DISCHARGE MEASUREMENT IN CIRCULAR PIPES USING CURRENT METERS

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The measurement of liquid or gaseous substances passing through a pipe-line constitutes an everyday problem in engineering practice. Continuous indications or even records of the volume conveyed in unit time are frequently required, while in other instances the momentary rate of flow is only of interest.

Venturi-tubes should practicably be used for the former purpose, since a significant part of the head-loss can be regained by the diffuser thereof. On the other hand, simple orifices are also satisfactory for periodical observations. However, straight pipe sections of sufficient length are required for the installation of measuring orifices, and such may not be available, or else the construction and installation of the orifice for a single observation may prove to be too expensive and time consuming, and finally, the presence of an orifice may appreciably influence the quantity to be measured.

For the above reasons the velocity-area method must in the majority of cases be resorted to, for determining the quantity of substance flowing in the pipe. This method, although less accurate than measuring orifice or Venturi-meter observations, is generally satisfactory for industrial purposes.

The velocity of flow at discreet points can be measured by direct-action meters (Woltmann wheel, anemometer), differential meters, Prandtl (Pitot) tubes, etc. Of these the Prandtl tube has most frequently been applied, which, owing to its small size can readily be installed into the pipeline to be examined (a 10 millimetre dia. hole is already sufficient), and does not result in any serious disturbance of flow.

The velocity head indicated by the Prandtl tube, in case of gas or air is frequently no more than 1 to 2 mm water column, and an accurate reading thereof in a conventional vertical U-tube is hardly possible. The bent pipe micromanometer with an empirical scale has been found most convenient for similar observations, since the relative error of this instrument can be maintained constant over a relatively wide range.

Velocity readings are taken at several points along a few straight lines suitably selected by adjusting the position of the instrument correspondingly.

Velocity distributions thus obtained for rectangular cross-sections are integrated, first in one direction and the resulting line-integrals (or line average velocities) are subsequently integrated, along lines perpendicular to the former.

In case of circular cross-sections the velocity distribution is determined along at least two diameters (perpendicular to each other). The arithmetic mean of four (or more) velocity values measured at equal distances from the center is considered representative for the annular layer defined by the radius. In other words, each observed velocity value is considered constant over a fourth (sixth, etc.) part of a circle.

The mean velocity for the entire cross-section is obtained as the mean of velocities pertaining to different radii weighted by the radii, thus:

$$\bar{v} = \frac{\int_0^R r\,v\,dr}{\int_0^R r\,dr} = \frac{2\int_0^R r\,v\,dr}{R^2}$$

whence the rate of flow

$$V = R^2 \pi \bar{v} = 2\pi \int_0^R r\,v\,dr\,,$$

where R is the radius of the pipe.

The above integral can be graphically solved by plotting values rv against r and by measuring the area under the curve by means of a planimeter. The area multiplied by the scale factors and by 2π yields the discharge conveyed in the pipe.

The method just described being somewhat lengthy, a more convenient, although less accurate solution is usually resorted to:

The cross-section is divided into several annular sections having equal areas and the arithmetic mean of velocities measured in each section is computed. Since individual velocity values should be measured at points (layers) dividing the actual section into two equal parts, the number of points selected along the radius should be twice as great as that of points in which observations are intended. Readings are taken at each odd point.

Dividing the cross-section into n equal sections (n being an even number), the area of the circle defined by the radius pertaining to point k is

$$r_k^2 \pi = \frac{k}{n} R^2 \pi\,,$$

and thus

$$r_k = \sqrt{\frac{k}{n}}\,.$$

For the points at which velocity observations are taken $1 \leq k \leq n-1$, an odd number.

The influence of the number of velocity observations on the error introduced remains to be investigated for different velocity distributions.

The difference between the actual velocity distribution along the radius corrected for axial symmetry and the flow pattern observed cannot be expressed mathematically, since velocity values between individual observation

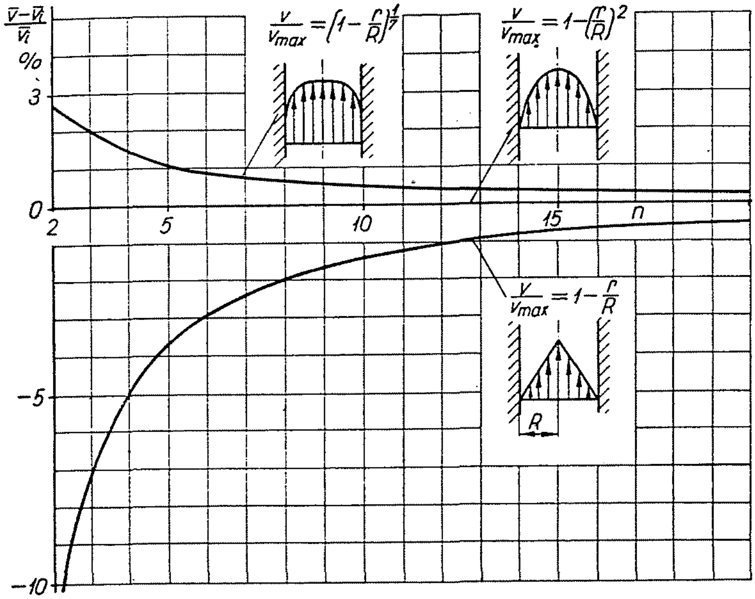

Fig. 1

points remain unknown. However, the actual velocity distribution along the radius, *i. e.*, the flow picture in a plane through the axis, is usually bounded by a curve showing no inflexions and thus a "smooth" distribution curve drawn through an adequate number of closely spaced points may be considered representative of the actual flow pattern. The same applies to velocity values between points of measurement lying on the same annual layer, as indicated by the fair agreement — usually within 1 to 2 per cent — between mean velocity values computed from observations at four and six etc. points, respectively.

Approximating the axially symmetrical flow picture by different power functions, the difference between the mean of velocities at the boundaries of annular sections having equal areas ($\bar{v}$) and the accurate mean velocity (v^2) divided by v_{max} is shown in Fig. 1 plotted against the number of annular sections (n).

In case of $n = 2$ the radius pertaining to the single boundary layer $r = 0,7071\ R$. The cross-section is usually divided into $n = 10$ annular sections, *i. e.*, velocity observations are taken at 5 points along a radius (at 10 points along a diameter). The spacing of the points of measurement for this case is shown in Fig. 2 in terms of the diameter.

In case of a parabolic velocity distribution described by the expression:

$$\frac{v}{v_{\max}} = 1 - \left(\frac{r}{R}\right)^2$$

the difference is uniformly zero for any number of annular sections.

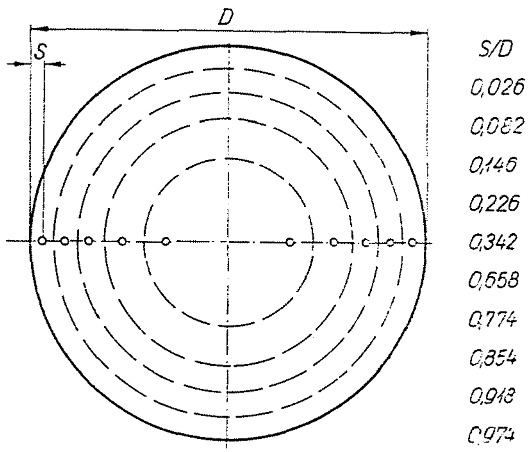

Fig. 2

In case of velocity distributions corresponding to the expression

$$\frac{v}{v_{\max}} = \left(1 - \frac{r}{R}\right)^{\frac{1}{p}}, \quad p \geqq 2$$

the mean of individual velocity values is slightly greater than the accurate mean velocity:

$$\frac{\bar{v}_i}{v_{\max}} = \frac{1}{R^2 \pi\, v_{\max}} \int_0^R 2 R \pi\, v\, d\, r = 2 \int_0^1 \frac{r}{R}\left(1 - \frac{r}{R}\right)^{\frac{1}{p}} d\, \frac{r}{R} = -\, 2 \int_s^0 \left(1 - \frac{y}{R}\right)\left(\frac{y}{R}\right)^{\frac{1}{p}} d\frac{y}{R} =$$

$$= 2\left[\frac{p}{p+1}\left(\frac{y}{R}\right)^{\frac{p+1}{p}} - \frac{p}{2p+1}\left(\frac{y}{R}\right)^{\frac{2p+1}{p}}\right]_0^1 = 2\left(\frac{p}{p+1} - \frac{p}{2p+1}\right).$$

The velocity distribution in developed turbulent flow can fairly accurately be approximated by the above power function using an exponent 1/7. *E. g.* the difference in case of $n = 10$ is but 0,52 per cent.

Adopting $n = 10$ as constant the difference has been plotted in Fig. 3 against the denominator of the exponent. In the neighbourhood of $p = 7$ the difference is about 0,5 per cent and varies but slightly.

Comparing the measured velocity distribution with the power function the computed discharge could be corrected on the basis of the above curve. By reducing the result obtained in case of turbulent flow generally by 0,5 per cent the error of the weighted mean computed along the radius may be assumed to be within $\pm 0{,}1$ per cent, and is thus negligible for all practical purposes.

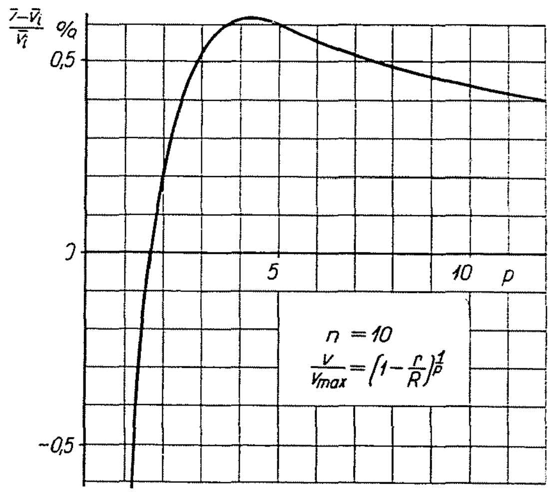

Fig. 3

Some information about the error introduced by the assumption of axial symmetry can be obtained from the difference between *e. g.* two mean velocities computed along two diameters.

Neglecting particular cases of specially located diameters (*e. g.* behind a bend in the plane of curvature) the difference between the arithmetic mean of the two above mean velocities and the actual mean velocity is not likely to be greater than that between the actual mean and the mean value computed along one of the diameters, *i. e.* than the half of the difference between the two mean values computed along the two diameters under consideration. Should this difference exceed a permissible limit, the measurement should be repeated along one or two other diameters.

The accurate setting of the measuring instrument is essential in ranges of high velocity gradients only (*e. g.* in the vicinity of the wall). However, any departure of the flow from the direction parallel to the pipe axis can introduce serious errors, the Prandtl tube being insensitive to deviations not exceeding about ± 15 degrees. Consequently the total velocity is indicated in this range by the Prandtl tube, while, as far as the rate of flow is concerned the component parallel to the pipe axis is of interest only.

The overall accuracy of the method is influenced, besides the error introduced by the computation of mean velocities, also by various regular errors such as : the scale factor of the micromanometer (difference in the specific weight of the measuring liquid), the error of observation, the chance error of the Prandtl tube or of any other meter, differences in the diameter of the pipe especially if the pipe is not exactly circular, error in the specific weight of the medium flowing in the pipe, etc. Under unfavourable conditions these constant errors may accumulate and thus the maximum error of observation is obtained as the properly weighted sum of all these errors, although the probable error is less. Flow-meter specifications define the "error to be contemplated", usually as the square root from the sum of the squared component errors.

The above "constant" errors being common with Venturi-tube of orifice observations as well, only the error involved in the mean velocity value computed from the observed velocity distribution is to be compared to the error introduced by the flow factors of flow meters. The accuracy of the area-velocity method is revealed by a similar comparison to be poorer by but 1 to 2 per cent than that of other methods, and while simple and fast, involves a maximum error generally not in excess of 4 to 5 per cent, an accuracy generally sufficient for industrial purposes.

Summary

1. The area-velocity method using 10 points along two diameters (Fig. 2) should be included in standard specifications of flow measurement.

2. In case of turbulent flow, the result obtained according to point 1 should be reduced by 0,5 per cent.

3. The accuracy limit of the mean velocity computed should be defined as the half of the difference between mean velocities computed from values observed along two diameters.

Bibliography

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