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RESEARCH ARTICLE

Analytical Study of Buckling of Hybrid Multilayer Plates

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Abstract
Thermal buckling behavior of hybrid multilayer plates was investigated in this paper. Different theories of plates taking into account or neglect the transverse shear are presented and the obtained results by these theories are compared. Nonlinear higher-order strain-displacement relations were considered. Using the principle of potential energy, the critical buckling temperatures are determined. Finally, a parametric study of the influence of various parameters such as: aspect ratios: b/a and a/h, thickness of metal, fiber angle and stacking sequence on the critical buckling temperature is shown and discussed. Numerical results indicate that the addition of metal to a composite material and the consideration of the transverse shear deformation have a significant effect on the thermal buckling behavior of simply supported hybrid multilayer plates.

Keywords
Thermal buckling, hybrid multilayer plates, transverse shear, critical buckling temperatures

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1 Introduction

Composite materials are attracting growing interest from many industry sectors, and their use is becoming widespread. This enthusiasm can be explained by their geometric structure designed to impart properties that their elementary constituents do not have individually, and allow them to fulfill many technical functions and achieve new levels of performance. The coupling between composites and metals gives rise to a range of so-called high-tech materials that can fulfill several tasks than traditional composites can't do include bolted joints. These materials can be tailored to suit a wide variety of applications by varying the fiber/resin system, the alloy type and thickness, stacking sequence, fiber orientation, surface pretreatment technique, etc. They are used specially for advanced aerospace structural.

Much research has been conducted to better understand the mechanical responses of these materials to applied loads [1-4]. However, little has been reported in the open literature on the composite plates buckling due to temperature changes. Biswa [5] solved thermal problems involving buckling of orthotropic plates. Similar problems for anti-symmetric and symmetric plates were resolved by Tauchert and Huang [6]. A more general formulation was proposed by Chen and Chen [7].

Many analytical studies have been performed on analyzing the behavior of functionally graded hybrid plates under different types of loading, such as: [8-11].

In this paper, we are interested by the deformation of these plates due at change of temperature. The equilibrium equations are obtained and used to calculate the thermal elastic buckling critical loads for a rectangular thin plate. To derive these equations, nonlinear relationships deformation-displacement was used. To introduce the transverse shear, U and V displacement components are approximated with expressions of high order.

2 Analysis

2.1 Displacement functions

The components of the displacement in the x, y and z directions can be written as follows:

$$
\begin{aligned}
U(x,y,z) &= u_0 - z\frac{\partial W}{\partial x} + f(z)\phi_x(x,y)\\
V(x,y,z) &= v_0 - z\frac{\partial W}{\partial y} + f(z)\phi_y(x,y)\\
W(x,y,z) &= w_0
\end{aligned} \tag{1}
$$

where u_0, v_0 and w_0 denote the displacements corresponding to the mean plane in the directions *x, y* and *z*. The function *f(z)* is given by the distribution of shear stress across the thickness of the plate. φ_x and φ_y are the rotations around the normal to the mid plane along the directions *x* and *y*, respectively.

2.2 Deformation-Displacement Relations

For this analysis, a rectangular plate with a constant thickness *h* was considered. Relationships Deformation- displacement are written:

$$
\varepsilon_{ij} = \delta\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} + \frac{\partial U_3}{\partial x_i}\frac{\partial U_3}{\partial x_j}\right) \tag{2}
$$

$\delta=1$ for $i\neq j$, $\delta=1/2$ for $i=j$

Where ε_{ii} are the normal strains and ε_{ij} are the shear deformations.

2.3 Stress-Strain Relations

The relation: stress-strain for each layer taken separately can be written:

$$
\{\sigma\} = [Q](\{\varepsilon\} - \{\alpha\}\Delta T) \tag{3}
$$

Where Q_{ij} are the stiffness constants of each layer. The linear coefficients of thermal expansion are represented by α_{ij} and ΔT means the elevation of the temperature in the plate.

2.4 Resultant Force and Moment

The resultant forces and moments acting on a laminate can be obtained by integrating the stresses through the thickness of the plate, as presented below:

$$
(N,M,P,R) = \int \sigma\left(1.z.f(z),\frac{\partial f(z)}{\partial z}\right)dz \tag{4}
$$

N_{ij} represent respectively the resulting normal stress along *x* and along *y* and shear stresses in the plane *(x, y)*. Similarly, R_{ij} are the resultant shear. Components M_x and M_y are the bending moments in the *x* and *y* directions, respectively; and M_{xy} component is the torsion moment. P_{ij} represents the efforts of a high order.

2.5 Equilibrium Equations

Using the principle of potential energy, the equilibrium equations are written as follows:

$$
\begin{aligned}
&\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0\\
&\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0\\
&\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + \frac{\partial\left(\frac{\partial w}{\partial x}N_x\right)}{\partial x} +\\
&\frac{\partial\left(\frac{\partial w}{\partial y}N_{xy}\right)}{\partial x} + \frac{\partial\left(\frac{\partial w}{\partial x}N_{xy}\right)}{\partial y} + \frac{\partial\left(\frac{\partial w}{\partial y}N_y\right)}{\partial y} = 0\\
&\frac{\partial P_x}{\partial x} + \frac{\partial P_{xy}}{\partial y} - R_{xz} = 0\\
&\frac{\partial P_{xy}}{\partial x} + \frac{\partial P_y}{\partial y} - R_{yz} = 0
\end{aligned} \tag{5}
$$

3 Case Study

We consider a multilayer composite plate with length *a*, width *b* and thickness *h*.

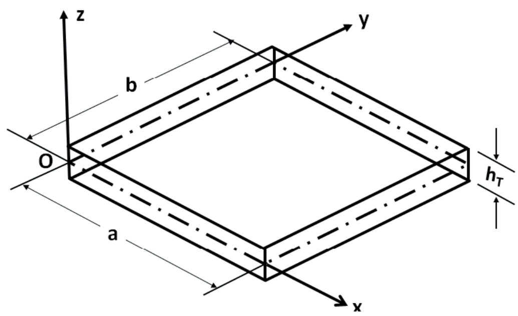


Fig. 1 Coordinates and geometry of a plate.

The function *f(z)* is related to the variation of the shear stress through the thickness. In what follows we will vary *f(z)* according to three different theories:

CPT [12]: $f(z) = 0$
FSDT [13] and [14]: $f(z) = z$
HSDT [15]: $f(z) = \sin\frac{\pi z}{h} e^{\frac{1}{2}\cos\frac{\pi z}{h}} + \frac{1}{2}\frac{\pi z}{h}$

The plate is assumed simply supported and displacements in the directions x and y are prevented.

By following the Navier solution procedure, the solutions to the problem are assumed to take the following forms:

$$
w = w_{mn}\sin\alpha_m x \sin\beta_n y \tag{6}
$$

Where: $\alpha_m = \frac{m\pi}{a}$ and $\beta_n = \frac{n\pi}{b}$

We considered that the temperature is uniform and by introducing (6) into (5), the equilibrium system can be written in the following form:

$$
[K_{ij}](u_{mn}, v_{mn}, w_{mn}, \phi_{xmn}, \phi_{ymn})^T = 0 \tag{7}
$$

Where $[K_{ij}]$ is a symmetric matrix

The critical value of ΔT is found by calculating the determinant $|K_{ij}|$ and it is:

$$\Delta T = K_{33} - \frac{\delta}{(A_{11}\alpha_1 + A_{12}\alpha_2)\alpha_m + (A_{12}\alpha_1 + A_{22}\alpha_2)\beta_n} \quad (8)$$

Where: $\delta = \frac{K_{34}(K_{34}K_{55} - K_{35}K_{45}) - K_{35}(K_{34}K_{45} - K_{44}K_{35})}{K_{44}K_{55} - K_{45}^2}$

4 Numerical Results

In what follows, we will present numerical results for the case of a multilayer hybrid plate: It is composed of thin layers of aluminum (0.3 to 0.5 mm) alternating with unidirectional layers of Glass/Epoxy (Fig. 2); we indicate the influence of transverse shear, aspect ratio and the number of layers on the buckling temperature.

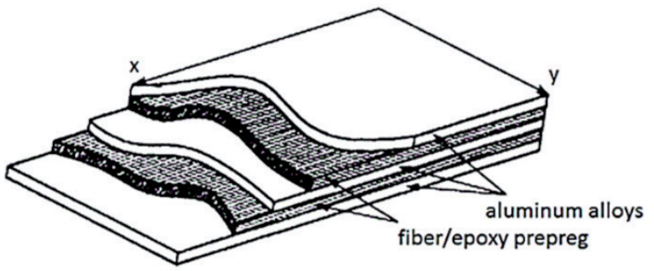


Fig. 2 Configuration of Glare

Table 1 Material Properties

	Epoxy/Glass	Aluminum
E_1 (GPa)	55	69
E_2 (GPa)	15	69
ν_{12}	0.26	0.33
$\alpha_1(10^{-6}\ K^{-1})$	11	23.6
$\alpha_2(10^{-6}\ K^{-1})$	15	23.6

In the second table, we calculate the critical buckling temperature for each material taken separately using the high-order deformation theory using the higher order deformation theory, with: $a/h_T=50,\ b/a=1,\ h_{Al}/h_T=0.15$($h_T$ is the thickness of the plate and h_{Al} is the thickness of Aluminum layer):

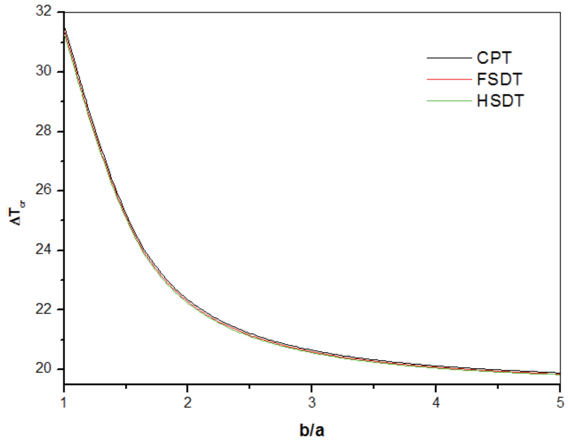
Table 2 Critical Temperture For Each Material

	Al	Epoxy/Glass	Glare
ΔT_{cr}	20.9152	30.5792	31.2481

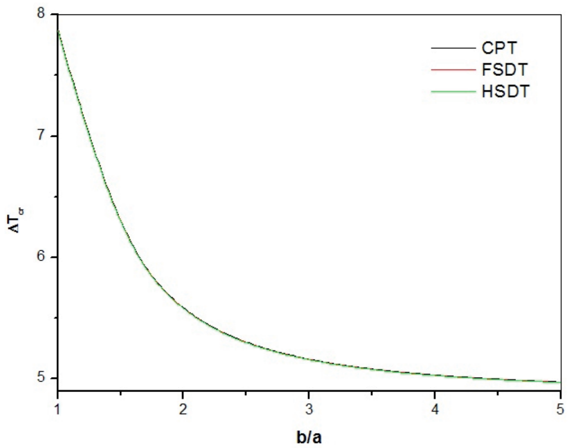
We can see that the Glare presents the highest critical temperature.

Next, we consider that we have [*Al/0°/Al/90°/Al*], the plate is rectangular with $h_{Al}/h_T=0.15$.

Figure 3 shows the plot of the critical buckling temperature of a simply supported rectangular plate with respect to the ratio *b/a*. The graph shows that increasing the ratio b/a leads to the decrease of T_{cr} until stabilization (for $b/a > 3$).



$a/h_T=50$



$a/h_T=100$

Fig. 3 Critical temperature VS *b/a* ($h_{Al}/h_T=0.15$)

In Figure 4 we plotted the critical temperature versus the aspect ratio a/h, we can see that there is a inverse relation between this quantities, the critical temperature decrease when a/h increase.

We can explain that by: when the plate become very large and so thin, the possibility of buckling of this plate will be great even under a little effort (in our case: temperature):

The critical charge depends on the flexural rigidity of the plate *D*, plate dimensions and buckling mode. For the same flexural rigidity of the plate and for the same buckling mode ($m=n=1$), the critical temperature decrease when the plate dimensions (length and large) increase. And for the same dimensions and same buckling mode, the critical temperature decrease when the flexural rigidity decrease, and this last decrease when the thickness *h* decrease.

We observe that the three theories (CPT, FSDT and HSDT) give close results (Fig. 3 and Fig. 4).

In Figure 5, the critical temperature was plotted as a function of the report h_{Al}/h_T. We see that when the Aluminum thickness

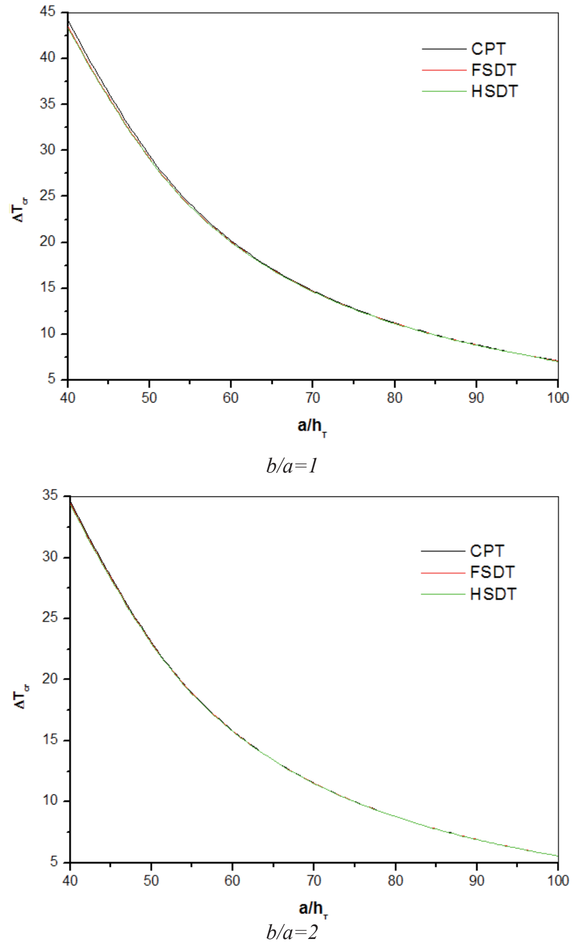


Fig. 4 Critical temperature VS a/h_T *(a=b, hAl/hT=0.15)*

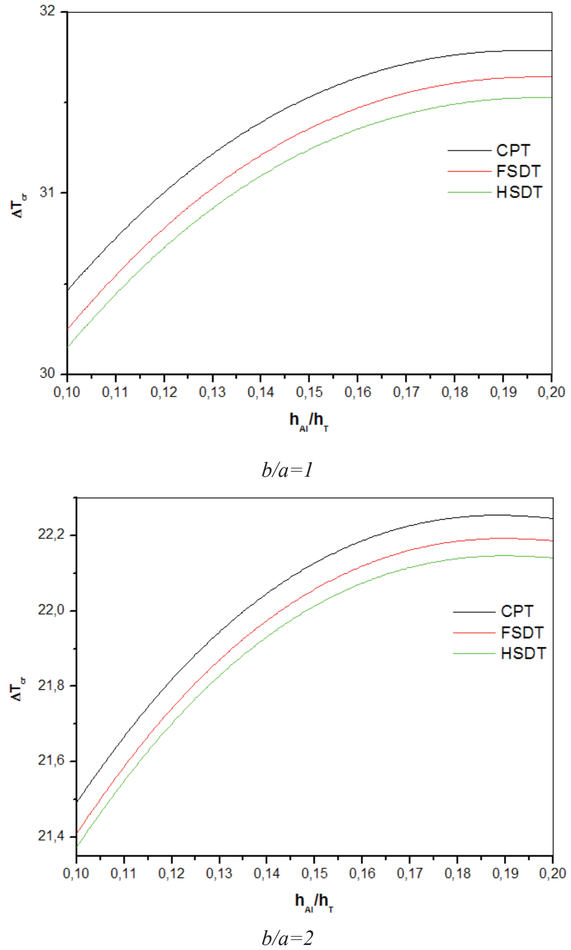


Fig. 5 Critical temperature VS h_{Al}/h_T $(a/h_T=50)$

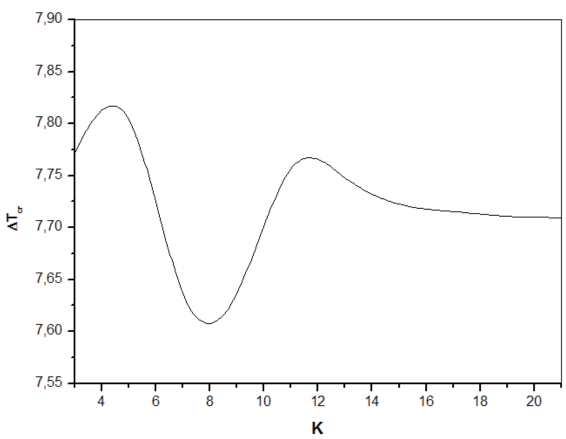


Fig. 6 Critical temperature VS K *(a=b, a/h_T=50, K= [Al/0°/Al/90°/Al/0°/...])*

increases, the critical temperature also increases. The results found by the three theories are distinct (the results given by the classical theory (CPT) are higher); this is due to the assumption of introducing or neglect the transverse shear.

Figure 6 shows the critical temperature T_{cr} of simply supported square plates for the first fundamental buckling mode with respect to number of layers K [*Al/0°/Al/90°/Al/0°/...*] using the high-order deformation theory. The material properties of the individual layers are given by Table 1. For small number of layers, the critical temperature is influenced largely by the number of layers and stacking sequences. However, for large number of layers, the critical temperature doesn't change and approaches a constant value. The feature of the anisotropy effects becoming stabilized as the number of layers increases can be noticed.

5 Conclusion

We made an analytical study on the behavior of buckling of hybrid multilayer plates. The study is based on three theories of the most commonly used by highlighting the similarities and differences points based on the assumption of introducing or neglect the transverse shear.

The study is made on rectangular plate by manipulating several parameters such as the dimensions, the number of layers and their orientation.

The introduction of the metal layers to the composite material leads to an increase in its resistance vis-a-vis the temperature.

The numerical results indicate that the deformation due to transverse shear has a significant effect on the thermal behavior of buckling of simply supported hybrid multilayer plates. The formulation lends itself particularly well in analyzing functionally graded materials in hybrid structure [16-18] which will be considered in the near future.

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