ABOUT ASPHERIC SURFACES USED IN OPTICAL SYSTEMS

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In manufacturing optical apparatuses, the production of wide-angle objectives of high luminous power, claims more and more attention, particularly when the demands for the latest advancements in photography are concerned. Up-to-date requirements can only be fulfilled by using elaborate systems comprising several lenses. The constructional elements of the optical systems, such as the radii of the lens surfaces, thickness of the members, and spacing of the lenses, in addition to the adequate choice of glass types, can be determined by calculating the optical system used in the various apparatuses, suitable for satisfying technical standards. In many cases mechanical requirements do not impose undue restrictions on the optical design, so that a wide range of optical systems, and of methods of calculation for their elements, is available.

The primary consideration in the computation of optical systems is to reduce aberrations to a minimum by adequate arrangement of the individual elements. Longitudinal aberrations, such as chromatic aberration, position aberration of the focii of elementary astigmatic beams, tangential and sagittal aberration, and transverse aberrations, like coma, distortion and chromatic aberration of magnification, should be taken into account. This is particularly so for apparatvses calling for optical systems of high requirements ; such as highly orthoscopic wide-angle apparatuses of a large aperture.

It has been found that adequately selected systems consisting of several cemented or air-spaced components are quite satisfactory for these requirements. Objectives of *e. g.* the triplet type give good results, when aberrations of the third order have not been considered. Beside an identical degree of correction for aberrations, the reduction of the number of refracting surfaces has always been a very attractive aim.

Refracting surfaces of optical systems are largely spherical. Transverse aberrations of the ray in striking one point of the spherical surface can be eliminated by appropriately varying the normal to the said surface point. A simple method therefore is, to substitute some other surface element for the concerned spherical surface element. The substitution of an aspheric surface element for a spherical surface element, whereby also the number of refractive surface elements can be reduced, is a widely followed practice. This problem has frequently been treated in literature ; various methods of computation are known for the beams passing through aspheric surfaces ; the problem of coefficients of third-order-aberrations has long been answered, but the introduction of aspheric surfaces still remains to be finally solved.

The optical characteristics of photographical systems are contradictory. Ever since Petzval elaborated the first photo-objective, a number of new designs were produced. Experiments in connection with these objectives have made it possible to generalize the correlations existing between the optical characteristics of photographic lens systems. Thus, the problem for simplifying the known modes of computation for the photographic optical systems having spherical surfaces was sought for. Computation becomes infinitely more simple when endeavouring overall correction only for axial aberrations. The concept of substituting aspheric surfaces for spherical ones arose in connection with the trend to simplify the system. The following problems are presented in this respect :

a) Which of the spherical surfaces should be replaced by aspheric ones?

b) What form should the aspheric surface have?

c) What is the influence of the aspheric surface on the entire optical system?

While researches so far carried out have not yet led to an uniform attitude in regard of these questions, one can safely establish that refracting surfaces of revolution whose axis of revolution is identical to the optical axis, can be used to advantage as aspheric surfaces. It must be noted that the trigonometric computation of optical systems having such aspheric surfaces is rather lengthy. This type of optical systems can be advantageously used in optical constructions which are expected to work with rays of high luminousity, but of very small fields of view, although the manufacturing methods for these systems are as yet not fully developed. Nevertheless, their use has been found advisable in many cases, especially for apparatus provided with projectors. However, adequate results have also been achieved with photo-objectives of similar design. The area in the vicinity of the diaphragm most readily lends itself for replacement by an aspheric surface, for, if so used, the aspheric surface will not affect the focal position of the astigmatic beams.

Image formation by rays passing through optical systems is always collinear between the object point $A = (\xi, \eta, \zeta)$ and the image point $A'(\xi', \eta', \zeta')$, so that

$$\begin{aligned}
\xi' &= \frac{a_1\,\xi + b_1\,\eta + c_1\,\zeta + d_1}{a\,\xi + b\,\eta + c\,\zeta + d} \\
\eta' &= \frac{a_2\,\xi + b_2\,\eta + c_2\,\zeta + d_2}{a\,\xi + b\,\eta + c\,\zeta + d} \\
\zeta' &= \frac{a_3\,\xi + b_3\,\eta + c_3\,\zeta + d_3}{a\,\xi + b\,\eta + c\,\zeta + d}\,.
\end{aligned} \tag{1}$$

When spherical surfaces are replaced by aspheric surfaces in optical systems, these latter are always surfaces of revolution, their axis being identical with the optical axis. Hence, image formation is axially symmetrical, and points A and A' lie in the plane intersecting the axis and point A. The correlation between object and image is, therefore,

$$\begin{aligned}
\xi' &= \frac{a_1\,\xi + b_1\,\eta + d_1}{a\,\xi + b\,\eta + d} \\
\eta' &= \frac{a_2\,\xi + b_2\,\eta + d_2}{a\,\xi + b\,\eta + d}
\end{aligned} \tag{1a}$$

When image formation is central, these formulae take the form of

$$\xi' = \frac{a_1\,\xi + d_1}{a\,\xi + d} \quad \text{and} \quad \eta' = \frac{b_2\,\eta}{a\,\xi + d} \tag{2}$$

The latter method presents a basis for computation of aberrations arising in connection with image formation.

For aspheric surfaces, only slightly diverging from spherical surfaces, it is easy to apply the method of computation based on a calculation utilizing the known values of remanent geometrical aberrations which correspond to the values of wave aberrations due to these surfaces. Nevertheless, the calculations for the optics of the system are rather lengthy, owing to the correction for aberration of broad oblique beams, and of axial aberration due to the introduction of aspheric surfaces. The pertinent work of V. N. Tschurilovskij [1] should be mentioned, in which the correction for transverse aberration is expanded in series according to a known parameter. In this case, each of the spherical coefficients of higher systems provides correction for the coefficients corresponding to the aberrations of the higher system.

Let us now follow the path of ray through an optical system of aspheric surface. Consider Fig. 1 and let the vector radiant starting from point be $\mathbf{A}$, where $|\mathbf{A}| = A$. If, furthermore, the normal of the spherical surface in the point of refraction of the vector radiant is $\mathbf{N}_1$, the normal of the aspheric surface in the point of refraction of the vector radiant is $\mathbf{N}_2$, the ray refracted at the spherical surface is $\mathbf{A}_1$, and the ray refracted at the aspheric surface is $\mathbf{A}_2$, the refractive indices are n and n', then, according to the known laws of refraction

$$\mathbf{A}_2 - \mathbf{A}_1 \cong \frac{n - n'}{n} (\mathbf{N}_2 - \mathbf{N}_1) \tag{3}$$

and the transverse aberration of the ray refracted at the aspheric surface is

$$|\mathbf{A}_2 - \mathbf{A}_1| \cong \frac{n - n'}{n} \Phi s \tag{4}$$

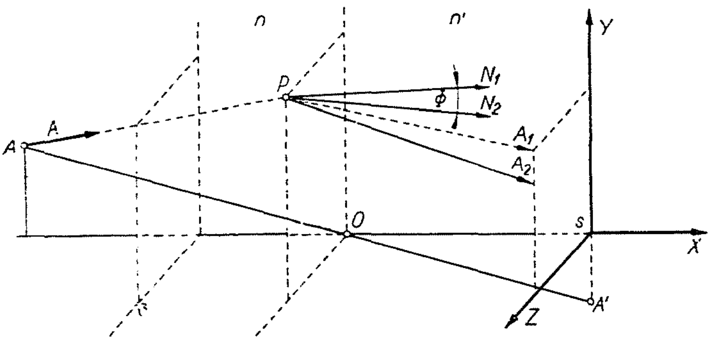

Fig. 1

where s is the distance of the Gaussian plane of the image from the aspheric surface.

It is worth mentioning, that in many instances the calculations of aberrations start with the primary condition for the substitution of aspheric surface elements for spherical surface elements that the two surfaces, or rather, surface elements, are in contact at the point in question ; furthermore, that their tangent planes and their curvatures are equal. This primary condition indeed holds good, but only along the optical axis of the system, that is, at the point where the axis intersects that spherical surface of the system which is to be substituted. Accordingly, as can be seen in Fig. 2, in point O, the origin of coordinates, the two surfaces have a common normal and a common tangent plane, and their curvatures are also equal. The tangential curve can preferably be taken as having the form

$$f(x) = y^2 = a_1 x + a_2 x^2 + a_3 x^3 + \ldots + a_k x^k \tag{5}$$

as has been done in various instances [2].

$$y = (s - x) \operatorname{tg} u \tag{6}$$

as in Fig. 1, where u denotes the angle of the vector radiant with the axis. In the equation (5), as in SCHWARZSCHILD [3],

$$a_1 = 2r \quad \text{and} \quad a_2 = -(1 + b) \tag{7}$$

where r is the radius of curvature and b the coefficient of deformation. The direct correlation between the ordinate l' on one hand, and the aberration

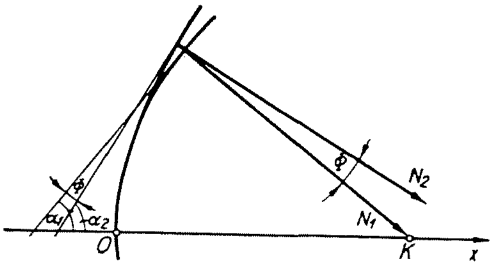

Fig. 2

of the ray, that is, the coefficients of the sequence (5) on the other, can now be established by means of the variations of the coefficients $a_3, \ldots, a_k$, in addition to the two fixed coefficients.

A useful, although trivial method for establishing the effect of the refracting surfaces of optical systems on aberration is the differential method, successfully applied by G. G. SLUSJAREV and a number of other opticians [4].

According to this method, a correlation exists between the partial differential quotients of ray aberration and the coefficients of the equation of the aspheric surface in the plane of the figure. This method permits exami-

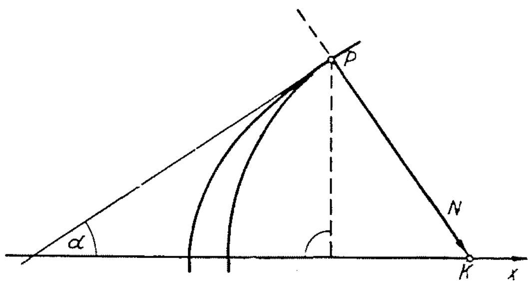

Fig. 3

nations in which, following the path of the beam through the optical system, the analysis of the coefficients of the Seidel-components gives the clue to the problem, which of the surfaces is responsible for the introduction of higher aberrations, in other words, which is the spherical surface to be replaced by an aspheric surface.

If in an optical system several spherical surfaces have to be replaced by aspheric surfaces, the differential method [7] yields the following equations:

$$
\begin{aligned}
\mathrm{d}l_1' &= \sum_1^k D_k^{(1)}\,\mathrm{d}a_k + \sum_1^n E_n^{(1)}\,\mathrm{d}b_n + \dots \\
\mathrm{d}l_2' &= \sum_1^k D_k^{(2)}\,\mathrm{d}a_k + \sum_1^n E_n^{(2)}\,\mathrm{d}b_n + \dots \\
&\vdots \\
\mathrm{d}l_N' &= \sum_1^k D_k^{(N)}\,\mathrm{d}a_k + \sum_1^n E_n^{(N)}\,\mathrm{d}b_n + \dots
\end{aligned}
\tag{8}
$$

where

$$D_n = \frac{\partial l'}{\partial a_n} \quad \text{and} \quad E_m = \frac{\partial l'}{\partial a_m} \tag{9}$$

are the partial differential quotients of the ray in the plane of the figure (l' is the ordinate of the intersection point in the optical system, and a_m and a_n are the coefficients of the mth and nth members, respectively, of the sequence (5)).

Determining the coefficients, the following equation results for the tangential curve of the aspheric surface :

$$
\begin{aligned}
y^2 &= a_1 x + a_2 x^2 + a_3 x^3 + \dots \\
y &= b_1 x + b_2 x^2 + b_3 x^3 + \dots
\end{aligned}
\tag{10}
$$

This mode of calculation has the advantage that after computations of the two starting partial differential quotients, the other coefficients of the sequence may be computed by a recurrence formula [7], *e. g.* where x_0 is the coordinate of the intersection of the ray and the surface.

Finally, it should be pointed out, that it is not advisable to choose the last surface of the system for the substitution of aspheric surfaces for spherical ones, as this surface would permit the increase of the relative aperture of the system. This, however, is not suitable for correction of oblique beam aberration.

Summary

A short survey over refractive surfaces applied in optical systems. The possibility of reducing the number of refracting surfaces by substitution of aspheric surfaces for spherical ones. Which spherical surface should preferably be replaced by an aspheric surface and what is the most advisable form to be given to the aspheric surface.

References

1. Чуриловский Д. Н.: О новом типе астрофотографического объектива с апохроматической апланатической, анастигматической и ортоскопической коррекцией. Тр. Инст. механ. и опт. 1940. т. I. в. 4.
2. Тудоровский А. И.,: Теория оптических приборов. II. § 299. (1952).
3. SCHWARZSCHILD, K.: Untersuchungen zur geometrischen Optik. I. § 6. S. 25 (1905).
4. Слюсарев, Г. Г.: Методы расчета оптических систем, Л.-М., 1937.
5. MERTÉ—RICHTER—ROHR: Das photographische Objektiv (1932).
6. LINFOOT, E. H.: Recent Advances in Optics (1955).
7. Волосов Д. С.: Методы расчета сложных фотографических систем. Огиз-Гос-Издат-техн. теор. лит., Л-М., 10. §. XIV. (1948).
8. COLEMAN, H. S.: Straylight in Optical Systems. J. of Opt. Soc. of Amer. V. 37, No. 6. p. 434 (1947).

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