Periodica Polytechnica
Mechanical Engineering

59(1), pp. 23-29, 2015
DOI: 10.3311/PPme.7540

RESEARCH ARTICLE

Modal Analysis of Cylindrical Gears with Arcuate Tooth Trace

Qinglin Chang[1 *], Li Hou[1], Bo Li[1], Fenglan Jia[1]

Received 29 May 2014; accepted after revision 08 September 2014

Abstract

In this paper, the forming principle, meshing features and tooth surface equation were introduced. And the modal parameters distribution of cylindrical gears with arcuate tooth trace was researched. The results show: 1. The modulus was the biggest impact factor for modal and natural frequency of cylindrical gears with arcuate tooth trace, then tooth width, and the radius of tooth line have the minimum influence; 2. When the modulus increased, natural frequency of cylindrical gears with arcuate tooth reduced rapidly; 3. When the tooth width increased, natural frequency of cylindrical gears with arcuate tooth has a tendency to rise except for first-order modal; 4. The influence of radius of tooth line can be basic ignored; 5. The second-order modal and third-order modal, fifth-order modal and sixth-order modal was very close. The research on cylindrical gears with arcuate tooth trace in this paper has a certain reference value on gear design and selection.

Keywords

Cylindrical gears with arcuate tooth trace, Finite method, Modal analysis, Matlab

[1] School of Manufacturing Science and Engineering, Sichuan University, No.24 South Section 1, Yihuan Road, 610065 Chengdu, China

[*] Corresponding author, e-mail: 1070776692@qq.com

1 Introduction

Gear drive is widely used in mechanical equipment and has the characteristic of compact structure. There are three form gear used regularly. But they all have some disadvantage because of the gear structure. So, the Japanese scholars, Kazuo Inoue successively have proposed finish machining method such as grinding teeth and burnishing teeth. Because of long contact line, high contact ratio, smooth transmission, high bearing capacity, good lubrication performance and a serial of advantages, cylindrical gears with arcuate tooth trace has been researched by many scholars all over the world [1-10]. Thus, there are many difficult with the processing and design, the cylindrical gears with arcuate tooth trace has not been widely applied [2-3].

Tseng has built mathematical model of arc gear, but not studied its bending characteristics with his partners and Wilcox et al have made different gears' stress analysis by finite method, but they don't specifically research that how the important parameters such as tooth line radius influence [4-7]. Professor Chen-min has made some breakthrough in analysis of arc gear forming principle, meshing performance and carrying capacity, but the modal was not analyzed [8-9]. Song Aiping from Yangzhou university china researched the meshing mechanism and characteristics on cylindrical gears with arcuate tooth trace, and put forward the device the four connecting rod translational processing mechanism [1, 10-13]. Shaojiang Wang and Huajun Xiao just derived mathematical equation of arc gear [14-15].

In this paper, how the modal distribution of cylindrical gears with curvilinear shaped teeth was researched. First, the 3-d model of cylindrical gears with curvilinear shaped teeth was established in UG NX8.0. Then, ANSYS14.0 Workbench was used to calculate 1-6 order modal of the gear under different parameters. At last, the analyze result was deal by Matlab2010a through curve fitting. Based on the analysis data and fitting curve, find cylindrical gears with curvilinear shaped teeth modal (frequencies) distribution.

2 Surface Equation and Modal Analysis

2.1 Forming principle and geometric parameters

The cylindrical gears with arcuate tooth trace can be processed by parallel linkage processing device as shown in Fig. 1 [13]. Geometric parameters on ketch plane of cylindrical gears with arcuate tooth trace were processed by parallel linkage as shown in Fig. 2. As shown in Fig. 2, the ideal geometric parameters of cylindrical gears with arcuate tooth trace is: the tooth line radius must equal between convex tooth surface and concave tooth surface; then the circumferential tooth thickness, circumferential tooth width and pressure angle also must equal on reference circle [12].

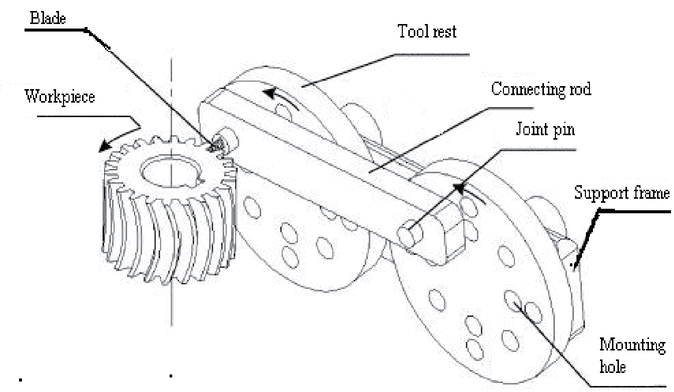


Fig 1 Parallel linkage processing device

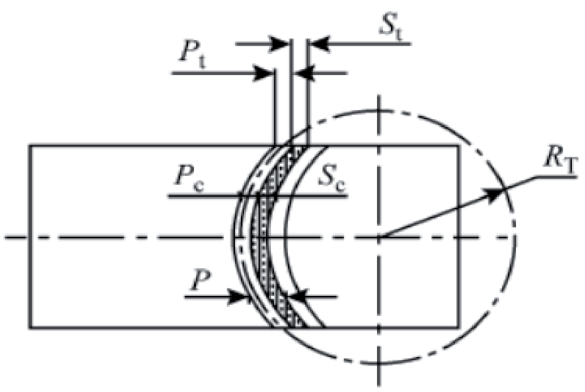


Fig. 2 The ideal geometric parameters of cylindrical gears with arcuate tooth trace

From Fig. 2: Because the radius of tooth line of convex tooth surface equal to concave tooth surface, a good wire mesh could achieve. The main geometric parameters of cylindrical gears with arcuate tooth trace were shown in Table 1.

Table 1 Main geometric parameters of cylindrical gears with arcuate tooth trace

Parameter name	Symbol	
Radius of tooth line	R_T	
Pressure angle on dividing circle	α	α =20°
Circumferential tooth thickness	s	$s=S_t=S_c=P/2$
Circumferential tooth width	e	$e=P_c=P_t=P/2$
Tooth width	B	

2.2 Tooth surface equation of cylindrical gears with arcuate tooth trace

As shown in Fig. 3, tooth flank ∑ is formed since involute tooth profile T_h of some radial cross-section scans along tooth line S of base cylinder. The coordinate systems $S_1(O_1\text{-}X_1Y_1Z_1)$ is established, the plane $X_1O_1Y_1$ through the middle cross-section of base cylinder and the Z-axis through the axis of base cylinder. $S_h(O_h\text{-}X_hY_hZ_h)$ is location coordinates systems of tooth line, and h is the distance from some point on the tooth line to the middle cross-section. R_b is the radius of base cylinder, and β is the position of arc tooth line angle. Section between plane $X_hO_hY_h$ and tooth surface ∑ is shown in Fig. 3 [1-3]. It's a involute. So the tooth surface equation could be represented by the following three steps.

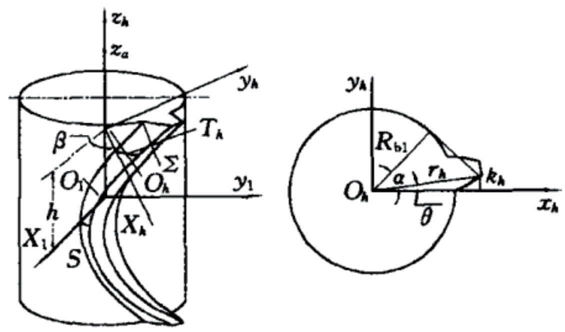


Fig. 3 Tooth profile of cylindrical gears with arcuate tooth trace

Step1: Within the coordinate plane $X_hO_hY_h$, the involute equation of convex tooth surface was shown as:

$$\begin{cases} \vec{r}_h = x_h i_h + y_h i_h \\ x_h = R_b \cos\alpha_h + \alpha_h R_b \sin\alpha_h \\ y_h = R_b \sin\alpha_h - \alpha_h R_b \cos\alpha_h \end{cases} \tag{1}$$

Step2: Transform the involute equation in coordinate $S_h(O_h\text{-}X_hY_hZ_h)$ to $S_1(O_1\text{-}X_1Y_1Z_1)$, tooth surface equation of convex tooth surface could be obtained:

$$\vec{r}_1 = M_{1h}\vec{r}_h \tag{2}$$

and,

$$[x_1, y_1, z_1, 1]^T = M_{1h}[x_h, y_h, z_h, 1]^T \tag{3}$$

The transformation matrix M_{1h} between coordinate S_h and S_1 is:

$$M_{1h} = \begin{bmatrix} \cos\beta & -\sin\beta & 0 & 0 \\ \sin\beta & \cos\beta & 0 & 0 \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$$\beta = (R_T - \sqrt{R_T^{\,2} - h^2}) / R_1 \tag{5}$$

Where:

R_1 —— the radius of reference circle;

b —— tooth width, $-b/2 \le h \le b/2$.

In the same way, tooth surface equation of concave tooth surface could get through transformation of coordinates.

2.3 Brief introduction to modal analysis

From the elastic mechanics, the differential equation of gear system is:

$$MX'' + CX' + KX = F(t) \tag{6}$$

Where:

X —— the displacement vector, $X = [x_1, x_2, \cdots x_n]^T$;

X' —— the velocity vector;

X'' —— the acceleration vector;

$F(t)$ —— the vibration force vector;

M —— the mass matrix;

C —— the damping matrix;

K —— the stiffness matrix;

If there is not vibration force ($F(t)$=0), it's a free vibration system. Because the damping force can be ignored at this time, so the vibration equation can be written as:

$$MX'' + CX' = 0 \tag{7}$$

The corresponding characteristic equation is:

$$(K - \omega_i^2 M)X = 0 \tag{8}$$

Where, ω_i is the natural frequency of the i-order modal for the system.

Vibration system generally has n individual natural frequency and main vibration mode. Each pair of frequency and vibration model represents a free vibration of single freedom system. The basic vibration characteristic of free vibration structure is called the modal of the structure. To multiple-degree-of-freedom system, free vibration can be decomposed into n harmonic vibration of single degree of freedom. It means that multi degree of freedom system in general is not a natural frequency of free vibration, instead of doing multiple harmonic vibration of composite natural frequency vibration [16-19].

3 Modal Analysis of Cylindrical gears with arcuate tooth trace

To research the modal of cylindrical gears with curvilinear shaped teeth with arcuate tooth trace and its impact factor, principle of single variable was used. The influence between modal and tooth line radius was researched at first, the tooth width was the next, and the modulus was the last one. In the process of the finite element analysis, the three-dimensional model of gears was always established in the environment of UG8.0, then the three-dimensional model will be import to ANSYS14.0 Workbench for finite element analysis. In addition to the three research factor, the other parameters are selected as shown in Table 2.

Table 2 Parameter table of cylindrical gears with arcuate tooth trace

Gear parameter	Parameter values
Number of teeth z	25
Pressure angle α(°)	20
Modification coefficient x	0
Tip clearance coefficient c^*	0.25
Addendum coefficient h_a^*	1
Diameter of axle /mm	45
Modulus of elasticity E/GPa	200
Poisson's ratio v	0.3

3.1 The modal influence of cylindrical gears with arcuate tooth trace by tooth line radius

To research on the relationship between modal of cylindrical gears with arcuate tooth trace and tooth line radius, take the module of gear $m = 4$, tooth width $B = 46mm$ and the tooth line radius as shown in Table 3.

Table 3 Different tooth line radius of cylindrical gears with arcuate tooth trace

No.	1	2	3	4	5	6	7	8	9
Tooth line radius (mm)	89	114.3	127	152.4	190.5	228.6	304.8	406.4	457.2

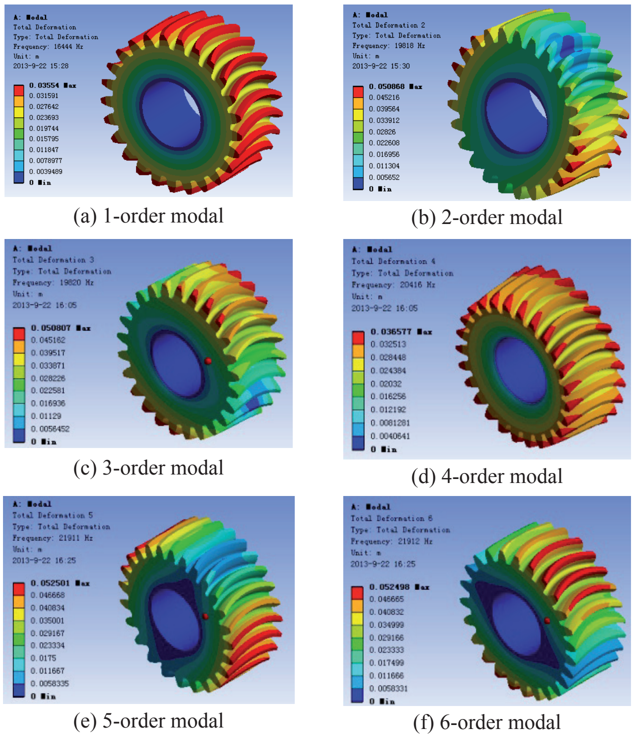
The 1-order modal to 6-order modal is shown in Fig. 4 where R=89mm. Gear modal distribute with tooth line radius is just shown in Table 4.

Table 4 Different modal distribution with tooth line radius

Tooth line (mm)	Modal frequency(Hz)					
	1-order	2-order	3-order	4-order	5-order	6-order
89	16444	19818	19820	20416	21911	21912
114.3	16442	19829	19830	20432	21906	21907
127	16439	19824	19827	20429	21910	21910
152.4	16436	19839	19841	20449	21902	21902
190.5	16437	19830	19831	20437	21906	21907
228.6	16436	19842	19845	20455	21900	21901
304.8	16435	19843	19844	20455	21900	21901
406.4	16436	19843	19843	20433	21903	21905
457.2	16437	19833	19834	20443	21906	21907

3.2 The modal influence of cylindrical gears with arcuate tooth trace by tooth width

To research on the relationship between the modal of cylindrical gears with arcuate tooth trace and tooth width, take the module of gear m = 4, radius of tooth line R_T=127mm and the tooth width as shown in Table 5.



(a) 1-order modal (b) 2-order modal (c) 3-order modal (d) 4-order modal (e) 5-order modal (f) 6-order modal

Fig 4 The modal distribution where *R*=89mm

Table 5 Different tooth width of cylindrical gears with arcuate tooth trace

No.	1	2	3	4	5	6	7	8
Tooth width (mm)	24	30	36	43	49	55	61	73

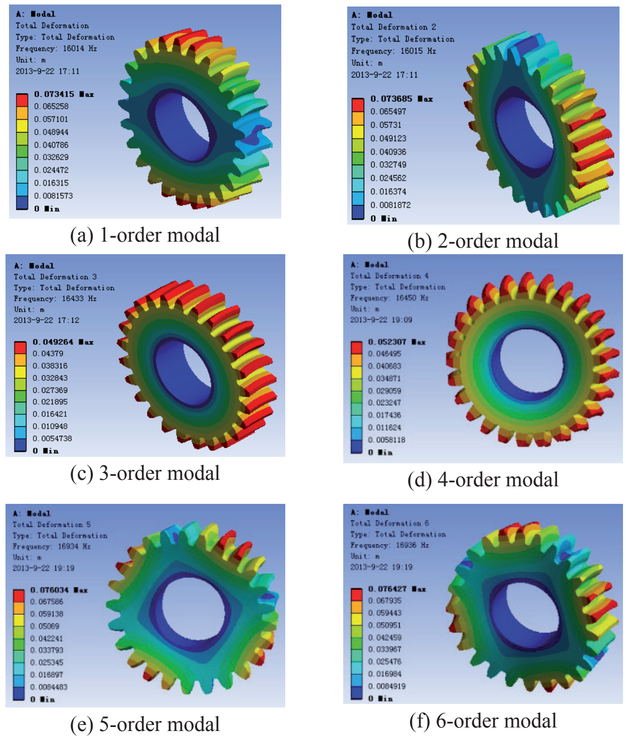
The 1-order modal to 6-order modal is shown in Fig. 5 where B=24mm. Gear modal distribute with tooth width is just shown in Table 6.

Table 6 Different modal distribution with tooth width

Tooth width (mm)	Modal frequency(Hz)					
	1-order	2-order	3-order	4-order	5-order	6-order
24	16014	16015	16433	16450	16934	16936
30	16435	17555	17560	18075	18835	18838
36	16437	18643	18643	19208	20318	20319
43	16438	19531	19533	20127	21673	21674
49	16439	20088	20089	20702	21908	21909
55	16442	20500	20502	21123	21905	21906
61	16443	20810	20813	21437	21906	21907
73	16443	21237	21238	21837	21919	21921

3.3 The modal influence of cylindrical gears with arcuate tooth trace by modulus

To research on the relationship between the modal of cylindrical gears with arcuate tooth trace and the modulus, take the tooth width of gear B = 46mm, radius of tooth line R_T=127mm and the modulus as shown in Table 7.



(a) 1-order modal (b) 2-order modal (c) 3-order modal (d) 4-order modal (e) 5-order modal (f) 6-order modal

Fig. 5 The modal distribution where B=2mm

Table 7 Different modulus of cylindrical gears with arcuate tooth trace

No.	1	2	3	4	5	6	7
Modulus (mm)	2.5	3	4	5	6	8	10

The 1-order modal to 6-order modal is shown in Fig. 6, where m=6. Gear modal distribute with modulus is just shown in Table 8.

Table 8 Different modal distribution with modulus

Modulus (mm)	Modal frequency(Hz)					
	1-order	2-order	3-order	4-order	5-order	6-order
2.5	49146	52489	52509	53860	55330	55347
3	35993	39692	39695	41540	41543	42169
4	16437	18643	18643	19208	20318	20319
5	9587.8	11673	11674	12368	13687	13687
6	6336.6	7645.6	7646.1	8357.5	9395.7	9396.0
8	3394.7	3947.8	3948.4	4571.1	5292.8	5292.9
10	2123.5	2372.2	2372.6	2876.9	3417.4	3417.5

4 Data Analysis and Discussion

Curve fitting method was taken to deal with the data in Table 4, Table 6 and Table 8. Fitting result is shown in Fig.7, Fig. 8 and Fig. 9. Fig. 7 use linear fitting, then Fig. 8 and Fig. 9 use 6 times polynomial fitting. In the figure, the 1-order modal was expressed by circle, the 2-order modal was expressed by cross symbols, the 3-order modal was expressed by fork symbols, the 4-order modal was expressed by prismatic, the

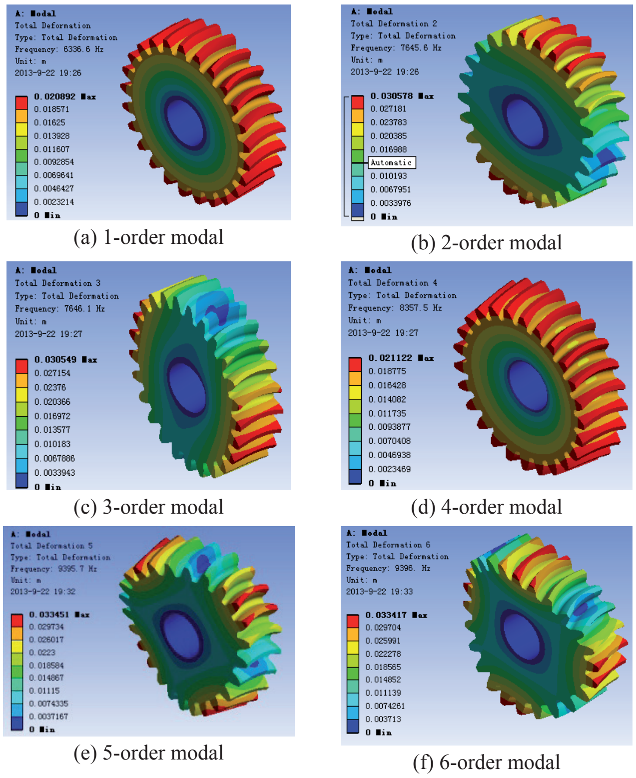

(a) 1-order modal
(b) 2-order modal
(c) 3-order modal
(d) 4-order modal
(e) 5-order modal
(f) 6-order modal

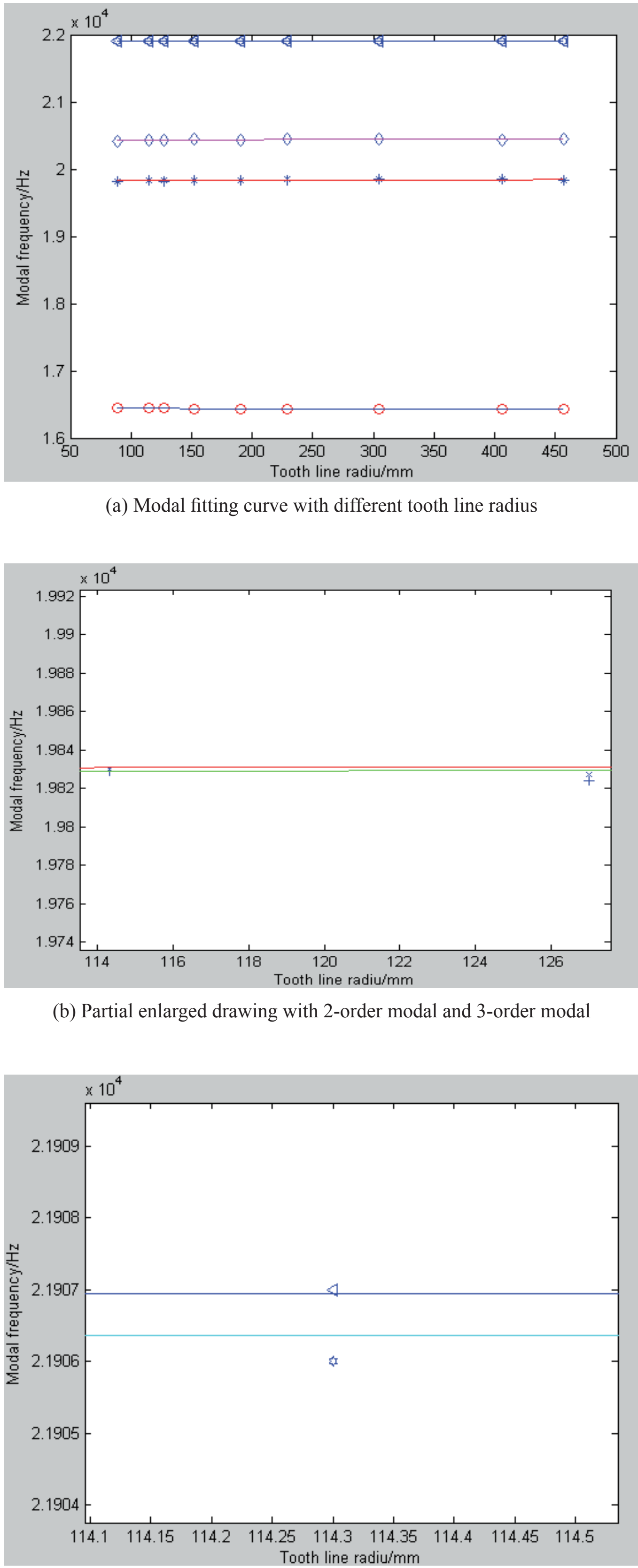
Fig. 6 The modal distribution where m=6mm

5-order modal was expressed by hexagram, and the 6-order modal was expressed by left triangle.

Figure 7(a) shows that the natural frequency and modal parameters have a few changes with different tooth line radius, tooth line radius have limited influence on modal of cylindrical gears with arcuate tooth trace. From the Fig. 7(b) (c), we can find: when tooth line radius changes, there are very approximate between 2-order modal and 3-order modal, also 5-order modal and 6-order modal

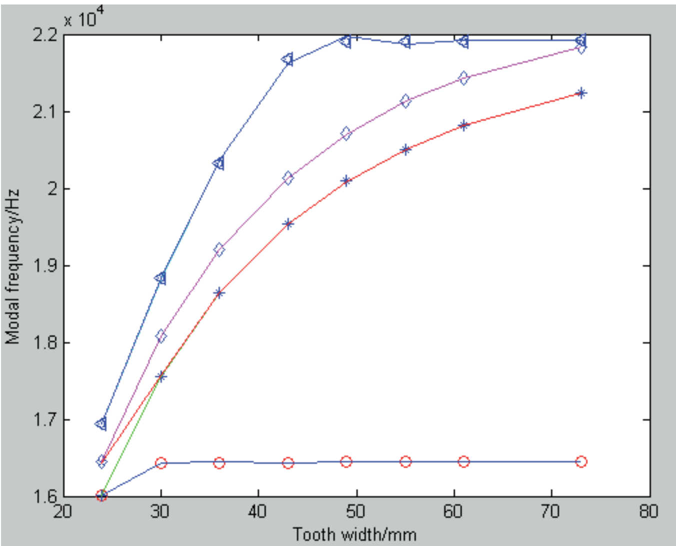
From the Fig. 8(a): 1-6 order modal of gears with arcuate tooth trace have a trend to larger with the tooth width increase. Tooth width has a big influence on 1-order modal when B<30mm. While the tooth width B>30mm, the tooth width has little influence on 1-order modal. The tooth width always has an obvious influence on 2-6 order modal. When B>45mm, the 5-order modal and 6-order modal no longer increases basic. From the Fig. 8(b)-(c), we also find: when tooth line radius changes, there are very approximate between 2-order modal and 3-order modal, also 5-order modal and 6-order modal.

From the Fig. 9(a): 1-6 order modal of gears with arcuate tooth trace have a trend to decline with the modulus increase. The modal falls faster where m<4, then the downward trend slow down while 4<m<8. The modal and natural frequency tends to stable while m>10. From the Fig. 9(b)-(c), we also find: when tooth line radius changes, there are very approximate between 2-order modal and 3-order modal, also 5-order modal and 6-order modal.

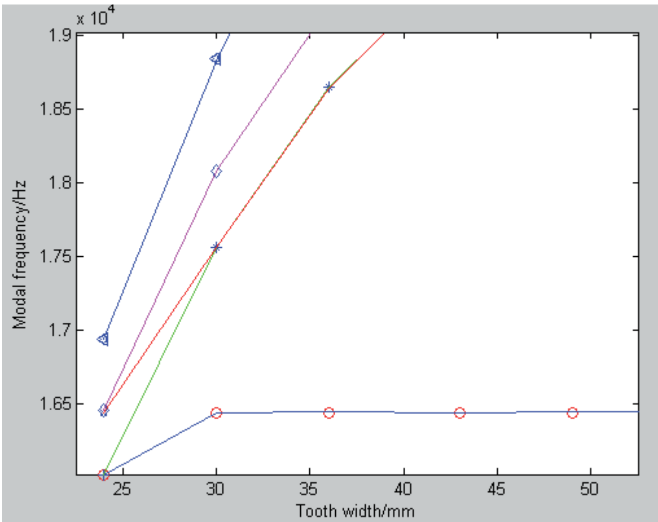


(a) Modal fitting curve with different tooth line radius
(b) Partial enlarged drawing with 2-order modal and 3-order modal
(c) Partial enlarged drawing with 5-order modal and 6-order modal

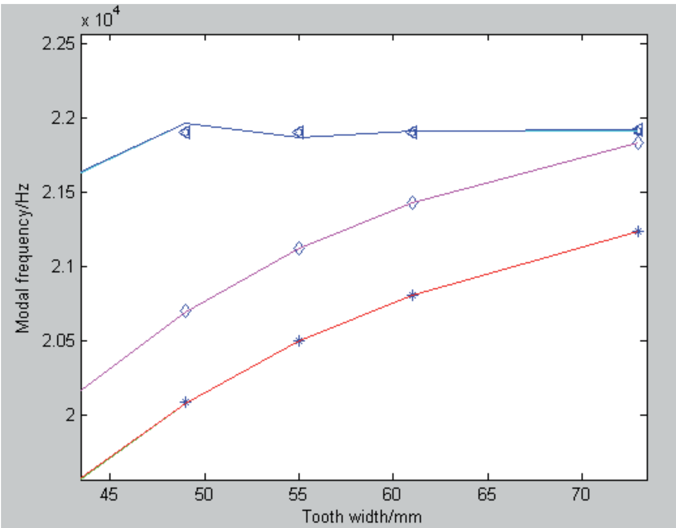
Fig. 7 The modal distribution figure with different tooth line radius



(a) Modal fitting curve with different tooth width

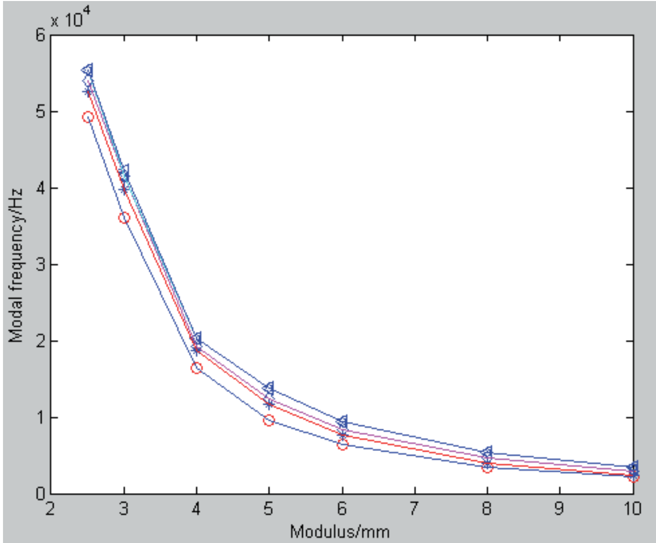


(b) Partial enlarged drawing with 2-order modal and 3-order modal

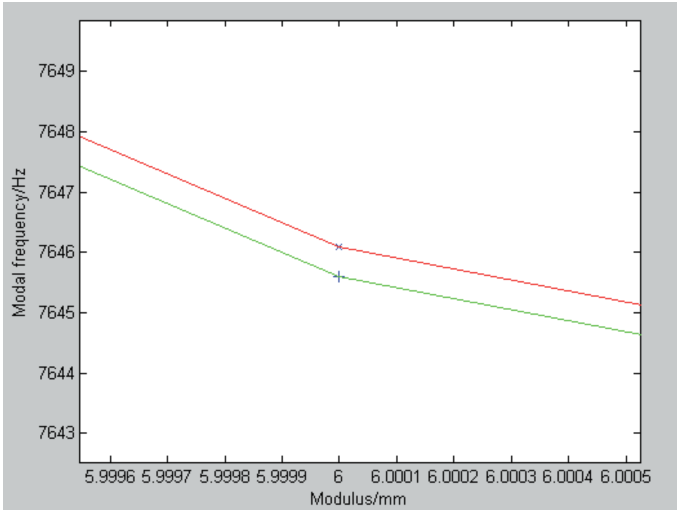


(c) Partial enlarged drawing with 5-order modal and 6-order modal

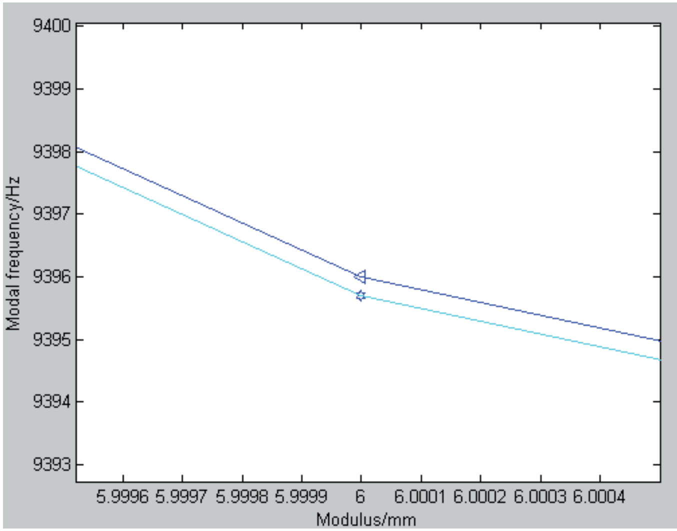
Fig. 8 The modal distribution figure with different tooth width



(a) Modal fitting curve with different modulus



(b) Partial enlarged drawing with 2-order modal and 3-order modal



(c) Partial enlarged drawing with 5-order modal and 6-order modal

Fig. 9 The modal distribution figure with different modulus

5 Conclusions

(1) Processing forming method, tooth surface equation and the modal analysis of cylindrical gears with arcuate tooth trace was basiclly introduced. The parameters of ideal cylindrical gears with arcuate tooth trace were the radius of tooth line must equal between convex tooth surface and concave tooth surface; then the circumferential tooth thickness, circumferential tooth width and pressure angle also must equal on reference circle.

(2) The 1-6 order modal was analyzed by finite element method for cylindrical gears with arcuate tooth trace with different parameters. And the Matlab2010a was used to deal the modal analysis data.

(3) Analysis shows that: The modulus was the biggest impact factors for modal and natural frequency of cylindrical gears with arcuate tooth trace, then tooth width, and the tooth line radius have the minimum influence. When the modulus increased, natural frequency of cylindrical gears with arcuate tooth reduced rapidly. When the tooth width increased, natural frequency of cylindrical gears with arcuate tooth has a tendency to rise except for first-order modal. The influence of radius of tooth line can be basic ignored. There are very approximate between 2-order modal and 3-order modal, also 5-order modal and 6-order modal.

Acknowledgements

This project is supported by National Natural Science Foundation of China (Grant No. 51375320).

The work was performed as part of the project “State Key Laboratory of Mechanical Transmission (Chongqing University) Open Foundation” (Code: SKLMT-KFKT-200901) and “Key Laboratory of Xihua University Open Foundation” (code szjj2011-041).

We also would like to thank to the reviewers for their encouraging comments and constructive suggestions to improve the manuscript.

References

[1] Aiping, S., Hong, Y., Wencheng, T., Zhonghua, N., Luyang, L. "Involute arc cylindrical gear and its mesh characteristics." *China Mechanical Engineering*. 9 (18). pp. 1888-1892. 2006.

[2] Wang, Z. "*Gearing mechanism and strength analysis of cylindrical gears with curvilinear shaped teeth.*" Yangzhou University. 2009.

[3] Wu, W. "*Research on process method and device for involute cylindrical gears with curvilinear shaped.*" Yangzhou University. 2010.

[4] Tseng R. T., Tsay, C. B. "Mathematical model and undercutting of cylindrical gears with curvilinear shaped teeth." *Mechanism and Machine Theory*. 36 (11-12). pp. 1189-1202. 2001. DOI: 10.1016/s0094-114x(01)00049-0

[5] Tseng, R. T., Tsay, C. B. "Mathematical model and surface deviation of cylindrical gears with curvilinear shaped teeth cut by a hob cutter." *ASME Journal of Mechanical Design*. 127 (5). pp. 982-987. 2005. DOI: 10.1115/1.1876437

[6] Wilcox, L., Coleman, W. "Application of finite elements to the analysis of gear tooth stress." *ASME Journal of Engineering for Industry*. 95 (4). pp. 1139-1148. 1973. DOI: 10.1115/1.3438262

[7] Oda, S., Nagamura, K., Aoki, K. "Stress analysis of rim spur gears by finite element method." *Bulletin of the Japanese Society of Mechanical Engineers*. 24. pp. 1273-1280. 1981. DOI: 10.1299/jsme1958.24.1273

[8] Di, Y., Hong, X., Chen, M. "Generation principle of arcuate tooth trace cylindrical gear." *Journal of Harbin Bearing*. 27 (3). pp. 58-61. 2006.

[9] Di, Y., Chen, M. "The generation principle and mesh characteristic of arcuate tooth trace cylindrical gear." *Mechanical Engineer*. 9. pp. 50-52. 2006.

[10] Wang, Z., Song, A., Chen, T., Wang, S. "Contact stress analysis of the involute arc cylindrical gear pair." *Machinery Design & Manufacture*. 12. pp. 188-191. 2008.

[11] Wu, W., Song, A., Wang, Z. "Research on the tooth root stress of the involute arc cylindrical gear." *Machinery Design & Manufacture*. 11. pp. 227-229. 2009.

[12] Song, A., Wu, W., Gao, S., Gao, W. "The Ideal Geometry Parameters of Arch Cylindrical Gear and Its Process Method." *Journal of Shanghai Jiaotong Uniwersity*. 44 (12). pp. 1735-1740. 2010.

[13] Song, A. "*Process method and device for cylindrical gears with curvilinear shaped*: China, ZL200410041297.2. 2008.

[14] Wang, S., Hou, L., Dong, L., Xiao, H. "Modeling and Strength Analysis of Cylindrical Gears with Curvilinear Shape Teeth for Manufacture." *Journal Of Sichuan University*. 44 (2). pp. 171-175. 2012.

[15] Xiao, H., Hou, L., Dong, L., Jiang, Y., Wei, Y. "Mathematical Modeling of Rotary Cutter Arc Tooth Line of Cylindrical Gear Shaped by Origin Face of Rotary Cutter." *Journal Of Sichuan University*. 45 (3). pp. 171-175. 2013.

[16] Chen, Q-S. "Finite Element Modal Analysis on Spur Gear." *Journal of Engineering Graphics*. 6. pp. 187-190. 2011.

[17] Wang, S., Wang, J., Cao, S., Zhang, C., Liu, J., Zhu, L. "Modal properties for bevel planetary gear trains." *Journal of Vibration Engineering*. 24 (4). pp. 376-384. 2011.

[18] Ma, H., Zhu, L., Wang, Q., Zhang, Y. "Modal Coupling Characteristic Analysis of a Helical Gear Rotor System With Parallel Shafts." *Proceedings of the CSEE*. 32 (29). pp. 131-136. 2012.

[19] Wang, Y., Sun, Z., Yang, Q., Du, Y. "Study on the Gear Mode and Resonance Reliability Sensitivity Based on Thermal Analysis." *Journal of Northeastern University*. 34 (3). pp. 408-412. 2013.