot \sqrt{\frac{G}{\Delta T} \cdot \frac{dQ}{dP}} . \tag{13}$$

We have thus arrived at the characteristic cross sectional area of flow, as the product of four factors. The first factor is a function of the properties of the flowing medium only

$$\sqrt{\frac{2}{g \cdot \gamma \cdot c_p}} = \varphi(\Phi)$$

where the dependence of the properties is denoted by the $\varphi(\Phi)$ symbol.

The second factor indicates the magnitude of the heat exchanging surface, — respectively, its square root — per unit length at unit cross sectional flow area. This factor is the function of the heat exchanger geometry only, since it is proportionate to heat exchanger surface per heat exchanger unit capacity:

$$\sqrt{F_0 \frac{dx}{dF}} = \varphi(\Gamma).$$

$\varphi(\Gamma)$ signifies the dependence on the geometric arrangement, on the material of the heat exchanger, and on its type.

The third factor contains the $\varepsilon \cdot D$ product, which obviously is the function of the exchanger geometry and the value: $\sqrt{f/N_{St}}$ which — pursuant from the theory of models — is the function of the Reynolds- and Prandtl-numbers. The Prandtl-number characterizes the physical properties of the medium, while the Reynolds-number is a function of the geometric conditions, of velocity, and of kinematic viscosity. Thus:

$$\sqrt{\frac{f}{\varepsilon \cdot D \cdot N_{St}}} = \varphi(\Gamma, N_{Re}, N_{Pr}).$$

The fourth factor is a function of the operating conditions only, respectively the initial design data of the heat exchanger:

$$\sqrt{\frac{G}{\Delta T} \cdot \frac{dQ}{dP}} = q_{F_0}. \tag{14}$$

Let us now introduce the N_{F_0}, a dimensionless term:

$$N_{F_0} = \sqrt{\frac{F_0}{\varepsilon D} \cdot \frac{dx}{dF}} \sqrt{\frac{f}{N_{St}}} = \psi(\Gamma, N_{Re}, N_{Pr}). \tag{15}$$

N_{F_0}, owing to its first radical, is obviously dependent on the geometry of the heat exchanger while, owing to its second radical, is dependent on the Reynolds- and Prandtl-numbers.

Thus, the authentic cross sectional area of flow is:

$$F_0 = \sqrt{\frac{2}{g \cdot \gamma \cdot c_p}} \cdot q_{F_0} \cdot N_{F_0} \tag{16}$$

or, expressed in a different form

$$F_0 = g_{F_0} \cdot N_{F_0} \tag{17}$$

where

$$g_{F_0} = \sqrt{\frac{2}{g \cdot \gamma \cdot c_p} \cdot \frac{G}{\Delta T} \cdot \frac{dQ}{dP}} \tag{18}$$

is the function only of the properties of the medium and the operating conditions, consequently it may be developed from the initial data. Denoting the dependence on the initial data, characteristic of the heat exchanger, by the symbol Ω, the following may be written:

$$g_{F_0} = \varphi(\Omega)$$

and

$$N_{F_0} = \psi(\Gamma, N_{Re}, N_{Pr}) .$$

Similar examinations may be made regarding the heat exchanging surface to be incorporated.

Comparing, namely, the equations (2), (3), and (4), we arrive at

$$dQ = \varepsilon \cdot c_p \cdot \frac{G}{F_0} \cdot N_{St} \cdot \Delta T \cdot dF .$$

Whence:

$$\frac{dF}{dQ} = \frac{1}{c_p} \cdot F_0 \cdot \frac{1}{G \cdot \Delta T \cdot \varepsilon \cdot N_{St}} .$$

Taking the (17) equation now into consideration,

$$\frac{dF}{dQ} = \frac{g_{F_0}}{c_p \cdot G \cdot \Delta T} \cdot \frac{N_{F_0}}{\varepsilon N_{St}}$$

from which

$$dF = \frac{g_{F_0}}{c_p \cdot G \cdot \Delta T} \cdot dQ \cdot \frac{N_{F_0}}{\varepsilon N_{St}}$$

respectively:

$$F = \int_0^Q \frac{N_{F_0}}{\varepsilon N_{St}} \cdot \frac{g_{F_0}}{c_p \cdot G \cdot \Delta T} \, dQ = \left[\frac{N_{F_0}}{\varepsilon N_{St}}\right]_m \int_0^Q \frac{g_{F_0}}{c_p \cdot G \cdot \Delta T} \, dQ . \tag{19}$$

As the value of $N_{F_0}/\varepsilon N_{St}$ does not undergo material changes along the length of the heat exchanger, by substituting it with mean parameters and considering it as constant, a fairly good approximation can be obtained.

Applying the same method, two characteristic numbers may be introduced. One is, again, dimensionless and the function of the heat exchanger type, the Reynolds-number and the Prandtl-number, while the other will be determined by the design conditions

$$N_F = \frac{N_{F_0}}{\varepsilon N_{St}} = \sqrt{\frac{F_0}{\varepsilon^3 D} \cdot \frac{dx}{dF}} \sqrt{\frac{f}{N_{St}^3}} = N_F(\Gamma, N_{Re}, N_{Pr}) \tag{20}$$

(see equation (15)).

On the other hand

$$g_F = \int_0^Q \frac{g_{F_0}}{c_p \cdot G \cdot \Delta T} \cdot dQ = \varphi(\Omega) = \frac{F}{N_F}. \tag{21}$$

The characteristic numbers thus introduced may naturally be expressed also in function of the friction work, the heat output per unit temperature difference, and the volume of flow.

The value of friction work, volume of flow, and heat output per unit temperature difference is expressed by the following equations:

$$dL = \frac{G \cdot dP}{\gamma} \tag{22}$$

where dL denotes the work put in by G quantity of medium, to overcome friction along the dx section, if the medium is considered as being incompressible.

$$V = \frac{G}{\gamma} \tag{23}$$

where V is the volume of the medium flow per unit time, expressed in m^3/h.

Denoting the heat quantity per unit temperature difference transferred per unit time by q kcal/h C°,

$$dq = \frac{dQ}{\Delta T}. \tag{24}$$

Considering the (18) equation and on ground of the above equations, we arrive at

$$g_{F0} = \frac{2V}{\sqrt{2g \cdot c_p}} \sqrt{\frac{dq}{dL}} \tag{25}$$

or, combining the equations (25) and (21):

$$g_F = \int_{x=0}^{x} \frac{2}{\gamma \cdot c_p \sqrt{2g \cdot c_p}} \sqrt{\frac{dq}{dL}}\, dq. \tag{26}$$

Integration must be carried out for the full length of the heat exchanger. In order to obtain a form easier for handling let us introduce the following modification:

$$g_F = \int_{0}^{x} \frac{2}{\gamma \cdot c_p \sqrt{2g \cdot c_p}} \sqrt{\frac{dq}{dL}} \cdot \frac{dq}{dx} \cdot dx. \tag{27}$$

It is obvious from the (27) equation that, assuming entirely identical structure throughout the full length of the heat exchanger, and constant heat transfer coefficient, respectively, medium properties — the integrand being also constant in this case — g_F may be calculated in the following manner:[1]

$$g_F = \frac{2}{\gamma \cdot c_p \cdot \sqrt{2g \cdot c_p}} \cdot \sqrt{\frac{q}{L}} \cdot q. \tag{28}$$

Since $dq = \varepsilon \cdot \alpha \cdot dF$ (see equations (2) and (24)), if ε and α are constant, then $q = \varepsilon \cdot \alpha \cdot F$. But, just in the above-outlined conditions for an optional heat exchanger — be it of the direct or counterflow type — it has been established[2] that

$$\varepsilon \cdot \alpha \cdot F = \frac{Q}{\Delta T_K} \tag{29}$$

where ΔT_K denotes the logarithmic mean of the temperature differences occurring in the heat exchanger.[3]

From the foregoing it will be apparent that the ΔT_K logarithmic mean temperature difference, characteristic of the whole heat exchanger, may well be applied here, instead of the ΔT local temperature difference, if the same conditions as usual with heat exchanger calculations, prevail. As, however, conditions are seldom fully identical with those as enumerated, theoretically

[1] Problems arising in practice can in most cases be solved by calculations with the arithmetic mean value of the extremes occurring along the full length of the heat exchanger.

[2] The ΔT_K so defined may be ascertained also for optional cross flow, by the introduction of an appropriate correction factor (see VDI Wärmeatlas, Table Ca 1—8).

[3] In the case dealt with, naturally only the part which falls on one half of the heat transmitting surface.

the integral mean value corresponding to the variable group included in the value of g_F should be assumed for the whole length of the heat exchanger. However, the arithmetic mean value in most cases fairly meets practical requirements.

Let us now put down the defining equation of the Reynolds-number:

$$N_{Re} = \frac{w \cdot D}{\nu} = \frac{G \cdot D}{F_0 \cdot \gamma \cdot \nu} \qquad \text{(see(4))}$$

where ν [m²/h] stands for the kinematic viscosity of the medium.

Comparing this equation with the (17), we arrive at

$$F_0 = g_{F_0} \cdot N_{F_0} = \frac{G \cdot D}{N_{Re} \cdot \gamma \cdot \nu}$$

whence

$$\frac{N_{Re} \cdot N_{F_0}}{D} = \frac{G}{\gamma \cdot \nu} \cdot \frac{1}{g_{F_0}} = \frac{V}{\nu \cdot g_{F_0}} = \Lambda. \qquad (30)$$

The (30) equation defines the Λ characteristic number.

While in (30) the $\dfrac{N_{Re} \cdot N_{F0}}{D}$ expression is a function only of the heat exchanger type and the Reynolds- and Prandtl-numbers, the $V/\nu \cdot g_{F_0}$ expression is dependent solely on the initial design conditions. The right side of the equation, obviously, cannot include absolute data characteristic of the absolute dimensions of the heat exchanger, only ratios, because otherwise the equation would contain contradictions. Actually, substituting the value of g_{F_0} — applying the (25) equation and considering those said in connection with (28) — we arrive at

$$\Lambda = \frac{V}{\nu \cdot g_{F_0}} = \frac{\sqrt{2g \cdot c_p}}{2\nu} \sqrt{\frac{L}{q}} \qquad (31)$$

(see equation (25)).

It is quite clear that apart from the material properties, the value of Λ depends solely on the L/q ratio.

Considering that

$$N_{F_0} = N_{F_0}(N_{Re}, N_{Pr}, \Gamma)$$

at a given Γ and likewise given N_{Pr}, N_{F_0} is the function of N_{Re} only. Considering further that $D = \varphi(\Gamma)$, it follows from the (30) equation that at a given type of heat exchanger and at nearly constant Prandtl-number, Λ is the function only of the Reynolds-number or, in other words, of N_{F_0} (with gas turbines this condition is fulfilled with fair approximation).

Comparing the (30) equation with the (20), it is obvious that

$$A = \frac{N_{Re} \cdot N_{F_0}}{D} = \frac{\varepsilon}{D} \cdot N_{Re} \cdot N_{st} \cdot N_F . \tag{32}$$

N_F being the only function of Γ, N_{Re} and N_{Pr}, the above conditions are valid also on N_F.

As a result it has been ascertained that at a given type of heat exchanger and given Prandtl-number (type of medium)

$$N_F = N_F(A) \qquad \text{and} \qquad N_{F_0} = N_{F_0}(A)$$

dimensionless terms, characteristic of the heat exchanging surface and the cross sectional area of flow, are functions of A only.

Some explanation should be given here as to the A number. A might be called the reciprocal of the characteristic heat exchanger dimension, expressed in 1/m. It is interesting to note that apart from the L/q ratio, which is generally considered to be the one characteristic of the operating conditions of the heat exchanger, A materially depends also on the properties of the flowing medium, taking part in the heat exchange process. In other words, the characteristic curve, resp. the economy of the heat exchanger is dependent not only on the L/q ratio but also on the quality of the medium taking part in the heat exchange, and on the value of its parameters. Thus, contrary to the concept as held hitherto, to be able to choose the most economical one from among various types of heat exchangers, it is not enough to know the L/q ratio, but the medium must also be known.

With the aid of the above analysis, any given heat-exchanging surface may easily be submitted to economic examinations. For example, investment items in many cases can be classified with good approximation under the following three groups:

1. Capital charges

Capital charges are nearly proportionate with the installed area and may be written as follows:

$$B_F = b_F \cdot F = b_F \cdot g_F \cdot N_F = b_F \frac{2}{\gamma \cdot c_p \cdot \sqrt{2 g c_p}} \sqrt{\frac{q}{L}} \cdot q N_F$$

(see equations (21) and (28)).

Since

$$\Delta = \frac{\sqrt{2g \cdot c_p}}{2\nu} \sqrt{\frac{L}{q}} \qquad \text{(see(31))}$$

this equation may be arranged in the following manner:

$$B_F = b_F \frac{q}{\gamma \cdot c_p \cdot \nu} \cdot \frac{N_F}{\Delta}.$$

Let us introduce the following new expressions:

$$\delta = \frac{c_p}{q} = \frac{c_p \Delta T}{Q} \tag{33}$$

$$n_F = \frac{N_F}{\Delta}. \tag{34}$$

Applying these formulae, the capital costs of the heat exchanger will be:

$$B_F = \frac{b_F}{\gamma \cdot \nu} \cdot \frac{n_F}{\delta}. \tag{35}$$

In this (35) equation b_F naturally represents the amortization cost of 1 sq. m of heat exchanger surface related to a certain given period (1 year, 25 years).

2. Operating costs

$$B_L = b_L \cdot L. \tag{36}$$

In (36) b_L is obviously the product of the utilization factor characteristic of the respective period and the monetary value of the work spent on circulating the medium.

From the (31) equation it may be written that

$$L = \frac{2\,\Delta^2 q \cdot \nu^2}{g \cdot c_p} = \frac{2\nu^2}{g} \cdot \frac{\Delta^2}{\delta}$$

whereby (36) will take the following form:

$$B_L = b_L \frac{2\nu^2}{g} \cdot \frac{\Delta^2}{\delta}.$$

3. Costs incurred by temperature differences taking place in the heat exchanger

Fig. 2 is the Ts-chart of a gas turbine cycle. It clearly illustrates that the temperature difference $\varphi \Delta T$ taking place in the heat exchanger affects the initial temperature in the combustion chamber. Thus, the $\varphi \Delta T$ heat gap prescribes the introduction of the additional heat quantity of $\varphi \cdot \Delta T \cdot c_p \cdot G$ without any changes in the work obtainable from the cycle. The fact that the entire temperature difference taking place in the heat exchanger is the sum of the temperature differences of both the colder and warmer sides, is being considered by the φ factor. Thus it is obvious that the heat quantity to be considered with the half under examination, is equivalent to just $c_p \cdot G \cdot \Delta T$. The product of this heat quantity with the utilization factor characteristic to the respective period and with the costs of the thermal power introduced[1] (the product of these latter two is denoted by b_q) will represent the expenses incurred by the temperature gap.

Thus

$$B_q = b_q \cdot c_p \cdot \Delta T \cdot G$$

whence, considering the (33) equation:

$$B_q = b_q \cdot \delta \cdot Q \cdot G. \tag{38}$$

Accordingly, the heat exchanger will be charged by the following costs:[2]

$$B = B_F + B_L + B_q = \frac{b_F}{\gamma \cdot \nu} \cdot \frac{n_F}{\delta} + b_L \frac{2\nu^2}{g} \frac{\Delta^2}{\delta} + b_q \cdot \delta \cdot Q \cdot G =$$

$$= \frac{1}{\delta g} \cdot \left[\frac{g \cdot b_F}{\nu \cdot \gamma} \cdot n_F + 2 b_L \nu^2 \Delta^2 \right] + b_q \cdot \delta \cdot Q \cdot G.$$

Let us now introduce the concept of specific charge, that is, the charge per 1 kcal/h:

$$\beta = \frac{B}{Q} \tag{39}$$

and substitute the value of δ (see (33)):

$$\beta = \frac{1}{c_p \cdot g \cdot \Delta T} \left[\frac{g \cdot b_F}{\nu \cdot \gamma} n_F + 2 b_L \nu^2 \Delta^2 \right] + b_q \cdot c_p \cdot \Delta T \cdot \frac{G}{Q} \tag{40}$$

[1] The cost of the introduced thermal power will have to include the cost of fuel, the amortization costs of the combustion equipment plus all other associated expenses.

[2] B_L and B_q should be calculated for a period, identical with B_F.

thus β has been derived as the function of Λ and ΔT. Let us determine the minimum value of β. For this reason we have to evolve both partial derivatives and make these equal to zero.[1]

$$\left[\frac{\partial \beta}{\partial \Lambda}\right]_{\Lambda=\Lambda_0} = \frac{1}{c_p \cdot g \cdot \Delta T} \cdot \left[\frac{g \cdot b_F}{\nu \cdot \gamma}\left(\frac{\partial n_F}{\partial \Lambda}\right)_{\Lambda=\Lambda_0} + 4 b_L \cdot \nu^2 \Lambda_0\right] = 0$$

respectively, since $1/\Delta T \neq 0$

$$\frac{1}{\Lambda_0}\left(\frac{\partial n_F}{\partial \Lambda}\right)_{\Lambda=\Lambda_0} = -4 \frac{b_L}{b_F} \frac{\nu^3 \gamma}{g} . \tag{41}$$

Λ_0 is the number which characterizes optimal heat exchanger dimensions at given operating conditions.

Since the left side of the (41) equation is a function of Λ to be graphically derived at any type of surface, and its right side is calculable, this (41) equation enables us to determine the optimal Λ_0 at a given ΔT, in the graphical way. We arrive at the very interesting result that the value of the optimal Λ is independent on the temperature difference.

Let us now evolve the partial derivative of β, according to $1/\Delta T$.

$$\frac{\partial \Lambda}{\partial (1/\Delta T)} = \frac{1}{g c_p}\left[\frac{g b_F}{\nu \cdot \gamma} \cdot n_F + 2 b_L \nu^2 \Lambda^2\right] - b_q \cdot c_p \cdot \Delta T^2 \frac{G}{Q}.$$

Making the equation equal to zero we arrive at

$$\Delta T_0 = \frac{1}{c_p} \sqrt{\frac{Q}{G}} \sqrt{\frac{b_F \cdot [n_F]_{\Lambda=\Lambda_0} \cdot g + 2 b_L \nu^3 \gamma \Lambda_0^2}{b_q \cdot \nu \cdot \gamma \cdot g}} . \tag{42}$$

Having computed the value of the optimal Λ_0 with the aid of the (41) equation, now it is possible to determine the value of the optimal ΔT_0, by the aid of (42). Thereafter the optimal N_F is also calculable, being the unique function of Λ. The determination of optimal g_F is likewise possible since, on ground of the (28) equation,

$$g_F = \frac{1}{c_p \cdot \gamma \cdot \nu} \cdot \frac{Q}{\Delta T} \cdot \frac{1}{\Lambda} . \tag{43}$$

Now optimum dimensions of the surface may also be determined for these are the product of g_F and N_F, calculated in the above-described manner.

[1] In connection with given N_F curves it has to be ascertained whether there is an extreme value and whether it is the minimum.

No difficulties will arise in determining the F_0 optimum of the cross sectional area of flow either, this being the product of g_{F0} and N_{F0} which can be determined, partly by the initial data, partly by Λ.

g_{F0} may best be determined by a slight modification of the (30) equation, with the aid of Λ, whence,

$$g_{F_0} = \frac{V}{\nu \cdot \Lambda} = \frac{\gamma \cdot G}{\nu \cdot \Lambda}. \tag{44}$$

The foregoing examinations are naturally only valid for given surfaces and are to be carried out for each side of the heat exchanger separately. The method outlined may be directly applied in such cases when the two surfaces can be altered independently; in cases, for instance, when the heat exchange is effected by the intermediary of a medium having very good heat transfer coefficient or else, when one side of the heat exchanger is ribbed. On the other hand, in case of conventional gas turbine recuperators, owing to technical-constructional reasons, there is generally some interconnection between the two surfaces and one side of the surface, to some extent, always determines the other side. This fact may be expressed mathematically by a functionality existing between the two surfaces and the two cross sectional flow areas.

If the functionality is not altogether close and the optimal dimensions for both sides of the heat exchanger can be realized in one single installation, our calculations can be directly applied. If, however, the relation is close, the problem becomes an optimum-calculation of a function of four variables where the functions relating to the technical feasibility establish some interconnection between the surfaces and cross sectional flow areas. Our calculation method may be applied even in such cases, however, with certain considerations.

In summing up it may be stated that this paper deals with a method which has been worked out to enable the evaluation of measurements of various types and ribbings of heat exchangers. On ground of the experimental data obtained, the $n_F(\Lambda)$ function can be evolved, and by this function the minimum calculation, as outlined, may be carried out graphically. This method at the same time enables the exact differentiation of the fields of application for various types of surfaces and their comparison, respectively, thereby making reliable heat exchanger design possible not only technically but also from the point of view of economy.

Summary

A theoretical method for the

1. comparison of economics of different heat exchanger types;
2. determination of the optimal type for various media and various operating conditions;
3. determination of the characteristics of the optimal type.

Nomenclature

b		
b^F		Capital charge per 1 sq. m. heat exchanging surface
q		Proportionality factor for computing charges associated with temperature difference
b_L		Proportionality factor for computing charges associated with friction work
c_p	kcal/kg, C°	Specific heat of the medium at constant pressure
f		Fanning friction factor
g	m/h^2	Gravitational acceleration
g_{F_0}	m^2	Characteristic number for computing the cross sectional area of flow
g_F	m^2	Characteristic number for computing the heat exchanger surface
q	kcal/h, C°	Heat quantity per unit temperature difference transferred per unit time
w	m/h	Velocity of medium flow in authentic cross section
B_F		Capital costs associated with heat transfer area
B_Q		Capital costs associated with temperature difference
B_L		Capital costs associated with friction work
D	m	Characteristic dimension perpendicular to flow (hydraulic diameter)
F	m^2	Heat transfer area
F_0	m^2	Authentic cross sectional area of flow
G	kg/h	Medium flow per hour
L	mkg/h	Work applied for circulating G kg/h quantity of medium
N_{Eu}		Euler-number
N_F		Dimensionless number, characteristic of heat transfer area
N_{F_0}		Dimensionless number, characteristic of cross sectional area of flow
N_{Pr}		Prandtl-number
N_{Re}		Reynolds-number
N_{st}		Stanton-number
P	kg/m^2	Pressure of flowing medium
Q	kcal/h	Heat quantity transferred to flowing medium per hour
ΔT	C°	Temperature difference between medium and theoretical boundary
ΔT_K	C°	Log. mean temperature difference
V	m^3/h	Volume of medium flow per unit time
α	kcal/m^2, h, C°	Heat transfer coefficient between medium and surface
β		Specific charge, $\beta = B/Q$
γ	kg/m^3	Specific weight of medium
δ		$\delta = c_p/q$
ε		Fin efficiency
Λ		Number, characteristic of heat exchanger
ν	m^2/h	Kinematic viscosity of medium
Γ		Symbol, denoting dependence on surface geometry
Φ		Symbol, denoting dependence on physical properties of medium
Ω		Symbol, denoting dependence on design data

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