BEHAVIOUR OF BRIDGE STRUCTURES STIFFENED BY FLOOR BEAMS AND LATERAL BRACINGS

By

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(Received June 2, 1957)

Cross bracings are usually applied if the opening to be spanned by the bridge is large. The number of main girders exceeds in most similar cases two. Lateral bracings acting in horizontal planes connect the main girders at the upper chord or at the floor (upper lateral bracing) and at the lower chord (lower lateral bracing) as shown in Figs. 1*a* and 1*b*. Cross bracings may be trussed girders (Fig. 1*b*), statically determinate or indeterminate, and again frames which are by nature statically indeterminate (Fig. 1*a*).

The investigation of transversal load distribution may be accomplished by one of two methods, depending on whether the lateral bracings are considered and designed to cooperate in the transversal load distribution produced by continuous floor beams and sway bracings, or only the main girders are considered.

Computation methods may further be distinguished according to whether the floor beams are assumed as elastic or as of infinite rigidity, and whether the torsion resistance of main girders are considered or neglected.

The present discussion will be restricted to the case when the torsion resistance of main girders are neglected, whereas the contribution of the lateral bracings are considered. Principles of the computation method making allowance for the torsion resistance of main girders have been discussed in the paper presented at the Congress of the I. V. B. M. in 1948 and reference is made to other literature on the subject.

A) Bracings of infinite rigidity

1. *General*

Bridge structures built over long spans with two or more main girders stiffened by lateral bracings and floor beams may be analyzed by assuming floor beams (sway bracings) having an infinite rigidity. The accuracy obtainable by this method is sufficient for most practical purposes. This generally encountered fact is due to the favourable circumstance that the rigidity of the comparatively short floor beams (bracings) are, relatively to that of the long main girders, very great. Therefore the unit-load deflections (e) and displacements characterising the flexibility of the main girders are relatively great in comparison to that of the deep, short floor beams (or bracings) applied between the closely spaced main girders. The flexibility of the latter, v, is very small. Therefore, the *grid-work rigidity factor* $z = \frac{e}{v}$ will be very great, i. e. practically infinite, or, conversely the *grid-work flexibility factor* $d = \frac{v}{e}$ will be very small, i. e. practically zero. (See point B. 4.)

The agreement between results to be obtained by the computation method base upon infinitely rigid bracings and actual conditions will improve as the

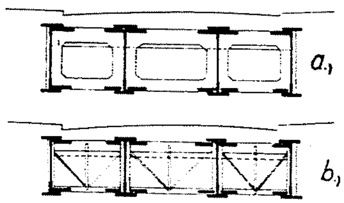


Fig. 1

distance between the main girders decreases and their span and rigidity of bracings increases.

At long span bridges it is, naturally, not necessary to apply bracings at every load-transferring vertical or floor beam, and the number of bracings may substantially be reduced to three or five intermediate cross-sections. At other load-transferring verticals the use of floor-beams of a rigidity inadequate for load-distribution are practicable.

2. *Basic principles of the method*

In order to develop general equations a simple bridge-structure will be analysed, the general cross-section of which is shown in Fig. 2. This structure is composed of four main-girders. The main-girders may be trussed or plate-girders, whereas the bracings may be trusses or frames.

Sign-rules are as follows (see Fig. 3) :

a) *Forces acting on joints* (X or R and Q) are positive as in Fig. 3 when acting upwards or from right-to left (i. e. forces acting upon bracings) and when loads of main-girders act downwards of from left to right;

b) *Load-forces* P are positive when acting in positive direction of Q forces;

c) *Couples* due to external forces are positive when acting countre-clockwise in the plane of the bracing.

Displacements in the plane of a girder, — working independently of bracings — due to unit load acting at the intersection of the plane of the main-girder and of that of the bracing should be denoted by "e" (dimension : cm/t, — cm/kg). These measure the "*flexibility of the main-girder*". The reciprocal value of "e", — that is a load acting at this point, and producing unit-displacement should be denoted by "p" and represents the "*rigidity of the main-girder*" (dimension : t/cm or kg/cm).

Of course

$$e = \frac{1}{p} \quad \text{and} \quad p = \frac{1}{e} . \tag{1}$$

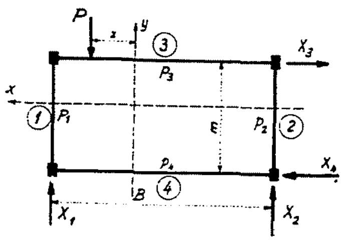


Fig. 2

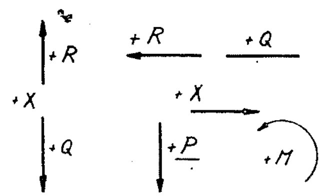


Fig. 2

Thus, the actual displacement in the plane of the main-girder due to force "X" (R, Q) acting upon the joint will be

$$\Delta = X \cdot e = \frac{x}{p} . \tag{2}$$

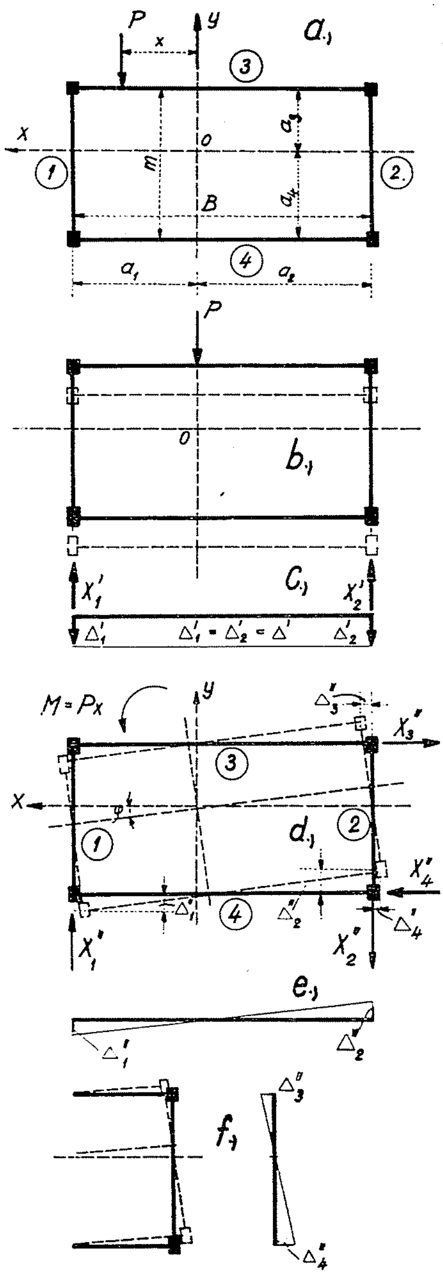
According to these notations, the rigidity of the main-girders should be p_1, p_2, p_3 and p_4 and their flexibility e_1, e_2, e_3, e_4, respectively.

The rigid cross-section under consideration will tend to rotate disc-like when subject to any assumed bending couple M. Let us therefore determine the center of gravity "O" of the system of main trusses illustrated.

With reference to Figs. 2 and 4, the distance of the centre of gravity measured from the planes of individual main trusses is :

$$\left.\begin{aligned} a_1 &= \frac{p_2}{p_1 + p_2} B \qquad & a_3 &= \frac{p_4}{p_3 + p_4} m \\ a_2 &= \frac{p_1}{p_1 + p_2} B \qquad & a_4 &= \frac{p_3}{p_3 + p_4} m \end{aligned}\right\} \tag{3}$$

Let this centre of gravity, as centre of rotation, be the origo of an x, y coordinate-system. Any vertical force acting along the y-axis, will cause a displacement Δ of the whole cross-section in vertical direction only, without any



P
y
x
a.)
3
X
O
1
m
B
2.
4
a1
a2
P
O
b.)
c.)
X1'
X2'
Δ1'
Δ1' = Δ2' = Δ'
Δ2'
M = Px
y
Δ3"
X3"
3
X
φ
2
d.)
1
X4"
4
Δ1"
Δ2"
Δ4"
X1"
X2"
e.)
Δ1"
Δ2"
Δ3"
f.)
Δ4"

Fig. 3

rotation about the centre of gravity. Therefore, according to this effect so-called joint-loads will act only upon main-girders 1 and 2, these being the elastic supports of the bracing. The resistance of main-girders 3 and 4 are assumed to be zero, because action is normal to their planes. Under a horizontal load acting along the "x" axis, only main-girders 3 and 4 will suffer displacement in their planes, and these will carry the entire load. Main-girders 1 and 2 will only displace in a direction normal to their planes ; their corresponding resistance in this direction is assumed to be zero, relative to that of the main-girders 3 and 4 in this case.

As the origo of the coordinate-system corresponds to the centre of gravity of the system, the assumed rigid cross-section, i. e. the bracing subject to couple M rotates around this centre of gravity by the angle "φ", and the main girders suffer simultaneously, relative displacements Δ_1, Δ_2, Δ_3 and Δ_4, respectively. The main-girders will, naturally, also rotate, yet their torsional resistance has been assumed to be zero, and thus this rotation will be without influence upon the load distribution. Movements of the main-girders are, therefore,

$$\Delta_1 = \varphi \cdot a_1 \qquad \Delta_2 = \varphi \cdot a_2 \,. \tag{4}$$

These are proportionate to the distances from the centre of gravity, i. e.,

$$\Delta_1 : \Delta_2 : \ldots = a_1 : a_2 : \ldots \tag{4a}$$

Ordinates of influence-lines of transversal load-distribution (q) will be next determined. The problem will be solved by determining the effect of a "P" force, which is parallel to the "y" axis and acts at any arbitrary point defined by the abscissa "x" (see Fig. 4).

As rigidity of the bracing is infinite, force "P" will be transposed to the centre of gravity of the cross-section (see Figs 4*b* and *d*).

Under the vertical force P — acting at the centre of gravity — the bracing will suffer a parallel displacement $\Delta' = \Delta_1' = \Delta_2'$ (see Fig. 4*c*). Therefore, the vertical force acting along the gravity-line will be distributed on main-girders *1* and *2* only (X_1' and X_2'). These joint-forces will act as active loads (Q) upon the main-girders, yet will act as reactions (R) upon the bracing. In the general analysis, forces arising in the planes of main-girders can be assumed to be *joint-forces* and denoted by "X". Their absolute values will be inserted into the equations, and sign will practicably be determined by inspection. For the sake of clearness it is deemed expedient to start from conditions of static equilibrium, wherefore joint-forces have been drawn as passive forces acting upon the bracing.

Since the displacement of main girders *1* and *2* under force "P" acting at the centre of gravity is equal, it may be written according to Equations (2) :

$$\Delta' = X_1' \cdot e_1 = X_2' \cdot e_2 = \frac{1}{p_1} X_1' = \frac{1}{p_2} X_2'$$

and as

$$x_1' + x_2' = P,$$

therefore, according to these two equations:

$$X_1' = \frac{p_1}{p_1 + p_2} P \qquad X_2' = \frac{p_2}{p_1 + p_2} P \tag{5}$$

Subject to couple $M = P \cdot x$, the rigid bracing will rotate around its centre of gravity together with the rigidly connected main-girders (see Fig. 4*d*). Therefore, in the planes of the main-girdes joint-forces will arise (X_1'', X_2'', X_3'' X_4'') according to the resistance due to rigidity, because rotation causes displacements in the planes of main-girders (see Figs 4f and f). (As before mentioned, forces induced by displacements normal to the planes of main-girders are negligible). Accordingly basic statical and deformation-equations will be:

$$X_1'' a_1 + X_2'' a_2 + X_3'' a_3 + X_4'' a_4 = P \cdot x = M$$

$$X_1'' = p_1 \Delta_1'' \qquad \Delta_1'' = \varphi \cdot a_1$$

$$X_2'' = p_2 \Delta_2'' \qquad \Delta_2'' = \varphi \cdot a_2$$

$$X_3'' = p_3 \Delta_3'' \qquad \Delta_3'' = \varphi \cdot a_3$$

$$X_4'' = p_4 \Delta_4'' \qquad \Delta_4'' = \varphi \cdot a_4$$

From these equations the absolute values are obtained as:

$$X_1'' = \frac{a_1 p_1}{a_1^2 \cdot p_1 + a_2^2 \cdot p_2 + a_3^2 \cdot p_3 + a_4^2 \cdot p_4} \cdot M = \frac{a_1 p_1}{\Sigma a_r^2 p_r} M$$

and

$$X_2'' = \frac{a_2 p_2}{\Sigma a_r^2 p_r} M \ldots \text{etc.}$$

Joint-forces due to assimetrically acting force "P":

$$\left.\begin{aligned}
X_1 &= X_1' + X_1'' = \left(\frac{p_1}{p_1 + p_2} + \frac{a_1 p_1}{a_r^2 \cdot p_r} \cdot x \right) P \\
X_2 &= X_2' + X_2'' = \left(\frac{p_2}{p_1 + p_2} + \frac{a_2 p_2}{a_r^2 \cdot p_r} \cdot x \right) P \\
X_3 &= X_3'' = \frac{a_3 p_3}{\Sigma a_r^2 p_r} \cdot x \cdot P \\
X_4 &= X_4'' = \frac{a_4 p_4}{\Sigma a_r^2 p_r} \cdot x \cdot P
\end{aligned}\right\} \tag{6}$$

In analogiam, joint-forces due to force "P" acting parallel to "x" -axis at a point defined by ordinate "y" will be :

$$\left.\begin{aligned} X_1 &= \frac{a_1 p_1}{\Sigma a_r^2 p_r} y \cdot P \qquad X_2 = \frac{a_2 p_2}{\Sigma a_r^2 p_r} y \cdot P \\ X_3 &= \left(\frac{p_3}{p_3 + p_4} + \frac{a_3 p_3}{\Sigma a_r^2 p_r} y \right) P \\ X_4 &= \left(\frac{p_4}{p_3 + p_4} - \frac{a_4 p_4}{\Sigma a_r^2 p_r} y \right) P \end{aligned}\right\} \tag{6a}$$

Sign of "a" in preceding equations should be in accordance with that of coordinate-axis "x" and "y", which again are in accordance with that of the couple. In this case sign of "X" will correspond to the sign-rule adopted in the foregoing.

In the general case, when a structure of "n" main-girders are analysed, where "n_x" and "n_y" are the number of main trusses parallel to the "x"-axis, and to the "y"-axis, respectively, and "k" denotes the main-girder considered. The corresponding joint-force will be

in case of vertical loading:

$$X_k = X_k' + X_k'' = \left(\frac{p_k}{\underset{n_y}{\Sigma} p_r} + \frac{a_k p_k}{\underset{n}{\Sigma} a_r^2 \cdot p_r} \cdot x \right) P \tag{7}$$

in case of horizontal loading:

$$X_k = X_k' + X_k'' = \left(\frac{p_k}{\Sigma p_r} + \frac{a_k p_k}{\Sigma a_r^2 \cdot p_r} \cdot y \right) P \tag{7a}$$

where sign of "x" and "y" coincide with that of the couple's sense of rotation, and sign of a_k corresponds to that of coordinate-system axes "x" and "y". In this case sign of X_k will be obtained by the sign-rule shown on Fig. 3.

3. *Influence-lines of transversal-distribution i. e. of reactions* (q_i, r_i)

These influence-lines will consist of a single straight line, because the rigidity of bracings is infinite. Therefore, the determination of any two arbitrary points, — for instance the ordinates of the two-end points or of one end-point, and of that corresponding to the centre of gravity, will be adequate. The method applied in the case shown in Fig. 5 is as follows :

a) Joint-forces due to unit-forces P_x and P_y acting at the centre of gravity shall be determined. According to notations adopted the transversal load-distribution factors will be :

in case of unit-force P_y: q_{y1} ; q_{y2} ; q_{y3} (see Fig. 5b)

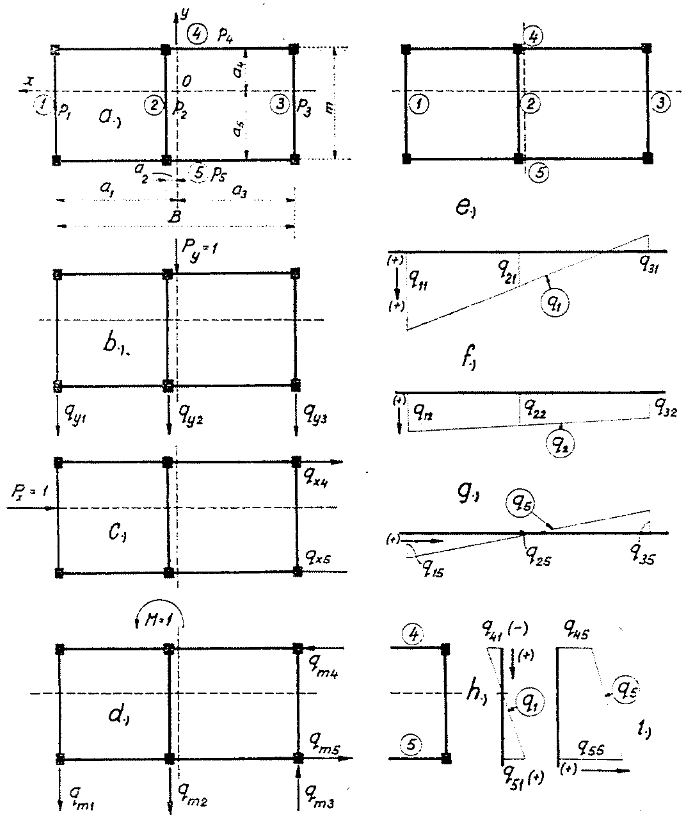


Fig. 4

where for instance

$$q_{y1} =: q_1' = \frac{p_1}{p_1 + p_2 + p_3}$$

in case of unit-force P_x : q_{x4} ; q_{x5} (see Fig. 5c)

where for instance

$$q_{x4} = q_{4'} = \frac{p_4}{p_4 + p_5} .$$

b) Joint-forces due to unit-couple M shall be determined (see Fig. 5d): For example:

$$q_{mk} = \frac{a_k p_k}{a_1^2 p_1 + a_2^2 p_2 + a_3^2 p_3 + a_4^2 p_4 + a_5^2 p_5} . \tag{8}$$

By application of above determined "q" values, — load-distribution values (q_{ik}) due to any arbitrary P_i unit-force may be determined for any main-girder. In general, for a unit-force P_i, if $x = a_1$,

$$q_{ik} = q_k' + a_1 q_{mk} \tag{8a}$$

Influence-lines of transversal distribution of the analysed system of main-girders is shown on Figs. 5e—5i.

Influence-lines of bracings coincide with q-diagrammes, yet are of opposite sign $(r_{ik} = q_{ik})$. In case of long spanned bridges, the bracings may be assumed as infinite but rigid, not only at midspan, but also near the supports, because the deflection of the main-girders at the latter point is still relatively much greater, than that of the bracings. Therefore, the application of average influence-lines of transversal load-distribution is justified in such cases. Yet, the relative rigidity of bracings and especially of those near to supports is small, if 1. the span of the bridge is short, 2. few main-girders are applied, and 3. the main-girders are spaced wide apart. All these circumstances tend to increase the relative span of the bracings. The relative rigidity of the bracings becomes small and the load-distributing effect will considerably alter. The case of elastic bracings arises, where the assumption of infinitely rigid bracings will not yield suitable results any more. Therefore, elastic bracings must be considered. If it is further remembered that bracings over piers and abutments repose upon fix supports, and load-distribution in such cases are identical with that of a normal continuous beam, even the determination of influence-planes may become indicated. It should be noted, however, that in many cases, where the height of the main-girders and bracings are small and especially if few main-girders are applied, the substitution according to the Leonhardt-principle of the bracing by an ideal floor-beam may be permitted. The flexibility factor "v" due to unit force of this beam may be determined by considering a bracing of double span (double distance of main-girders). Further computations may be carried out by taking these floor beams into consideration, instead of the original bracings (Leonhardt-principle).

4. *Numerical example*

The bridge-structure is composed of three main girders and two wind-trusses stiffened by bracings of the Vierendeel type (Fig. 6). Vertical main-girders are marked 1, 2 and 3, lateral ones by 4 and 5.

Rigidity of main-girders

$$p_1 = p_3 = 1{,}0 \text{ t/cm}\,; \;\; p_2 = 0{,}8 \text{ t/cm}: \;\; p_4 = p_5 = 1{,}2 \text{ t/cm}.$$

Factors of transversal load-distribution :

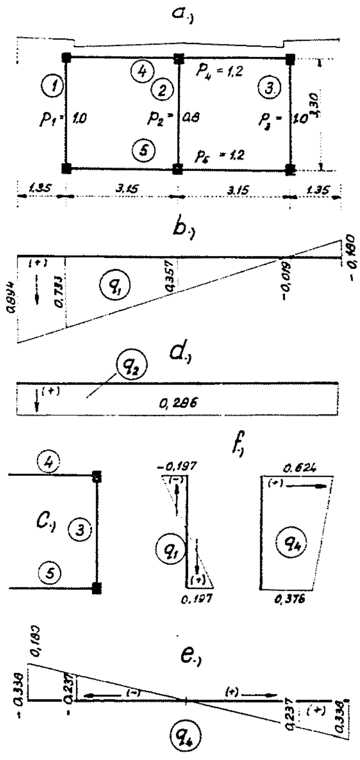


Fig. 5

1. For vertical forces :

$$q_{1,1\,;\,3,1} = \frac{1}{2{,}8} \pm \frac{2}{5{,}32} = 0{,}357 \pm 0{,}376 = \begin{matrix} +\,0{,}733 \\ -\,0{,}019 \end{matrix}$$

$$q_{1,2\,;\,3,2} = \frac{0{,}8}{2{,}8} = \qquad = 0{,}286$$

$$q_{1,4\,;\,3,4} = \mp \frac{1{,}26}{5{,}32} = \qquad = \mp\, 0{,}237$$

2. *For horizontal forces* :

$$q_{4.1\,;\,5.1} = \mp \frac{1{,}05}{5{,}32} = \qquad\qquad = \mp 0{,}197$$

$$q_{4.2\,:\,5.2} = 0 \qquad\qquad = \quad 0{,}00$$

$$q_{4.4\,;\,5.4} = \frac{1}{2} \pm \frac{1{,}32}{0{,}64} = 0{,}500 \pm 0{,}124 = \begin{cases} 0{,}624 \\ 0{,}376 \end{cases}$$

Influence-lines of transversal load-distribution are shown in Figs. 6b, c, d, e, f.

B) Elastic bracings

1. General

The exact analysis of multi-girder structures stiffened by several bracings require much computation work, and in practice is hardly feasible. The analysis will become extremely cumbersome, if the main girders are outwardly statically indeterminate structures and the outwardly statically indeterminate disposition of bracings due to continuity is further complicated by their inwardly statically indeterminate character.

In principle there is no obstacle of mathematically exact computations, however, the evaluation of results obtained at load tests of existing structures led to the conclusion that the introduction of certain simplifying assumptions are permissible, without impairing the required accuracy.

The analysis including the elastic behaviour of the bracings, and to be described subsequently, is based on the following assumptions :

a) Torsional resistances will be neglected.

b) The rigidity of main girders in directions normal to their planes is zero.

c) The load-distributing effect of a single "ideal bracing" will be analyzed only, the ideal, equivalent rigidity of which will be increased according to *Leonhardt's* principle. The average value of this increasing factor is, in case of

one or two bracings	= 1,0
three or four bracings	= 1,6
five or more bracings	= 2,0

d) *The rigidity of main girders* (p) respectively their *flexibility* (e), — which both depend on support-conditions of the main girders will be determined at midspan by the unit-displacement or unit-load applied in the plane of the main girders at the place of bracing.

e) The effect of more than one bracing upon the behaviour of the main girders will not be considered. Yet, if necessary, i. e. in case of short main girders stiffened by a few bracings only, this will be allowed for by approximation.

2. *Basic principles of the method*

Elastic supports of the bracings are the vertical and horizontal main girders, the ideal scheme of which in case of a four main-girder system is shown in Fig. 7 where the conventional marking of elastic supports is also to be seen.

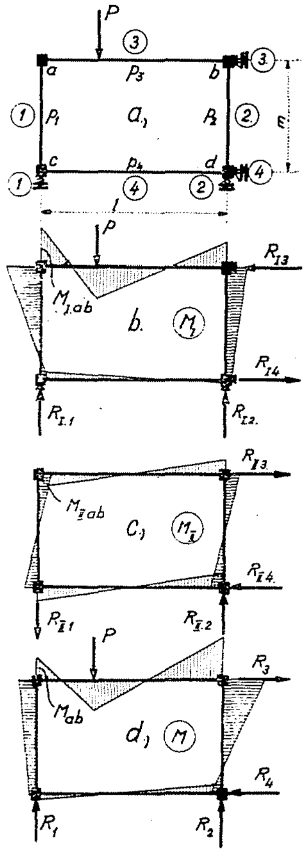


Fig. 6

In order to avoid the direct solution of an inconventiently great number of equations the computation will be resolved into several phases.

In the *initial stage*, the elastically supported bracing will be transformed into a rigidly supported structure by an assumed supporting bar at the place, and in the plane of elastic supports. Stress distribution of this rigidly supported bracing can be determined for the acting external force by any known method. *This is the first phase of computation*, the results of which will yield the M_{Ik} end-moments acting at fixed ends, and supporting axial forces R_{Ik} acting in the assumed supporting bars and also axial forces $S_{I\,i}$ acting in the bars, when the bracing is a trussed girder. (Index I marks the first phase of computation.)

A bracing of the *Vierendeel*-type will be analysed in the following (see Fig. 7). The same principles logically apply to the investigation of trussed bracings as well.

In the expression M_{Ik} the index "k" will be replaced by symbols denoting the two ends of the supporting bar, according to the support under consideration. For instance at the cross-section joining the support "a" the notation $M_{I,ab}$ will be used, if the opposite end of the bar is denoted by "b". In case of the supporting force acting in the assumed bar, the latter, i. e. the replaced main girder will be denoted by "k". Correspondingly, in the present example, notations 1, 2, 3 and 4 (see Fig. 7b) will be used. Supporting forces (R_I) marked in this way, represent those required to establish equilibrium conditions, i. e., reactions acting upon the bracing. Forces contrary to the afore-mentioned ones will act as active loads (Q_1) upon the elastic supports i. e. upon the main girders.

In the *second phase* of computation, effects resulting from elastic displacements of supports (M_{II}, R_{II}, Q_{II}) will be considered. Summation of forces obtained in phases one and two of the computation will yield the final forces (M, R) acting upon the bracings and also the loads (Q) acting upon the main girders, according to the well-known equations

$$\left.\begin{aligned} M &= M_I + M_{II} \\ R &= R_I + R_{II} \\ Q &= Q_I + Q_{II} \end{aligned}\right| \qquad (9)$$

Computation of the *second phase* will again be divided into two parts. In the first part essentially preparatory investigations will be carried out on the rigidly supported girder by calculating internal M_{ki} and external supporting (R_{Ki}) forces due to unit displacement ($\Delta_k = 1$) applied upon the assumed supporting bar. In the analysed case the supporting forces of the bracing due to unit-displacement of the assumed bars 1, 2, 3 and 4 can generally be expressed (see Figs. 8b—e) as :

$$R_{k1},\ R_{k2},\ R_{k3},\ R_{k4},$$

if the unit-displacement $\Delta_k = 1$.

In the *second part* of computation the elastic behaviour of the supports will be considered. For this end the actual displacements Δ_i of the elastic supports will be determined by the condition that the resulting force R_i at the support will be the sum of force (R_{Ii}) acting on the rigidly supported girder and of actual forces $R_{ki\cdot k}$ acting on support "i", which are due to an actual displacement k. Therefore

$$R_{IIi} = \Sigma R_{ki} \cdot \Delta_k \tag{10}$$

and

$$R_i = R_{Ii} + \Sigma R_{ki} \cdot \Delta_k \tag{11}$$

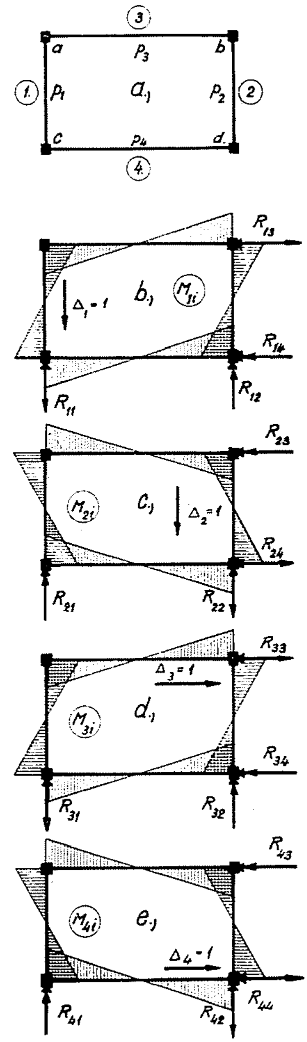


Fig. 7

A further condition to be satisfied is that at places of redundant restraints — i. e. at assumed supports — the sum of all forces due to external loads and redundancy must be zero. Thus:

$$R_{Ii} + \Sigma R_{ki} \cdot \Delta_{\;} - R_i = 0. \tag{12}$$

Due to the linear relationship $R_i = p_i \Delta_i$ between actual displacement and actual supporting force, — e. g. for support 1 (see Fig. 8) — this may be written as : (13)

$$R_{11} \Delta_1 + R_{21} \Delta_2 + R_{31} \Delta_3 + R_{41} \Delta_4 + R_{11} - p_1 \Delta_1 = 0 \tag{11a}$$

where R_{11} is the supporting force of the rigidly supported structure due to external loads. I_n this case four linear equations are necessary to determine the actual Δ_i values at the supports 1, 2, 3 and 4. Therefore,

$$\begin{array}{llll} (R_{11} - p_1)\,\Delta_1 + & R_{21}\,\Delta_2 + & R_{31}\,\Delta_3 + & R_{41}\,\Delta_4 + R_{11} = 0 \\ R_{12}\,\Delta_1 + & (R_{22} - p_2)\,\Delta_2 + & R_{32}\,\Delta_3 + & R_{42}\,\Delta_4 + R_{12} = 0 \\ R_{13}\,\Delta_1 + & R_{23}\,\Delta_2 + & (R_{33} - p_3)\,\Delta_3 + & R_{43}\,\Delta_4 + R_{13} = 0 \\ R_{14}\,\Delta_1 + & R_{24}\,\Delta_2 + & R_{34}\,\Delta_3 + & (R_{44} - p_4)\,\Delta_4 + R_{14} = 0 \end{array} \tag{14}$$

In these equations values of Δ_i are the only unknown quantities.

Having determined values of Δ_i, the joint-forces may be obtained from Eqs. (13) or (11).

Internal forces acting upon the bracing may be directly determined by the following expression bearing a strong resemblance to Eq. (11) :

$$M_i = M_{Ii} + M_{IIi} = M_{Ii} + \Sigma\, M_{ki} \cdot \Delta_k \tag{15}$$

The only load member differring from zero in condition Equations (14) will occur at the support subject to unit load, if our aim is to compute influence lines of transversal distribution only. In this case Eqs. (11), resp. (13) yield the influence-factors ($-q_{ki}, r_{ki}$) directly. The symmetry of both bracing and rigidity conditions of the main girders about the longitudinal axis, together with the condition of statical equilibrium as regards forces acting upon the bracing, permits the intoduction of considerable simplifications. Owing to the condition of statical equilibrium, e. g. under the load $P_1 = 1$ in the case shown in Fig. 8, by applying the theorem of projections :

$$r_{11} + r_{12} = 1$$

and employing the double index also for Δ-s,

$$p_1 \cdot \Delta_{11} + p_2 \cdot \Delta_{12} = 1$$

and thus

$$\Delta_{12} = \frac{1}{p_2} - \frac{p_1}{p_2} \cdot \Delta_{11} \,. \tag{16}$$

Furthermore,

$$r_{13} + r_{14} = 0$$

which means that

$$p_3 \cdot \Delta_{13} + p_4 \cdot \Delta_{14} = 0,$$

whence

$$\Delta_{14} = - \frac{p_3}{p_4} \Delta_{13} . \tag{16a}$$

According to the theorem of moments and with due regard to the sign convention adopted it may be written :

$$(q_{11} - 1) \, l - q_{13} m = 0$$

or

$$q_{12} \cdot l + q_{13} \cdot m = 0$$

that is

$$\Delta_{13} = - \frac{1}{p_3} \cdot \frac{l}{m} + \frac{p_1}{p_3} \cdot \frac{l}{m} \cdot \Delta_{11} \tag{16b}$$

or

$$\Delta_{13} = - \frac{p_2}{p_3} \cdot \frac{l}{m} \cdot \Delta_{12} . \tag{16c}$$

Similarly in case of force $P_3 = 1$

$$\Delta_{32} = - \frac{p_1}{p_2} \cdot \Delta_{31}. \tag{17}$$

$$\Delta_{34} = \frac{1}{p_4} - \frac{p_3}{p_4} \cdot \Delta_{33} . \tag{17a}$$

According to the theorem of moments :

$$q_{31} \cdot l + (1 - q_{33}) \, m = 0$$

from which

$$\Delta_{31} = \frac{p_3}{p_1} \cdot \frac{m}{l} - \frac{m}{p_1^l} \cdot \Delta_{33} . \tag{17b}$$

Applying the above equations of statical equilibrium and substituting corresponding Δ-values into Equations (14), the number of independent equations to be solved will always be reduced by three. Thus in the problem shown on Fig. 8 a single Δ is to be determined only, corresponding to the single Equation (14). The others may be determined from the conditions of statical equilibrium. For example in case of unit-load P_1, with values Δ_{12} Δ_{13} and Δ_{14} having been determined by applying Equations (16) from among equations of type (14) only the first, — corresponding to point "1" will be required for the computation of Δ_{11}. Having established Δ_{11}, values of Δ_{12}, Δ_{13} and Δ_{14} may be computed by Equations (16).

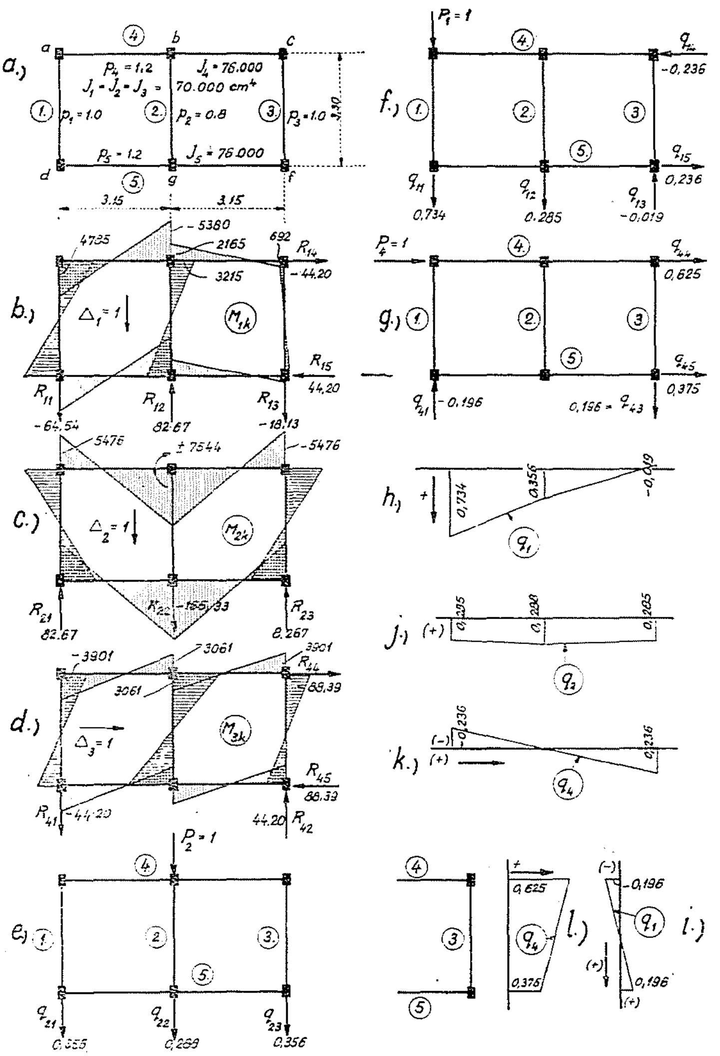


Fig. 1

3. *Numerical example*

The problem to be solved has been described in the foregoing under point A. 3 (Fig. 9). Basic data:

$$I_1 = I_2 = I_3 = 70\,000 \text{ cm}^4$$

$$I_4 = I_5 = 76\,000 \text{ cm}^4$$

$$p_1 = p_3 = 1{,}0 \text{ t/cm}$$

$$p_2 = 0{,}8 \text{ t/cm}$$

$$p_4 = p_5 = 1{,}2 \text{ t/cm}.$$

Both the main girders and the bracing are steel-structures. $E = 2100$ t/cm^2.

Results of auxiliary computations belonging to second group of second phase are to be seen on Figs. 9b, c and are as follows :

— in Fig. 9b M_{1k} and R_{1k} due to $\Delta_1 = 1$;

— in Fig. 9c M_{2k} and R_{2k} due to $\Delta_4 = 1$ and

— in Fig. 9d M_{3k} and R_{3k} due to $\Delta_3 = 1$.

Dimensions of M-values are tcm and of R-values t.

The left side of condition equations of deformation — according to formula (14) are :

$$-65{,}54\,\Delta_1 + 82{,}67\,\Delta_2 - 18{,}13\,\Delta_3 - 44{,}20\,\Delta_4 + 44{,}20\,\Delta_5 \tag{1}$$

$$82{,}67\,\Delta_1 - 166{,}13\,\Delta_2 + 82{,}67\,\Delta_3 + 0 + 0 \tag{2}$$

$$-18{,}13\,\Delta_1 + 82{,}67\,\Delta_2 - 65{,}54\,\Delta_3 + 44{,}20\,\Delta_4 - 44{,}20\,\Delta_5 \tag{3}$$

$$-44{,}20\,\Delta_1 + 0 + 44{,}20\,\Delta_3 - 85{,}59\,\Delta_4 + 84{,}39\,\Delta_5 \tag{4}$$

$$44{,}20\,\Delta_1 + 0 - 44{,}20\,\Delta_3 + 84{,}39\,\Delta_4 - 85{,}59\,\Delta_5 \tag{5}$$

Actual displacements and forces due to unit-load $P_2 = 1$ (see Fig. 9*e*).
Actual displacements and forces due to unit-load $P_1 = 1$ (see Fig. 9)*f*.
Actual displacements and forces due to unit-load $P_4 = 1$ (see Fig. 9*g*).

Owing to symmetry and load-conditions :

$$p_1\,\Delta_{41} + p_3\,\Delta_{43} = 0$$

$$p_4\,\Delta_{44} + p_5\,\Delta_{45} = 1$$

$$p_1\,\Delta_{41}\,2\,l + (1 - p_4\,\Delta_{44})\,m = 0.$$

and accordingly

$$\Delta_{41} = -\Delta_{43}$$
$$\Delta_{45} = 0{,}8333 - \Delta_{44}$$
$$\Delta_{42} = 0$$
$$\Delta_{44} = 0{,}8333 + 1{,}5909\ \Delta_{41},$$

Thus, the condition equation applying to point 1 assumes a form after rearrangement:

$$188{,}05\ \Delta_{41} = 36{,}84$$

whence

$$\begin{array}{lllll}
\Delta_{41} = & -0{,}196 & q_{41} = & -1{,}0 \cdot 0{,}196 = & -0{,}196 \\
\Delta_{42} = & 0 & q_{42} = & 0 \cdot \qquad = & 0 \\
\Delta_{43} = & 0{,}196 & q_{43} = & 1{,}0 \cdot 0{,}196 = & 0{,}196 \\
\Delta_{44} = & 0{,}521 & q_{44} = & 1{,}2 \cdot 0{,}521 = & 0{,}625 \\
\Delta_{45} = & 0{,}312 & q_{45} = & 1{,}2 \cdot 0{,}312 = & 0{,}375
\end{array}$$

Influence-lines of q_1, q_2 and q_4 are shown on Figs. 9h, i, j, k, l. As compared to results comuted for a bracing of infinite rigidity the differences are insignificant.

4. *Practical remarks*

In order to obtain information as to whether the computation of an elastic or of an infinitely rigid bracing will be necessary, it is advisable to determine approximatingly the rigidity-factor "z" or flexibility-factor "d" of the gird-work.

As already mentioned: $z = \frac{e}{v}$ and $d = \frac{v}{e}$.

The factor of flexibility of the main girder "e" is the reciprocal value of its rigidity-factor "p". Considering an average value of "e"

$$e = \frac{e_1 + e_2 + \ldots e_n}{n} = \frac{1}{n}\left(\frac{1}{p_1} + \frac{1}{p_2} + \ldots + \frac{1}{p_4}\right).$$

In practical approximative computations of grid-works rigidity, "v", should be determined by considering about twice the sum of the two chord's moments of inertia, or the average moment of inertia multiplied by four. According to definition "v" — the flexibility of the bracing — is the deflection of a girder of „2 l" span due to unit-load (1 t) acting at midspan.

The case of an infinitely rigid bracing should be considered if the approximate value of the grid-work rigidity (z) equals or exceeds 10.

References

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Summary

The determination of transversal distribution-characteristics in multi-girder bridge-structures stiffened by bracings and wind-trusses is discussed, and a method to enable the computation of loads carried by individual main girders is given for cases of both infinitely rigid and elastic bracings.

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