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RESEARCH ARTICLE

Continuummechanics – Heat Conduction – Cognition

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Abstract
Starting from the basic equations of solids we have shown one possible construction of the modification method of the law of heat conduction. Based on this method the steps of development can be outlined, the sequence and role of the tasks can be established. As an additional result we managed to highlight some aspects of the cognitive process, which emerge in the course of natural scientific research.

Keywords
continuummechanics, heat conduction, Fourier-law

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1 Introduction

The scientist dealing with thermo-elasticity sooner or later is faced with questions involving the modification of the law of heat conduction. In the case of analysing dynamical processes he either accepts one of the modified laws of heat conduction [1] or he may try to solve this problem relying more or less on his own resources [2]. This is however where the scientist will find himself in the midst of investigations related to continuum-physics. He started out from continuum-mechanics [3] his research led him to thermodynamics and now he has to move on. He has a relatively long way and a vast area to cover, on his way he encounters a lot of things, thus the idea of the cognitive process also emerges. The principal and additional results of such an investigation will be shown in the paper to follow. Starting from the basic equations of solids we study the construction of the modification method for the law of heat conduction and the interdisciplinary problems it involves. The role of Mathematics and the intriguing question of the "freedom" of research will also be discussed.

2 Basic equations of solids

The study of the coupled mechanical and thermo-dynamical processes of solid continuum can be done with the help of the basic equations. These, in one-dimensional cases look as follows:

$$\sigma_x = \rho v_t \tag{1}$$

$$\rho u_t = \sigma \varepsilon_t + \lambda h_x \tag{2}$$

$$q \le T s_t \tag{3}$$

$$\varepsilon_t = v_x \tag{4}$$

$$q_t = h_x \tag{5}$$

$$\phi(\dots) \equiv 0 \tag{6}$$

$$\tilde{\varphi}(\dots) \equiv 0 \tag{7}$$

$$\psi(\dots) \equiv 0 \tag{8}$$

$$F_i(\dots) \equiv 0 \tag{9}$$

In these basic equations the following basic functions appear:

$$\sigma, v, u, \varepsilon, h, q, T, s \ ,$$

And the subscripts denote partial derivatives. The first group of basic equations is made up by the laws of nature: the equations of motion (1), the equation of energy (2) and the 2nd law of thermo-dynamics (3). The second group contains the equations of kinematics: the geometric equations (4) and the equation of thermo-kinematics (5). The third group comprises the constitutive equations that is the constitutive law (6), the equation of state (7) and the law of heat conduction (8). The argument of theses contains the basic functions and their derivatives. The fourth group is made up by the initial and boundary conditions.

If we study mechanical processes only in that case equations (1), (4) and (6) are to be used (obviously the appropriate equations of group (9) are necessary). If we are only concerned with thermo-dynamical processes in that case we have to deal with equations (2), (7) and (8). In this case the $\sigma\varepsilon_t$ member should be omitted from (2). As can be seen equation (3) does not necessarily appear at the study of either the mechanical or the thermo-dynamical process. Similarly, the use of equation (5) is not essential either. This gives rise to the thought that the 2nd law is not as organic part of the construction as are the other laws of nature. This idea is reinforced by the fact that and 2nd law is an inequality the "practical value" of which for the researcher of exact natural sciences is smaller than that of the equation. Another idea occurs at this point: the relations can be examined on the basis of certain groups of the basic equations e.g. selecting function σ, v, ε, (mechanical process) to close the system one single constitutive equations is necessary to equations (1) and (4): this is the law (6) (e.g. $\sigma = E \cdot \varepsilon$). Or selecting functions u, h, (thermo-dynamical process) two constitutive equations are necessary for the equations (2) to make the system manageable: the equations of state (7) and the law of heat conduction (8) (e.g.: $u = c \cdot T$ and $h = \lambda \cdot T_x$).

Thus we have reached the question of examining the constitutive equations as a basic research problem in the field of continuum-mechanics. The research done in the area of all three constitutive equations mentioned has been sufficiently documented, numerous method are known. Let us now turn our attention to the examination of the law of heat conduction. As apart from the last decades, Fourier's law of heat conduction served perfectly despite the challenges of time, and as in the overwhelming majority of the tasks it still works perfectly the researches related to the law of heat conduction are manifested in an effort geared at modifying Fourier's law.

3 Modified equations for heat conduction

The classical theory of heat conduction is based on the equation:

$$\mathbf{q} = -k\nabla T \ , \tag{10}$$

for the heat flux **q**, which combined with the conversation of energy equation:

$$\rho\frac{\partial e}{\partial t} + \nabla\mathbf{q} = 0 \ , \tag{11}$$

and the equation of state:

$$e = cT \ , \tag{12}$$

we get the well-known equation for T:

$$k\nabla^2 T = \rho c\frac{\partial T}{\partial t} \ . \tag{13}$$

This equation is a parabolic-type partial differential equation that allows an infinite speed for thermal signals. During the last six decades nonclassical theories free from this paradox have been published. These new models are mostly modified versions of the classical Fourier's equation and consequently involve hyperbolic-type heat transport equations admitting finite speed for thermal signals. According to these theories heat transport is a wave phenomenon rather than a diffusion type. These new theories are referred as theories with finite wave speeds or theories with second sound (the first sound being the usual sound).

The first model to remove the above mentioned paradox was proposed by Carlo Cattaneo and Pierre Vernotte in 1958:

$$\tau\frac{\partial q}{\partial t} + \mathbf{q} = -k\nabla T \ . \tag{14}$$

The modified Fourier equation coupled with the energy balance equation leads to a hyperbolic heat equation:

$$k\nabla^2 T = \rho c\left(\frac{\partial T}{\partial t} + \tau\frac{\partial^2 T}{\partial t^2}\right) . \tag{15}$$

Now the new material parameter appearing in the heat equation which referred as thermal relaxation time.

Some of other modification of the Fourier equation are the following:

$$\tau\frac{\partial q}{\partial t} + \mathbf{q} = -k\nabla T + a_1\nabla^2\mathbf{q} + a_2\nabla^2\mathbf{q} \ , \tag{16}$$

$$\tau\frac{\partial q}{\partial t} + \mathbf{q} = -k\nabla T + l\nabla\frac{\partial T}{\partial t} \ , \tag{17}$$

$$\mathbf{q} = -k\nabla T + a_2\nabla^2\mathbf{q} \ . \tag{18}$$

Here (16) is the Guyer-Krumhansl equation, (17) is the Jeffreys type equation, (18) is the Green-Naghdy type equation of heat conduction, l , α_1, and α_2 are material parameters. In [13] was derived these models with the help of irreversible thermodynamics and the relations of the different terms was showed with their dissipative and nondissipative nature.

Another possible solution is the modifying the equation of state [14]:

$$e = cT + c_1 T_t \,, \tag{19}$$

where $c_1 = \tau c$. It results the (15) as well.

Since the thermal relaxation time is found to be very small, the variety of the researchers' opinion has wide range. Many authors have argued that the last term in (15) may be ignored in many practical problems. But some experts have employed the modified equation of heat conduction in their calculations to study some practically relevant problems and have found that, in heat transfer problems involving high heat fluxes or short time intervals, the hyperbolic heat conduction model gives significantly different result than the parabolic equation. Based on these studies we can conclude that, the modifying the Fourier's law is not negligible particularly when the elapsed time during a transient is less than about 10^{-5} sec or when the heat flux involved is greater than 10^3W/mm^2. Such conditions are not impossible for example in the nuclear industry. In our previous work [7] we showed that by numerical modeling of the Cattaneo-Vernotte equation, depending on the value of relaxation time the difference between the solution of Fourier modeling and solution obtained by the modified equation can be significant.

4 Physical meaning and mathematical model of thermal relaxation time

The material parameter τ appearing in (15)-(17) has a definite physical interpretation. It results from the phase lag between the heat flux vector and the temperature gradient in a high-rate response. In other words τ represents the relaxation time or build-up period for the initiation of heat flow after a temperature gradient has been imposed at the boundary of the domain. It states that heat flow does not start instantaneously, but rather grows gradually with a relaxation time τ after the application of the temperature gradient. Thus, there is a phase lag for the disappearance of the heat flow after removal of the temperature gradient. So the relaxation time is associated with the linkage time between phonons (phonon-phonon collision) necessary for initiation of the heat flow and is a measure of the thermal inertia of the medium.

The above mentioned delayed behavior can be expressed as:

$$\mathbf{q}(r, t+\tau) = -k\nabla T(r,t) \tag{20}$$

where r is the position vector of the elementary volume under observation [9]. In the absence of a delayed response, (20) reduces to the classical Fourier equation. After application of the linear Taylor series expansion to **q**, we obtain the Cattaneo-Vernotte equation (15).

By focusing our attention on the mathematical representation of thermal relaxation time after some derivation of the modified heat equation (15) we get the following form:

$$\nabla^2 T = \frac{1}{\mathrm{D_T}}\frac{\partial T}{\partial t} + \frac{\tau}{D_T}\frac{\partial^2 T}{\partial t^2} \,, \tag{21}$$

where $\mathrm{D_T} = k/\rho c$ the thermal diffusivity. Focusing on the terms containing $\nabla^2 T$ and $\partial^2 T/\partial t^2$ we obtain:

$$\frac{\partial^2 T}{\partial x \partial x} + \ldots = \ldots + \frac{1}{\frac{D_T}{\tau}}\frac{\partial^2 T}{\partial t^2} \,. \tag{22}$$

It is obvious that $\frac{D_T}{\tau}t^2$ must have a dimension of square of a length. It suggests that the ratio $\sqrt{\frac{D_T}{\tau}}$ must be a velocity-like quantity. Thus

$$v_t = \sqrt{\frac{D_T}{\tau}} \tag{23}$$

with being v_t the thermal wave speed in the medium.

Both D_T and τ are intrinsic thermal properties of the medium. The resulting thermal wave speed v_t, therefore, is also an intrinsic thermal property. It characterizes the thermal wave propagation the same way as the diffusion behavior characterized by the diffusivity. As termed by Chester, the reciprocal of the relaxation time, $f = 1/\tau$ is the critical frequency assign the activation of thermal wave behavior. When the collision frequency among molecules exceeds such a threshold, the wave behavior in heat conduction dominates over diffusion [9]. According to some recent works correlating the thermal wave theory to the microscopic model, the thermal wave speed v_t is related to the coupling factor G of the electron-phonon collisions and the volumetric heat capacities of the electrons C_e, and the metal lattice C_m through the equation [9]:

$$v_t = \sqrt{\frac{kG}{C_e C_m}} \,. \tag{24}$$

The thermal relaxation time with the microscopic parameter is:

$$\tau = \frac{1}{\mathrm{G}\left(\frac{1}{\mathrm{C_e}} + \frac{1}{\mathrm{C_m}}\right)} \,. \tag{25}$$

The coupling factor of electron-phonon interactions depends on the number density of electrons, speed of sound, thermal conductivity, and Boltzmann constant. It is clear that the determination of the relaxation time and the thermal wave speed in microscopic point of view is impossible without some knowledge in solid state physics.

5 Experimental methods on Non-Fourier heat conduction

In developing a suitable model for describing certain phenomena in engineering, establishment of a rigorous physical basis and comparison with experimental observations are equally important. Experimentally, the second sound was first detected by Peshkov in liquid helium in 1944 [8]. He has found the thermal wave speed to be equal to 19 *m/s*. Later, second sound was also detected in solid helium at temperatures about 0,6 K. Subsequently the phenomenon was also detected in NaF, NaI, and in some other crystals. A typical value of the thermal relaxation time for metals at ambient temperature has been reported to be of the order of 10^{-11} sec [9]. In 1993 was reported by Majumdar that diamond has a high relaxation time ($\tau = 10^{-3}$ sec) at 77K and exhibits hyperbolic heat conduction in a macroscopic sample. Due to the absence of a table for the thermal wave speed, table for the relaxation time for engineering materials is still absent at this point. The reason is that it is difficult to measure the thermal wave speed because of the very short time intervals. We have to sample the quantities in a very high frequency range and ensure some special conditions e. g.: high temperature gradient or very low temperature.

Due to the above mentioned reasons very few experimental method was published on the validation of the modified law of heat conduction or experimental determination of the relaxation time. In ref [10] the theory and implementation of an experiment aimed at studying heat behavior on a short time scale was explained. By this measurement based on the forced Rayleigh scattering method, the Fourier law was well confirmed. The second sound phenomenon was investigated by Szekeres et. al. [6] with thermal shock of long bar. They obtained that the thermal relaxation time lies at the order of magnitude of 10^{-1} sec, which considerably differs from the value given in literature.

6 Construction of the modification method for the law of heat conduction

The necessity and some methods of the modifications of Fourier's law of heat conduction as well as the results of these methods and the application of these results are well known from the technical literature. Let us now introduce a possible construction of the modification method. This takes into account the earlier methods as well as the means at hand. Let us examine the phenomenon of heat conduction from a *macro-structural* aspect first, i.e. using the basic equations of *thermo-dynamics*. In this case equations (2), (7) and (8) describe the process with the help of u, h and T basic functions. Supplementing the investigation from a different point of view, the *physics of solids* indicates from a *micro-structural* aspect what the nature of the law of heat conduction and the equation of state could be like due the material property. From a *practical* point of view the task can be examined from several viewpoints. First of all as *Fourier's* law of heat conduction has served the purpose well it should appear in each modification as a special case. The results must be verifiable *experimentally* (numerically and physically) and *applicable* in practice (thermo-elasticity, thermo-energetics). The exact delineation of the conditions is done on the language of *mathematics*. As we are seeking a constitutive equation thus it must contain the derivatives of all the basic functions.

The form of (8) for example is:

$$\psi\left(u,h,T,u_x,h_x,T_{x,}u_t,h_t,T_t\ldots\right)\equiv 0 \qquad (26)$$

Obviously, for the sake of feasibility in practice we only use a limited number of derivatives. Time and place cannot be present explicitly in a constitutive equation, or those variables can be filtered through its argument which is physically not objective. Since the phenomenon propagates with finite velocity and there must be receding and recurring waves the governing equation system should be hyperbolical independently from the boundary and initial conditions. Combining this with the requirement of retaining Fourier's law of heat conduction the task is to produce a law of heat conduction with which in the case of a fast process the system is hyperbolical in the case of decelerating process the system is moving towards the parabolic whereas in the case of a stationary process it is exactly parabolic. For this the coefficients of the law of heat conduction e.g.: the relaxation time can be given in the form of some sort of series:

$$\tau=\tau_0\cdot\left(e^{\sum_i a_i\frac{\partial T}{\partial t^i}}-1\right).$$

Based on the considerations summarized above we can define the differential equation system describing the phenomenon. From the analytical numerical or combined solution of the differential equation system conclusions can be drawn with relation to the construction of the modified law of heat conduction. The modified law of heat conduction is put to practical test by the methods mentioned earlier. (See summary in Table 1 and Fig. 1.) Reviewing the method outlined above it is clear that as it forms an interdisciplinary field of several areas the cooperation of the specialists of thermo-dynamics, solid physics, thermo-energetics, mechanics, mathematics, experimental physics or mechanics and the computer scientist is needed. These results problems of interdisciplinary nature which will be discussed in the following chapter only minor comments are made at this point. Let us picture Fig. 1 in a case where the phenomenon is examined in its complete continuum-physical reality. Imagine how many fields would then be involved in the investigation of the phenomenon!

7 Thoughts pertaining to interdisciplinary problems

Let us try to generalize what has been said above on interdisciplinary questions in view of continuum mechanics. The development of sciences – and in this respect continuum-mechanics is no difference – has a twofold – intensive and extensive – direction. As far as intensity or depth is concerned this is a much discussed issue. Let us now deal with the extensive development i.e. the broadening of the disciplines. This

Table 1

I. Macro-structural (thermo-dynamical) equations

II. Micro-structural (solid physics) relations

III. Practical conditions

   a. Preservation of Fourier's

   b. Experiment

      i. Numerical

      ii. Physical

   c. Application

      i. Thermo-elasticity

      ii. Heat-energetics

IV. Mathematical tools

   a. Mathematical formulation of conditions

      i. Conditions set by the constitutive equations

         1. All derivatives of all variables

         2. No explicit place and time

         3. Physical objectivity

      ii. Condition of hyperbolicity

      iii. Preservation of Fourier's

   b. Solution of differential equation system

      i. Method of solution

      ii. Analytical solution

V. Computing technics

   a. Numerical solution

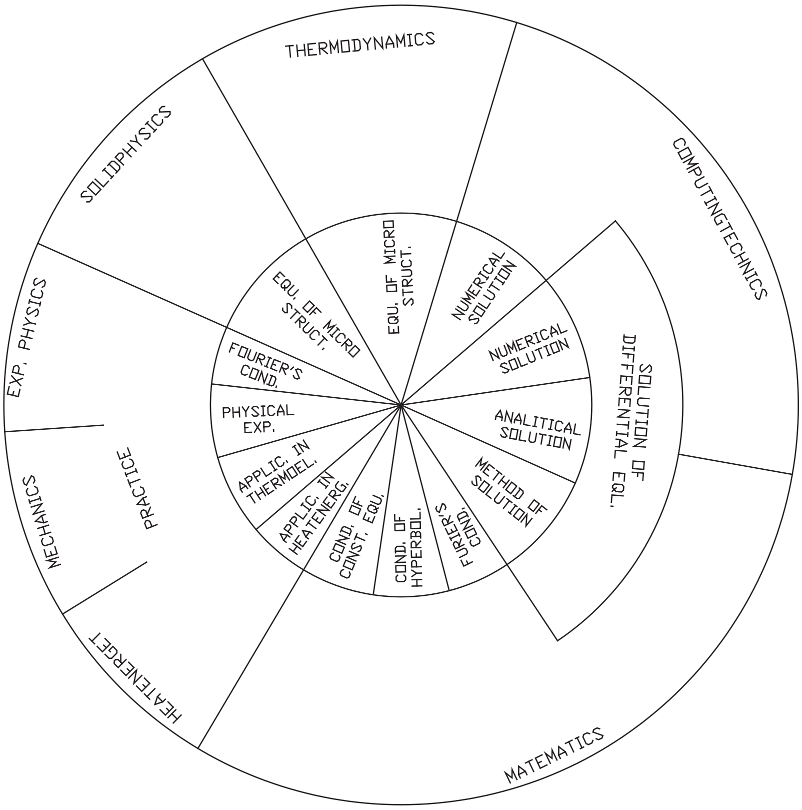

Fig. 1

development leads to and beyond interdisciplinary areas. As deformation phenomena due to the very nature of physics, are or can be accompanied by thermo-dynamical, magnetic and electric or other phenomena as well at a certain level of the development the necessity to connect these phenomena arises naturally. Looking at this from a somewhat wider prospective we must rather say that in special cases these connections can be disregarded. From this point of view we must however talk about continuum-physics a specific specially defined part of which is continuum-mechanics.

It should be noted that the most common connection is that of the fields of deformation and temperature. This may have more profound reasons stemming from the nature of the material it may however also be simply due to the limits of our cognitive skills: motion and temperature can be experienced better than the electric or magnetic phenomena. The concept of cognition leads however to another viewpoint. If we aim at explaining the above phenomena sooner or later we will have to deal with the material properties that is whereas so far we have devoted all our attention to the nature of the macro- structure now we have to turn to the micro-structure of the materials. This has led us right through to another chapter of physics namely the solids state physics. The role of mathematics in physics is too obvious to be mentioned. And here an interesting leap may come: so far we have looked at how the examination of one physical phenomenon leads us through to the examination of another related phenomenon. Or what partial phenomenon should we devote our attention to in order to get a better understanding of the whole. Or what auxiliary tool – mathematics – is necessary for the examination of the phenomenon. Let us now have a look at how the potential granted by the means affects our understanding of the originally examined phenomenon. Let us, for example see the early time of the digital computers. At the outset we computed the tasks with calculators. Subsequently mechanized methods came into being which directed the attention to certain tasks. This development however was far from being a smooth one. Middle aged or older researchers may easily recall the times when a research topic with the title "solving mechanical tasks with digital calculators" seemed to make little sense. Today in turn the FEM (or the CAD, CAM) are research topics in their own right even among researchers of mechanics who are seeking the widest possible application.

As for the conservative approach, other examples may be mentioned as well of course. Not too long ago – if we measure time in decades – it seemed without a purpose to couple macro- and micro-structural investigations. Today the use of these is evident.

As continuum-mechanics is a strongly theoretical discipline, let us focus our attention on the practical side. Here we are faced with interdisciplinary fields once again on the one hand through the users and utilizers, on the other hand because of the necessity to provide experimental verifications. Obviously the effect is reciprocal here as well. Experience influences theory. Thus elasticity strength calculations and thermo-dynamics use and inspire the results of continuum-mechanics. On the other hand those conducting the experiments must be prepared to examine these phenomena taking into account the special demands. Here interdisciplinary questions arise due to the analogies, e.g.: heat conduction, diffusion, phenomenon described with the telegraph equation.

8 The role of mathematics

As mathematics has been mentioned in the previous chapters a number of times, it seems reasonable to devote a separate section to the discussion of its role. Let us however, reach a little further back in time. A few years ago the highly regarded and world famous scientist. Janos Selye made a rather curious statement. He argued that the physician does not need mathematics as the logic of mathematics is harmful to medical thinking. This We have ever since regarded as nonsense and have some ill feelings towards Janos Selye. Is it possible that he has retained his aversion towards mathematics from childhood all the way until becoming a world renowned scientist? Who knows?

Mathematics is a tricky issue. There is no other science which would divide so markedly into two groups those getting in touch with it. Those who need mathematics, having gotten to do with it, cannot imagine their work without it. Other people however, consider it completely superfluous and object to it. This division is for sure not coincidental, the explanation for it lies in the very nature of mathematics. It is an extremely abstract complex science which is better to avoid. On the other hand it is hardly possible to avoid, as it defines the most basic laws of the universe, thus one bumps into it in all areas. And this has been so for thousands of years! As Feynman put it very nicely, nature talks to us in the language of mathematics.

"The whole of modern science starting with Da Vinci and Copernicus was born under the auspices of Pythagoras, Plato and especially Archimedes. Pithagorism advocated that: everything can be derived from the number and the proportion: this was the first panmathematism. The Pythagorean Philolaios said that the nature of the number, similarly to harmony does not bear anything false, as that is not its own. Truth is inherent to the number. … Archimdes otherwise considered himself a Platonist and Pythagorean. Galilei talks about Archimedes with fascination: the superhuman Archimedes, whose name I can never utter without deep emotions [5]."

I feel that mathematics and languages are to some extent related. Those who have lived their whole life without ever speaking any other language than their mother tongue, will never have this sense of having missed something. Those, however, who start learning foreign languages, will sense a growing urge to learn further and further languages.

Let us now ponder a little bit about the relationship of mathematics with the engineering and natural sciences. The more basic relations can easily be described with words, the more complicated ones, however call for mathematical apparatus.

The constitutive law of linearly elastic, homogeneous, isotropic bodies, for example, has first been defined by Hooke in a very simple form. According to him stress and strain are in proportion, the proportion factor is characteristic of the material. If we think of the simple tension test this is correct. Introducing the notions of stress and strain, Hooke's law can be written in the following form:

$$\sigma = E \cdot \varepsilon \,,$$

where E is the proportion factor and can be obtained from the experiment. It is to be assumed, that Hooke summarized his revolutionary observation in words or in this simple formula first. The first problem presents itself if we take a closer look at the experiment, as the strain occurs transversely as well. Thus the transversal size is reduced. It may be started that the transversal size alteration is in proportion with the longitudinal one, and the proportion factor is characteristic of the material in this case as well. Based on the above:

$$\varepsilon_k = -\nu \cdot \varepsilon \tag{27}$$

where ν is the other proportion factor that can be established from the measurement. Let us now move on. Let us this time imagine a twisted rod instead of a tension one. In this case torsion moment appears instead of tension and naturally the phenomenon is twisting instead of tension. Introducing the specific quantities similarly to the previous case the

$$\tau = G \cdot \gamma \tag{28}$$

relation results, where G is another material characteristic which can also be obtained through experiments. Let us make yet another step forward. Instead of a rod let us now picture a three-dimensional body, and let us assume that we load this body with forces and moments working along all three axes. In this case a relationship between the tensions and deformations (strains and twists) cannot be described with words, only with an equation system of six unknown. Up to this point the mathematical skill acquired in high school will be sufficient. Knowing matrix calculation, however, this can well be condensed:

$$\underline{\underline{\varepsilon}} = \frac{1+\nu}{E}\left(\underline{\underline{\sigma}} - \frac{\nu}{1+\nu} \cdot \sigma_I \cdot \underline{\underline{I}} \right) \tag{29}$$

$$G = \frac{E}{2 \cdot (1+\nu)} \,. \tag{30}$$

This comprises everything but requires advanced mathematical skills. Let us now take another example: as Fourier put it in the law of heat conduction, heat flows from the warmer towards the colder spot and its intensity is proportionate with the difference in temperature:

$$q = -k \cdot \frac{T_2 - T_1}{\Delta x} \tag{31}$$

where k proportion factor is the coefficient of heat conduction. At this point several questions arise. One is how should Δx be: It is obvious that the notion of the dT/dx temperature gradient occurs immediately. The other problem is that the above is only true in the case of isotropic materials. In the case of anisotropic the phenomenon can only be described with the relation

$$\underline{h} = \underline{\underline{k}} \cdot grad\ T \,. \tag{32}$$

Here it becomes obvious that the simple picture described by Fourier does not hold true, h and grad T are not parallel and there are several heat conduction factors. This phenomenon cannot be either understood or described without the appropriate mathematical apparatus. The series of examples could be continued or extended. Describing certain phenomena in the language of mathematics may show that the phenomena are related. This leads us to analogies, which provide special experimental and computer technical opportunities e.g.: the analogy of mechanical and electrical vibration systems). Or through the examination of the mathematical apparatus (model) describing the phenomenon, we may actually reach the criticism of the original physical model. (E.g.: modified law of heat conduction [8]).

9 The "freedom" of research

A number of ideas have come up in the previous chapters which can be assigned to the group of questions the "freedom" of research. The reason why the word freedom is between quotation marks is that it is used in anusual sense here. It rather refers to the requirement that the thought must be independent. What we exactly mean here will be shown through examples. A few years ago the first author was once having a discussion with the reviewer of one of his papers to be published. The topic of our discussion was the method and result of the modification of Fourier's law of heat conduction. The reviewer, an acknowledged scientist in the field, asked for physical interpretation with regard to the member to appear in the modified law. The discussion was rather awkward, for how can a modification be explained with the notions to be modified. In other words the discussion of the issue slipped out of the field of physics and into the plane of cognition. If we accept Fourier's law of heat conduction then we cannot interpret the modified law introduced instead of the former one. In our thoughts we need to get detached from the notions we are already accustomed to. This is easier if the necessity of modification is well founded the sphere of application of the old law to be modified is clearly delineated.

Our other example is related to the thermal shock of long rod. Calculations have shown that sudden – shock-like – warming causes cooling in the rod. This might appear as nonsense at first hearing. Assuming that the calculations were correct, the explanation for the results needs to be and has been found. Deacades ago there were still researchers in Hungary, who questioned the necessity to modify the law of heat conduction. One of them became renowned professor in Canada. This researcher was a heat energetician, who obviously never or hardly ever encountered cases where Fourier's law of heat conductions was not good for the purpose. This may have been the main reason! Another reason may have been that this would have made it necessary to modify – at least in principle – the system elaborated by him (heat technical calculations). A third reason may be that the need – demand – to modify the law came up from the field of thermo-elasticity, that is mechanics, which is related to interdisciplinary problems. Finally a fourth reason has to do with the "freedom" of research, maybe in the classical sense. The person in question was a Hungarian scientist with excellent qualifications, thus he could not and did not let others – qualified or not – intrude into his field, as this is an existential question. The two – personal existence and freedom – are incompatible in the field of scientific research.

Based on the above it seems noteworthy to point out how and why the principle of the freedom of the researcher is violated:

- Due to erroneous technical (conceptual) reflexes, a good example of which is the physical interpretation demanded in connection with the modified law of heat conduction or the unacceptability of the temporary cooling at the thermal shock of the long rod.
- Due to incomplete technical and professional information, an example for which may be that the case mentioned has not come up in the experience of the researcher.
- The reason may also be simply laziness: it is more simple to deny a phenomenon than to take it into account maybe with a lot of work.
- Interdisciplinary conflicts may arise if somebody from a different field slips over to my area, which I am unwilling to accept.
- The reason may finally be a simple existential question as well. "I am the qualified researcher, leading professor, and nothing and nobody can question this."
- Or simple professional stinginess, an example for which could be the exclusion of computer science and of the micro-structural examination. We may, of course call this another type of interdisciplinary problem as well. Whereas in the previous case the researcher was not let into the interdisciplinary field, now he can refused to step out there.

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