SUGGESTIONS ON THE DETERMINATION OF THE SAFETY ZONE α IN THE ISO RECOMMENDATION FOR LIMITS AND FITS

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1. *The ISA Tolerance System* has been established in the ISA Bulletin 25 as the result of the international standardizing activity for the time after World War I. This system, adopted by various nations proved to be satisfactory; consequently in the new era the ISO working bodies decided with only slight modifications to adopt it. The main problem is its extension in both directions *i. e.* for dimensions below 1 mm and for those above 500 mm. On the other hand, a new and reasonable interpretation of the tolerance prescribed is found to be necessary because of recent developments in measuring methods.

I

The influence of the error of measurement on the setting of tolerance values

2. *The safety zone.* When adopting and explaining the system of tolerances we have to adhere to the principle of considering the standard limits as within which the actual dimensions of the working pieces are to be safely maintained, in consequence of which the manufacturer is bound to take a possible degree of uncertainty of the measurements into account [1]. The necessity for making regulations concerning the tolerable errors of measurement of the industrial measuring instruments arises, *viz.* to provide the safety zones that represent the utmost margin up to which the prescribed limits can be approached when measuring with the usual device.

It must be borne in mind that the usual measuring instruments are different and are always chosen according to the actual working grade; again, the range of the errors of measurements varies with the different kind of instruments; also the mistakes of form of the work-pieces are different. Maybe, the latter ones can be eliminated when applying the Taylor's principle; however, in most of the cases, we are not able — as is well known — fully to adhere to this rule.

The standard values of the safety zone to be prescribed strictly depend on two factors: on the size of the dimension to be measured and on the grade of tolerance applied. It is obvious that the values of the safety zone have to be set by experiences on hand *i. e.* through gathering and evaluating actual data of measurements. As the present paper could not be based on recent experimental results, the actual task has been confined to develop a few theoretic statements with reference to the desirable mathematic formula suitable to establish the safety zone a as a function of the dimension D and of the grade of tolerance q.

3. *Mathematic conception of the safety zone.* The tolerance T can be conceived as the maximum allowed variation of the dimension of the product. Supposing the manufacturer could make full use of the tolerance zone, the variation of the dimensions will show the maximum frequency in the middle of this zone, whereas at its both ends (at the limits) the frequency will be near to zero. In practice, when setting the actual tolerance values, this character of the manufacturing variation used to be most frequently assumed. Especially, in the case of transition fits, we used to ascribe this kind of statistical character to the results of manufacturing; otherwise we could not obtain a tight fit, even with tolerance zones that may, in extreme cases, yield a clearance especially in consequence of the actual overlapping of the two zones. By accepting the nature of the allowed manufacturing variation G as exposed, and supposing an accidental error of measurement M, it is the additive rule of accidental variations which shall be applied when determining the ideal tolerance limit T:

$$T^2 = G^2 + M^2 \tag{I}$$

The meaning of this formula may be stated as follows:
Supposing a manufacturing variation G, and a full use of the tolerance field by the manufacturer, the size of this field shall be set as T, when the probable error of measurement will be M. Respectively, if the ideal limits T should be maintained, the effectively measured limits should not surpass G. Accordingly, the formula of the safety zone will be:

$$2\,a = T - G = T - \sqrt{T^2 - M^2} \tag{II}$$

In this conception, a represents the width of zone which has to be kept as a distance each limit of tolerance.

4. *The unit of the uncertainty of measurement included in the unit of tolerance, and the unit of the safety zone.* In the system of ISA limits and fits, the method was adopted for the sake of simplicity — at any rate in the field of the coarser tolerances above the grade $IT5$ — to set the tolerance and the safety zone as the product of a grade-factor q and a unit i, respectively, a_0. By dividing the equation (II) with q we have:

$$2\,a_0 = i - g_0 = i - \sqrt{i^2 - m_0^2} \tag{III}$$

Here, m_0 is the unit of the error of measurement ; further a_0 and i, respectively g_0 represent the respective units according to ISA statements, *viz.* :

$$\frac{T}{q} = i = 0{,}45\, D^{1/3} + 0{,}001\, D = \text{tolerance unit ;}$$

$$\frac{G}{q} = g_o = 0{,}45\, D^{1/3} = \text{unit of the allowed manufacturing variation ;}$$

$$\frac{2a}{q} = 2\, a_0 = 0{,}001 D = \text{unit of the safety zone;}$$

$$\frac{M}{q} = m_0 = \text{unit of the error of measurement.}$$

5. *How the error of measurement depends on the size of diameter.* In the ISA system the tolerance unit serves to establish a uniform rule for all grades as far as the dependence of the tolerance on the size of the diameter is concerned. However, the application of the empiric formula of the safety zone, according to the ISA specification ($2a_0 = 0{,}001D$), is restricted to the values of dimension only within the range $180 < D < 500$ mm. This empiric equation does not seem to be applicable, at least not in the range of dimensions tending towards zero ; this is obvious at first sight on the right side of the equation (III) (s. next paragraph).

When trying to find an exact mathematic formula expressing the error of measurement as a function of the size of the diameter [$m_0 = f(D)$], great difficulties are encountered. Our measuring methods are quite different when measuring large or small diameters ; consequently the realization of a continuous function is hardly to be expected. Nevertheless, compelled by the imperious necessity to establish a function which is of some way comprehensible, conventionally we used to apply a linear formula as a first approximation. The same practice has become customary in papers dealing with the errors of measuring instruments. Moreover, in both the ISA and ISO systems this linear conception has found its way as a formula of tolerance for the finest grades. Namely, in the case of so close limits the tolerance value depends, not on the variation of manufactured dimensions, but only on the errors of measurement. In the equation (I), with $G = 0$, the result is $T = M$. Effectively, for the grade *IT*1 of ISA and for the grades *IT*01, *IT*0, and *IT*1 of ISO, the fundamental tolerance values are simply in linear proportion to the actual diameter. The second part of this study's task is to make suggestions as far as new parameters should be included in the formula, in order to elaborate a more precise expression.

6. *Interpretation of the error of measurement as a linear function.* As expressed in the linear formula : $M = A + B \cdot D$, the error of measurement

increases in proportion to the dimension measured ; again, the importance of the absolute term is clear ; this term "A", being independent of the diameter, represents the minimum value, below which no measurement is possible with the respective device. This term is not included in the adopted ISA and ISO formulae and this is the reason why the application can not be extended to the direction of smaller diameters towards zero.

The value of the absolute term in the finest tolerance grades $IT1$ of ISA is $A = 1{,}5\ \mu$; again, for the finest ISO tolerance grade IT01 the respective value is : $A = 0{,}3\ \mu$. This difference is comprehensible when considering the exigencies of the recent technical development of measuring. Further when interpreting the unit of error of measurement m_0 the absolute term has a quite special and different meaning. The product obtained by multiplying the absolute term by the grade factor has to be taken into particular consideration, when weighing its significance as regards the measuring method applied in the manufacturing process which varies in the several grades of tolerance. In this connection the quotient A/B deserves our special attention, because it does not depend on the grade factor. In the system ISA this quotient is equal to 100, whereas in the system ISO its value is about 40 ; from this fact it can be concluded that the threshold value A has become smaller in consequence of better and more exact measuring methods, although, the value B could at the same time have been diminished.

Determination of the safety zone values as a function of the error of measurement and in analogy to the tolerance unit

7. *The a_0 values provided in system ISA.* As already mentioned in paragraph 5, these values are fixed only in the range of diameters $180 < D < 500$ mm, as shown in column 4 of the table in the supplement ; these values correspond to the grade $IT16$, dividing the original values of a_{16} by the grade factor, $q = 1000$; $\left(a_0 = \dfrac{a_{16}}{1000}\right)$. By introducing the tolerance unit, the dependence of the tolerance values on the dimension D became uniform in all grades of tolerance. Moreover, when having the tolerance unit multiplied by the grade factor, both the manufacturing variation and the error of measurement increase in the same proportion. However, this analogy does not seem to be justified in all cases. May be, *e. g.*, when passing from some one grade to another, the proportion of changing of the manufaturing variation will be different from that of the uncertainty of measurement.

This will be obvious when considering the fundamental tolerances in the finer grades, *i. e.* those beneath the grade $IT5$; in this range the grades $IT1$ and $IT01$, resp. $IT0$ are in linear proportion to D ; in other words, in these grades the term $G = 0$, and likewise the factor of g_0 is equal to zero.

In the coarser grades, the grade factor of g_0 increases and for the grade *IT*5, this factor equals the factor of m_0. Besides there are also reverse examples. In the coarsest grades the grade factor of m_0 will not change much. Considering the fact that according to the standard prescriptions for the range of the coarser grades, the grade factor g_0 and m_0 are identical, so it seems right to fix the desirable value of the safety zone in such a manner that the diminution of the value of T has to be set equal to the addition added to the value of G when the value of T was predetermined. Thus, when expressing the tolerance T as a product of the grade factor q multiplied by the tolerance unit and when adhering to express the safety zone a as a product proportional to the factor q, ($a = q \cdot a_0$), this zone is bound to equal the same quantity as the supposed error of measurement, when predetermining the value of tolerance.

Below it will be shown that this zone-width has to be considered as a maximum and it can be set narrower by taking the development of more exact measuring methods into consideration.

8. *The safety zone in the system ISA as a function of the error of measurement* m_0 *included in the unit* i. As shown in § 6, the formula given in the ISA Bulletin as a simple linear proportion ($2a_0 = 0{,}001\ D$) cannot be considered as formally right. Now, when we substitute the original formula by the equation (III), we will be able to build a new formula for the error of measurement derived from the actual values given for the range in which the law maintains its validity, by assuming that these values can be calculated on ground of the new formula. Supposing the error of measurement as a simple binomial expression, the possible solutions are manifold, not only because of the nature of this problem, but also on account of the rather large variation of the data which serve as base. When considering the 5th column of the table, we find the binomial expression :

$$m_0 = 0{,}45 + 0{,}00315\, D$$

that represents a fair average of the ISA values (see column 6).

9. *Extension to the range* D $<$ 180 *mm of the values of the safety zone.* In considering the function of m_0 above given as valid without the original range (180. . .500), the a_0 values may be calculated by a simple extrapolation of the formula (III). The zone values obtained in this way are to be considered, in accordance with § 7, as limits of the manufacturing variation that have served as base when determining the tolerance values, since the uncertainty of measurement is rightly given by the function m_0. The same method can be applied when dealing with the dimensions below 1 mm, by accepting the value of i = const = 0,6 μ as given in the recommendation ISO. On this base the calculated values of a_0 are given in column 6 ; their deviation from

the function values enumerated in column 7 (which are not suitable for the desired extension) deserves special attention. One may observe that the change of a_0 is not monotonous with the size D; on the contrary, the changing of a_0 comprises some extreme values, the origin of which may be found in the given form of the functions i and m_0. The mistakes are analogous to those of the functional form of i (see § 6).

10. *Extension to the range* D > 500 *mm of the values of* a_0.

In the system ISO there is a new formula for the tolerance unit ($I = 2{,}1 + 0{,}004\,D$) as valid in the range of dimensions over 500 mm; this can be used as basis for calculating the values of a_0 (see column 6).

The determination of the values of the safety zone as a function of the manufacturing variation

11. *The function of the manufacturing variation.* In order to obtain a determination of the a_0 values especially extended to the dimensions over 500 mm the method based on an accepted form of the function of the allowed manufacturing variation seems to be a better way and a more secure one than the former based on an accepted binomial form of the value m_0. By the formula (III) it is easy to understand that the value of a_0 can be determined by means of the function of g_0 as well. Now the function of g_0 is known through the formula of the tolerance unit according to the ISA Bulletin in the range of dimensions 1 ... 500 mm (see § 4). The only question is, whether the function $g_0 = 0{,}45D^{1/3}$ may be considered as valid in the range $D > 500$ mm. In this respect the collection of statistical data gathered by M. SIEMENS [2] is positively appreciated. In this paper arguments are given according to which this expression of the third root is suitable to express the effective variations experienced in shop practice in the range of $D < 2000$ mm, with the only modification of an additional term being included. A further study [3] on this collection makes it clear that this additional term is identical to the threshold value of the simple measuring method which as an average can be considered as constant.

12. *The calculation of the safety zone by means of an accepted manufacturing variation.* As exposed in § 5 and § 6 this method is applicable only within the range $180 < D < 500$. For dimensions over 500 mm the tolerance units I come into validity (see § 10). The a_0 values calculated by this method are shown in column 7 of the table. In the range of $D > 500$ mm, the a_0 values calculated by the same method are somewhat larger than those in column 6; it is to be mentioned that this method has been already recommended by M. TÖRNEBOHM. The larger values may be explained by assuming that the error of measurement M_0 as virtually included in the ISA tolerance unit I, should surpass the unit of error m_0 enumerated in column 5.

13. *The function of the error of measurement* M_0 *as a term included in the tolerance unit of the system ISO.* In analogy to the formula (III), the tolerance unit I will be expressed as follows :

$$g_0^2 = I^2 - M_0^2 \tag{IV}$$

Since the third root expression of the variation g_0 has been accepted, this formula can serve to calculate the values of the error of measurement M_0 in a reverse way. By assuming the quantity M_0 as a simple binomial expression, the two parameters can be calculated on hand of the values of M_0.

As already stated in § 8 as regards m_0, the same applies to the multifarious choice of the parameters of M_0, that all can fairly well represent the error values. The respective values of M_0 as enumerated in the table, are calculated by the formula :

$$m_0 = 0{,}45 + 0{,}00\,315\,D$$

the parameters being chosen as to obtain the best approximation to the values given in the ISA Bulletin. This binomial expression comprises an absolute term (threshold value) and a quotient A/B (= 143), both of a fairly high value (see § 6). The following formula with somewhat different parameters gives an approximation not worse than the former one :

$$m = 0{,}355 + 0{,}00\,325\,D$$

In this formula the quotient A/B equals 109, *viz.* nearly the same as in the $IT1$ grade in the ISA Bulletin (see § 6). In this connection it is advantageous that all the grades from $IT1$ to $IT6$ and above, will be in good conformity to each other (*cf.* the remarks on the grade factors of m_0 and g_0 in § 7).

The parameters of the binomial expression determining the quantity M_0 as included in unit I, may be set as follows :

$$M_0 = 0{,}315 + 0{,}004\,D \tag{V}$$

Herein, the quotient $A/B = 79$; the values of M_0 are in 8th column and the respective a_0 zone values in column 9.

The determination of the safety zone and of the tolerance unit by a uniform setting in all ranges

14. *The uniform formula for the tolerance unit.* Starting from the formula (IV) and using the parameters given above we have :

$$I_1 = \sqrt{(0{,}45\,D^{1/3})^2 + (0{,}315 + 0{,}004\,\mathrm{D})^2} =$$
$$= \sqrt{0{,}1 + 0{,}2\,D^{2/3} + 0{,}0025\,D + 16 \cdot 10^{-6}\,D^2} \tag{VI}$$

In substituting the values of M_0 by m_0 the resulting values according to the formula (VI) are equally suitable to yield tolerance units for both ranges below and above 500 mm, as seen by inspecting the values in the column 3 and 10. As in practice the tolerance values are applied in rounded values and steps, for calculating the tolerances we can apply one single equation with only 3 parameters instead of the ISO system using 3 equations and 5 parameters.

As shown above, the binomial expression according to the equation (V) may be regarded as the uncertainty of measurement serving as base for the tolerances in every range of dimension.

15. *Summary: The maximum specific values of the safety zone (per tolerance unit)* a_0 *for all diameters.* The advantage of this value can be proved when we intend to set the safety zone proportionally to the grade factor q, simply for its being logical to have it calculated at the same size as when setting the tolerance values. Concretely this is :

$$a_0 = \frac{1}{2}\left[\sqrt{0{,}2\,D^{2/3} + (0{,}315 + 0{,}004\,D)^2} - 0{,}45\,D^{1/3}\right] \qquad \text{(VII)}$$

The numeric values are enumerated in column 11 of the table.

II

16. *The revision of the values of the safety zone.* Already in the ISA Bulletin 25 we find the clear expression of the firm tendency, to consider "nominal limits as final" (see § IV. A. 4) whereas the safety zone values are regarded as items that have to be set ever narrower in conformity with modern technical developments in order to facilitate manufacturing methods and for economical reasons (*cf.* § 1). In other words, the applicability of the formula (V) and the products obtained by multiplying the respective values by the grade factors, have to be checked by means of measuring experiments for all grades of tolerance. The aim of this paper is not to execute such a large task, it is to check the problem by fixing some guiding principles as far as formal features are concerned.

17. *Equality of the safety zone and of the error of measurement;* $2a = M$. The position of the manufacturing variation showing an axis of symmetry coinciding with the middle of the tolerance and having limits identical to the allowed ones, is hardly to be expected during the actual production of the working pieces. This ideal distribution of frequency is only an assumed position of the allowed maximum variation, presupposed when predetermining the tolerance values. During the production, the middle of the manufacturing variation may be situated at any possible level of the tolerance

Maximum values (in μ) of the safety zone a_0 as included in the tolerance unit

Diameter step in mm	D	$i = \frac{T_6}{10}$	$a_0 = \frac{a_{16}}{1000}$	$m_0 = 0,45 + 0,00315\,D$	$a_0 = \frac{i - \sqrt{i^2 - m_0^2}}{2}$	$a_0 = \frac{i - 0,45\,D^{1/3}}{2} = 0,0005\,D$	$M_0 = 0,315 + 0,004\,D$	$a_0 = \frac{i - \sqrt{i^2 - M_0^2}}{2}$	$I_1 \sqrt{0,2\,D^{2/3} + M_0^2}$	$a_0 = \frac{I_1 - 045\,D^{1/3}}{2}$
1	2	3	4	5	6	7	8	9	10	11
— 3	1	0,6		0,453	0,104	0,0005	0,319	0,046	0,55	0,05
> 3— 6	4,24	0,8		0,463	0,080	0,0021	0,335	0,037	0,80	0,036
> 6— 10	7,8	0,9		0,474	0,068	0,0039	0,346	0,035	0,95	0,03
> 10— 18	13,4	1,1		0,492	0,058	0,0067	0,369	0,0325	1,12	0,025
> 18— 30	23,3	1,3		0,523	0,056	0,012	0,408	0,0275	1,34	0,025
> 30— 50	39	1,6		0,572	0,054	0,020	0,470	0,035	1,59	0,03
> 50— 80	63	1,9		0,65	0,056	0,032	0,568	0,046	1,87	0,04
> 80— 120	98	2,2		0,76	0,069	0,049	0,707	0,057	2,18	0,055
> 120— 180	147	2,5		0,91	0,087	0,074	0,903	0,09	2,53	0,075
> 180— 250	212	2,9	0,11	1,12	0,113	0,106	1,16	0,12	2,91	0,12
> 250— 315	281	3,2	0,14	1,33	0,146	0,140	1,44	0,16	3,26	0,13
> 315— 400	355	3,6	0,18	1,57	0,18	0,178	1,74	0,23	3,61	0,205
> 400— 500	447	4	0,22	1,86	0,23	0,223	2,10	0,3	4,02	0,29
		$I = \frac{T_{11}}{100}$			$\frac{I - \sqrt{I^2 - m_0^2}}{2}$	$\frac{I - 0,45\,D^{1/3}}{2}$		$\frac{I - \sqrt{I^2 - M_0^2}}{2}$		
> 500— 630	561	4,4		2,22	0,31	0,345	2,56	0,41	4,49	0,39
> 630— 800	710	5		2,69	0,40	0,495	3,15	0,56	5,08	0,54
> 800—1000	894	5,6		3,27	0,52	0,63	3,90	0,775	5,80	0,72
>1000—1250	1120	6,6		3,97	0,66	0,96	4,79	1,035	6,67	1,00
>1250—1600	1410	7,8		4,9	0,87	1,38	5,97	1,38	7,80	1,39
>1600—2000	1790	9,2		6,08	1,14	1,87	7,47	1,925	9,23	1,81
>2000—2500	2240	11		7,49	1,47	2,55	9,26	2,54	10,9	2,54
>2500—3150	2805	13,5		9,28	1,89	3,58	11,53	3,25	13,1	3,43

field ; consequently the accidental error of measurement should be taken into account at both limits in its full size. This kind of frequency is not suitable to apply the quadratic combination of the variations.

Now, let us withdraw from the method of determination of tolerance values as exposed in the first part of this paper ; doing so it is our task, on the base of competent experiments, to assume an error of measurement set as narrow as possible and to take it into account at its full size. Opposite to the former task as to keep an ideal manufacturing variation G (*cf.* § 3) our new problem consists in maintaining a theoretic limit T by making the largest advantage possible of it.

18. *The variable character of the safety zone.* Aiming to establish a safety zone for practical purposes as narrow as possible, this may be fixed at different sizes in compliance with various conditions. There are various measuring methods and devices differing as measurements are made either on the GO side, or else on the NOT GO side ; consequently, the error of measurement also varies. Likewise, the error may be different for the internal and for the external gauging. Thus, there are already four different cases for the safety zone and the error of measurement, respectively :

$$\left.\begin{array}{ll}\text{checking the GO-side :} & a_G = \dfrac{1}{2} M_G \\[2ex] \text{checking the NOT GO side :} & a_N = \dfrac{1}{2} M_N\end{array}\right\} \text{ by internal gauging}$$

$$\left.\begin{array}{ll}\text{checking the GO-side :} & a_{1G} = \dfrac{1}{2} M_{1G} \\[2ex] \text{checking the NOT GO-side :} & a_{1N} = \dfrac{1}{2} M_{1N}\end{array}\right\} \text{ by external gauging}$$

Besides, within the same tolerance grade, variations of the error of measurement may occur as the measuring methods and measuring devices change.

19. *Analysis of the error of measurement of gauging.* The stronger our efforts to set the required error of measurement at the smallest size possible, the more desirable it is to analyse it. The total error is the result of some accidental causes ; these again should be combined by calculating the resulting value according to the law of quadratic addition of accidental errors. One of these accidental errors is the inexactitude of the measuring instrument M_i ; as it is quite reasonable, in the system ISA there are prescriptions for the finest grades, referring to the proper manufacturing tolerances of the fixed gauges ; furthermore, the measuring results are influenced by the total

M_m *i.e.* the sum of some factors that may intervene during the measuring operation ; these are *e.g.* some incidental mistakes caused by the sensitive imperfection of the operator, the variations of the measuring force, of the reference temperature, etc. ; finally, in the system ISA there is a prescription according to which the Taylor principle has to be adhered to as far as possible ; yet this cannot always be strictly applied, but only in part. Therefore the error originating from these conditions is M_t, the 3rd component of the resulting error ; on closer inspection this partial error is essentially the effect of an inadmissible mistake of form, which cannot be revealed by the gauge, because of the insufficiency of the measuring surfaces. The total variation combined from these partial errors is :

$$M^2 = M_i^2 + M_m^2 + M_t^2 \qquad \text{(VIII)}$$

20. *An empiric formula for the error of measurement* M. Concluding from the preceding and based on a more particular analysis, this empiric formula can not be formed as a simple binomial expression (*cf.* § 5) ; even if the components M_i, M_m and M_t could be expressed in a simple binomial form, their quadratic summarization would be a trinomial expression, with 3 parameters, *e. g.* :

$$M = \sqrt{a + b \cdot D + c \cdot D^2}$$

In fact the relations become more complicated as the partial error M_t, as prescribed in the specification, is identical to some *IT* grade, which in itself — as is known — is not a simple binomial expression.

21. *The proper manufacturing error of the measuring device* M_i. In the ISA specification the manufacturing tolerances of the fixed gauges are marked H and H_1 and are prescribed by applying the grades *IT*1 . . . *IT*7. As already shown in § 7, the dependence of these grades on the dimension D is of a nature that leaves no chance to set an adequate binomial expression. Let us quote the original text in ISA Bulletin 25 :

> „On account of the general tendency for errors to occur in the most precise measurements, these errors increasing proportionally with the diameter, the values of the tolerances for *IT*1 increase in a linear manner. *IT*2 to *IT*4 have been arranged in a geometric series between *IT*1 and *IT*5." (S. II. A.)

Accordingly, the grade-factor assigned to the unit g_0 of the manufacturing variation increases step by step from 0 to 7 in the grades *IT*1 — *IT*5 ; and in the formula of these grades, the newly included term is of a lower degree then the linear one *viz.* a term of $D^{1/3}$ as follows :

$$M_i^2 = a + d \cdot D^{1/3} + b \cdot D + c \cdot D^2 \qquad \text{(IX)}$$

As already shown in [5] this term ($D^{1/3}$) is indispensable mainly with the lower values of the dimension D. With the larger values of D, the influence of this member on the character of the function is almost nullified. For practical purposes this term could be formed as $D^{1/2}$, an expression that is more consistent with the sequence of powers in the equation :

$$M_i^2 = a + d \cdot D^{1/2} + b \cdot D + c \cdot D^2 \tag{X}$$

22. *The error of measuring by gauges* M_m. Well perused, the ISA Bulletin 25 contains suggestions on the values of this error ; namely prescriptions are given referring to the relation between manufacturing tolerance of snap and reference gauge. The variation of the working measure of a snap gauge H_1 should be considered as a resulting variation of the reference gauge H_p and the error of measurement M_m, *viz.* :

$$H_1^2 = H_p^2 + M_m^2 \tag{XI}$$

This relation is clearly exposed in the Bulletin (IV. E. 3. c.) : "The difference between the limits given by H_1 for the snap gauge and H_p for the reference gauge, represents a safety zone on both sides for errors in measurements in the same way as a and a_1 does for working pieces over 180 mm."

The formula (XI), essentially identical to formula (I), could be applied for the calculation of the function M_m. Only the dependence of H_1 on H_p is fixed by the standard specification and not by experimental data ; consequently, the formula (XI) can be considered but as the standard prescription for the maximum values of M_m :

$$(M_m)_{\max} = \sqrt{H_1^2 - H_p^2} \tag{XII}$$

23. *The formula for the error of measurement* M_m. On starting from the equation (XII), the formula can be derived from the difference between two *IT* grades. By the graduation of the grade factors as characterized above, the importance of the term $D^{1/3}$ will be predominant in the formula for M_m^2. Only, such an extreme relation does not seem probable at all ; in order to explain it, the standard prescriptions should be considered as arbitrary.

24. *The error of measurement* M_t *caused by the deviation from the Taylor principle.* The size of this error depends on the actual mistakes of form of the work-piece and on the degree these mistakes could be registered or neglected by the gauge. The latter again depends on the extent of the gauge surfaces covering the surfaces of the piece. This component of the error M_t could be determined only by experiments. Therefore a standard specification of the

measuring surfaces of the gauge is badly needed. In the Hungarian standard specification there is a fundamental prescription referring to this problem. [6].

25. *The empiric formula for the values of the safety zone.* In consequence of the preceding one of the equations (IX) or (X) seems to be applicable to represent the empiric values of the zone :

$$\alpha = \frac{1}{2} M = \frac{1}{2} \sqrt{a + d \cdot D^{\frac{1}{2}} + b \cdot D + c \cdot D^2} \qquad \text{(XIII)}$$

As shown in the first part of this paper, 4 parameters are sufficient to represent this function with a suitable exactitude in the whole range of the dimension D. Taking into account all the relations exposed above, an expression with the square root seems to be clearly the most appropriate to represent the true character of the function.

26. *Practical data serving to determine the error of measurement.* Among the documents ISO, a collection of measured data collected by the RIV Works has been published [7]. Herein, the variations of measured dimensions are given, obtained by internal gaugeing carried out on work-pieces of the size within the range $D = 310 \ldots 1200$ mm produced, according to the tolerance grades $IT6 \sim IT7$. In this collection clear distinction is made between personal errors of operators, further, errors evoked by different conditions of measuring, errors varying with the various devices ; besides, the measurements were made at six different spots of the same piece and consequences can be drown thereof, as far as the mistake of form of the piece is concerned. Nevertheless, the collection of data was evaluated only with regard to the variety of devices, and two different binomial expressions were elaborated to represent the size of the errors of measurement. To set the first binomial formula, readings on a comparator of the vertical type served as basis ; this resulted in the following binomial expression :

$$M = 1 + 0{,}01\ D$$

The second formula is based on measurements, carried out by a micrometer :

$$M = 10 + 0{,}01\ D$$

27. *The empirical formula as compared with the safety zone expressed in relation to the tolerance unit.* The values of the safety zone (unit) α_0 as calculated by the formula (VII), are to be multiplied by the factor $q = 10$ resp. 16, according to the prescriptions for $IT6$ resp. $IT7$. The values of α resulting from this multiplication can be considered as the maximum width of the safety zone. Now, in the formula (VII) the error value is equal to that of the

formula (V); in order to make a comparison possible, the equation has to be multiplied by the grade-factors as follows :

$$M_6 = 3{,}15 + 0{,}04\,D$$

$$M_7 = 5 \quad + 0{,}064\,D$$

When comparing these values resulting from the two latter formulae with the empirical values as quoted above, it can be stated that the empirical ones are far less, *viz.* they are only fractional parts of the maximum value, obviously, only in the range of dimensions in which the experimental measurements took place. Consequently, based on the experimental results, the binomial expressions are not susceptible to an extension to the lower range of $D < 310$ mm. On the contrary, based on the formula (XIII), the same binomial formulae may be, with more confidence considered as suitably applicable in the range of $D > 1200$ mm, because the function according to the formula (XIII) assumes a linear character in the range of large diameters.

28. *Consequences rendered possible by a synoptical evaluation of the measurements carried out in Italy.* By a suitable grouping of the data we can fix not only the different errors of the various measuring methods, but we can distinguish the variation M_m arising in the method itself from those evoked by the variation M_t of the mistakes of form when applying statistical mathematical methods. By using the same measuring method in the lower and in the higher ranges of the dimension D, formulae can be derived from new data similar to the formula (IX) and (X).

The manufacturing variation of the device M_i has to be prescribed as formerly, by specifying the IT grades accordingly. In the case of internal gauging the safety zone for the NOT GO side, and for the GO side can be expressed in connection to this function as follows :

$$a_N = \frac{1}{2}\sqrt{M_m^2 + M_{ij}^2}$$

$$a_G = \frac{1}{2}\sqrt{M_m^2 + M_i^2 + M_t^2}$$

Internal dimensions are measured by gauges with external dimensions and inversely, external dimensions are measured by gauges with internal dimensions. Therefore, these two kinds of measuring do not differ in principle. In other words, in the case of external gauging the same formula can be applied. The difference is only in the sizes of tolerances. For internal gauging the manufacturing tolerance $M_i = H$ is to be substituted in the formulae for a_N and a_G, whereas for external gauging the manufacturing tolerance $M_i = H_1$ is to be substituted in the formula for a_{1N} and a_{1G}.

Summary

This study is to find an empiric formula suitable to express the safety zone as a function of the dimension to be measured. The tolerance is expressed as a statistical summarization according to the law of probabilities, the components being the allowed variation of the dimension and the probable error of measurement. It is reasonable to state that, by setting the safety zone, the same size of error should be deduced from the tolerance as had been assumed when establishing the tolerance values. In this way, a maximum width of the safety zone is determined. — In order to set the width of the safety zone on the basis of empirical measuring, in this study an empiric formula is recommended that complies well with the application of the law of summarizing probabilities.

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