# CONSIDERATION OF A SURVEYING TELESCOPE OBJECTIVE IN THE INITIAL STAGE OF OPTICAL DESIGNING, TAKING THIRD ORDER ABERRATIONS INTO ACCOUNT 

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(Received November 19, 1958)

Various points of view have to be considered in the design of objectives for surveying telescopes. The requirements thus arrived at are often opposed to one another, and the resulting solutions are due to the preponderance of certain of these considerations.

In the present paper the author wishes to set forth the principal features of the results of examinations carried out in the initial stages of designing


Fig. 1. Schematic diagram of a simple teleobjective
with a view to restricting third order aberrations in such a manner that the individual lenses should make but slight contribution to the value of resultant spherical aberration and coma. Such examinations are motivated by the fact that in many cases the final degree of possible correction is predetermined by the initial data of the optical system. Hence, the subsequent application of the methods of computation, by means of which the designer skilled in the art, can determine or correct the aberrations of an optical system taken owing to certain considerations, is not sufficient.

In order to keep the weight and dimensions of surveying instruments within reasonable limits, let us accept the practice that the telescope objective is of the tele type, comprising a stationary converging member and a travel-
ling diverging member, as represented in Fig. I. Let $e$ designate the distance of the converging and diverging lens for a telescope adjusted to infinity, $s^{\prime}$ the image distance behind the diverging member, $L$ the optical tube-length, $F$ the resultant focal length of the teleobjective, and $h_{1}$ the semi-diameter of the converging member.


Fig. 2. Variation of focal lengths for the positive and negative components ( $f_{1}$ ) and ( $f_{2}$ ) respectively of the teleobjective, depending on $q$ and $Q$

Taking $F$, a series of solutions may be obtained for the focal lengths $f_{1}$ and $f_{2}$ of the converging and diverging lenses, respectively, depending on $e, s^{\prime}$ and $L$, with the aid of the equations (1) and (2), where

$$
\begin{align*}
f_{1} & =\frac{F \cdot e}{F-s^{\prime}}  \tag{1}\\
f_{2} & =\frac{e \cdot s^{\prime}}{L-F} \tag{2}
\end{align*}
$$

The high number of possible variations can, however, be substantially lessened by reasonably selected restrictions. In the present case preference will be given
to constructional data which satisfy the following two requirements the best :
a) The Petzval sum shall be as low as possible. If this requirement is met, then, in the case of small astigmatism, it is possible to decrease the curvature of the field. In short :

$$
\sum P \approx 0
$$

This aim can be realized, if

$$
\left|\frac{f_{1}}{f_{2}}\right| \approx 1
$$

It is permissible here to neglect the difference existing between the refractive indices of the individual lenses.
b) The absolute values of $f_{1}$ and $f_{2}$ should be as large as possible, in order to ensure small coefficients for the lenses.

Two parameters, $Q$ and $q$, have been introduced for effecting the examination, denoting the following quotients:

$$
Q=\frac{L}{F}, \quad q=\frac{e}{s}
$$

The focal lengths $f_{1}$ and $f_{2}$ have been computed depending on $q$, choosing the value of $Q$, and taking $F=100 \mathrm{~mm}$.

The results are illustrated in the following three tables and in the diagram represented in Fig. 2 :
I. Let $Q$ be 0.6

| $q$ | 0.5 | 1 | 2 |
| :---: | :---: | :---: | :---: |
| $f_{1}$ | +33.3 | +42.8 | +50.0 |
| $f_{2}$ | -20.0 | -22.5 | -20.0 |

II. Let $Q$ be 0.7

| $q$ | 0.5 | 1 | 2 |
| :---: | :---: | :---: | :---: |
| $f_{1}$ | +43.7 | +53.8 | +60.9 |
| $f_{2}$ | -36.3 | -40.8 | -36.3 |

III. Let $Q$ be 0.8

| $q$ | 0.5 | 1 | 2 |
| :---: | :---: | :---: | :---: |
| $f_{1}$ | +57.2 | +66.6 | +72.7 |
| $f_{2}$ | -71.0 | -80.0 | -71.0 |

A comparison of the results shows that with the increase of $Q$ and $q^{r}$ within the range considered $f_{1}$ and $f_{2}$ also increase. As the increase of $f_{2}$ ismore rapid, there is a range in the ambiance of $Q=0.8$ and $q=2$ where-

$$
f_{1}=-f_{2}
$$

However, knowing these figures, it does not yet clearly define the form of realization of the telescope objective. Obviously, both the converging and diverging members may comprise several converging or diverging lenses each, and the shape, distance, sequence, focal length and material of such lenses may vary within broad limits.

Again, one has to introduce reasonably selected considerations in order to limit the wide range of possible variations. Our example aims at arrangements and decompositions in which the shape factor of the lenses shall not substantially differ from the shape factors associated with the minimum spherical aberration of the individual lenses. The adoption of this principle has proved to be very advantageous from the viewpoint of both designing and manufacture. Designing is convenient, since it is in the ambiance of the minimums that the results of third order analytical computations give the best approximations to trigonometrical calculations, while manufacture is made easier owing to the large tolerances permitted by this method.

Let us now describe a numerical example, with the following data in millimetres, making use of the last column in Table III :

$$
\begin{aligned}
F & =180 \\
L & =144 \\
f_{1} & =131 \\
f_{2} & =-128 \\
e & =96 \\
s^{\prime}= & 48 \\
h_{1}= & 16
\end{aligned}
$$

In the simplest case the objective may comprise one converging and one diverging lens only. This arrangement (Fig. 1), however, does not comply with the requirement referred to in the introduction with regard to the shape of the lenses, as will be seen from the following.

Let us employ the Coddigton-Taylor method, writing the spherical aberration ( $\Delta S^{\prime}$ ) and coma ( $\Delta K^{\prime}$ ) in millimetres, depending on the shape factor $\delta_{1}$, in the following manner :

$$
\begin{aligned}
-2.77 \delta_{1}^{2}+4.83 \delta_{1}+0.015 \delta_{2}^{2}+0.165 \delta_{2}-4.72 & =\Delta S^{\prime} \\
-1.75 \delta_{1} & -0.035 \delta_{2}+1.53
\end{aligned}=\Delta K^{\prime}
$$

The refractive index of each lens has been taken as 1.62, and $\Delta K^{\prime}$ denotes the difference ( $\Delta F-\Delta S^{\prime}$ ).


Fig. 3. Structure of the teleobjective described in the numerical example with thin lenses

Let us now compute for both lenses the shape factors ( $\delta_{i M}$ ) associated with the minimum spherical aberration which is $\delta_{1 M}=+0.870$ for the first lens and $\delta_{2 M}=-5.50$ for the second lens. Solving the equations $\Delta S^{\prime}=0$ and $\Delta K^{\prime}=0$, the following results are obtained :

$$
\begin{array}{ll}
\delta_{1}^{\prime}=+0.691 & \delta_{1}^{\prime \prime}=+1.29 \\
\delta_{2}^{\prime}=+9.00 & \delta_{2}^{\prime \prime}=-20.8
\end{array}
$$

Comparison of these with the shape factors $\delta_{1 M}$ and $\delta_{2 M}$ shows rather large deviations, in particular for the second lens.

If the converging member of the objective is decomposed into two converging lenses, this deviation is lessened, while upon decomposition into three converging lenses, as represented in Fig. 3, the following equations can be written for $\Delta S^{\prime}$ and $\Delta K^{\prime}$ :

$$
\begin{gathered}
-0.102 \delta_{1}^{2}+0.178 \delta_{1}-0.101 \delta_{2}^{2}+0.530 \delta_{2}-0.100 \delta_{3}^{2}+ \\
+0.866 \delta_{3}+0.015 \delta_{4}^{2}+0.166 \delta_{4}-2.47=\Delta S^{\prime} \\
-0.194 \delta_{1} \\
-0.193 \delta_{2}
\end{gathered}
$$

$$
-0.190 \delta_{3}
$$

$$
-0.035 \delta_{4}+1.50=\Delta K^{\prime}
$$

Calculating $\delta_{i M}$ and substituting them into $\Delta S^{\prime}$ and $\Delta K^{\prime}$, we get :

$$
\begin{gathered}
\delta_{1 M}=+0.870 \delta_{M M}=+2.62 \delta_{3 M}=+4.33 \delta_{4 M}=-5.53 \\
\Delta S_{M}^{\prime}=-0.18 \mathrm{~mm} \Delta K_{M}^{\prime}=+0.19 \mathrm{~mm}
\end{gathered}
$$



Fig. 4. Curves 1 to 4 illustrate the variation of spherical aberration for the individual components of the teleobjective illustrated in Fig. 3, while curves (1) to (4) refer to the variation of coma

1. $-0.102 \eta_{1}^{2}+0.178 \delta_{1}$
2. $-0.194 \delta_{1}$
3. $-0.101 \delta_{2}^{2} \div 0.530 \delta_{2}$
4. $-0.193 \delta_{2}$
5. $-0.100 \delta_{3}^{2}+0.866 \delta_{3}$
6. $-0.190 \delta_{3}$
7. $+0.015 \delta_{4}^{2}+0.166 \delta_{4}$
8. $-0.035 \delta_{4}$

Taking into consideration that the value of the quotient $\frac{\lambda}{\sin ^{2} \beta^{\prime}}$ serving as a basis for the tolerance of the permissible errors is about $\pm 0.07 \mathrm{~mm}$ in our example, one can see that $\Delta S_{M}^{\prime}$ and $\Delta K_{M}^{\prime}$ are in the neighbourhood of the ideal. Hence, correcting them will not substantially change the values of $\delta_{i M}$.

Correction has been carried out with the aid of the diagram in Fig. 4, whence the variation of spherical aberration and coma for the individual lenses can be read, depending on the alterations of the shape factor. It is clear that when the shape factor of the three converging lenses is maintained as, and the absolute value of $\delta_{4 M}$ is lessened, the bending of the diverging member alone


Fig. 5. The lens system as shown in Fig. 3, after correction and substitution of thin lenses by thick lenses
will improve the values of $\Delta S_{M}^{\prime}$ and $\Delta K_{M}^{\prime}$. We have chosen $\delta_{4}=-2.5$ as a first trial.

Fig. 5 represents the data for the objective in which thin lenses have been replaced by thick ones. A trigonometrical test on the data yields the following errors :

$$
\begin{aligned}
& \Delta S^{\prime}=+0.023 \mathrm{~mm} \\
& \Delta K^{\prime}=-0.008 \mathrm{~mm}
\end{aligned}
$$

It is not here envisaged to provide correction for chromatic aberration, as the latter can be easily counteracted by dividing the lenses with adequate radii, that is, by cementing lenses made of glasses having different dispersion but identical refractive indices at a wave length $d$, without substantially changing $\Delta S^{\prime}$ and $\Delta K^{\prime}(n=1.62)$.

Geodetical telescopes are often employed for distance measurement. Let us, therefore, consider the course of the anallatic point in our example. This is illustrated in Fig. 6, where the range-finding errors resulting from the travelling of the anallatic point are plotted as ordinate against the object
distance as abscissa. The results indicate that the objective is suitable for range-finding purposes. Several calculations have further been carried out int other cases, for testing the travelling of the anallatic point in objectives where $Q$ lies in the range of 0.6 to 0.8 , and $q$ between 1.8 to 2.0 . The results obtained were found to be satisfactory.

The author would like to stress his view that lens designing is not merely a process of calculations which, based on data such as the focal length, the relative aperture of the objective and the field of view, would lead to a single solution. When designing has reached a certain stage, it is the approach of the


Fig. 6. Graph representing the range-finding error resulting from the displacement of the anallatic point $A \mathcal{A}$, depending on the object distance
designer to the problem, or his "optical instinct" that will determine his choice between the various possible solutions. Obviously, the final solution will be due to a compromise between the different considerations which have to be taken into account.

## Summary

Examinations have been made as to the variations experienced by the focal lengths of the converging and diverging lenses in teleobjective-type surveying telescope objectives depending on optical tube-length and separation of the lenses. In order to reduce aberrations, large partial focal lengths and small Petzval sums have been aimed at. An apparently appropriate solution has been chosen, illustrated by a numerical example.

## References

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