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RESEARCH ARTICLE

Determination of Stress Intensity Factor in Concrete Material Under Brazilian Disc and Three-Point Bending Tests Using Finite Element Method

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Abstract

Concrete is a brittle material with high compressive strength, low tensile strength and poor toughness, where cracks of different degrees and different forms can be developed, which compromises the durability and the lifetime of concrete structures.

The crack starts to propagate in concrete when the crack tip stress intensity factor reaches the fracture toughness value (critical stress intensity factor K_C or fracture energy G_C). Many studies were carried out on the fracture of concrete to measure K_C and G_C based on LEFM (Linear Elastic Fracture Mechanics).

This paper uses tow approaches to analyze the crack propagation of cracks three-point bending concrete beam and Brazilian disk with initial crack based on determination of stress intensity factor analytically starting from weight functions and numerically using the finite element software platform Abaqus.

Keywords

Propagation of the cracks, stress intensity factor, three-points bending, Brazilian disc, fractures mechanics, finite element method

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1 Introduction

Concrete as a modern building material is the most widely used. As concrete have the advantages with abundant availability, simple process, well fireproof, lower cost and wide adaptability [1], concrete has been widely used in our countries. However, concrete is a brittle material with high compressive strength, low tensile strength and poor toughness, it makes that cracks of different degrees and different forms occur in using process and even in construction process which makes the durability of concrete structure deteriorates and lifetime shortens. In modern times profuse experiments and engineering practices show that: cracking of concrete structure is almost inevitable; according to current economy and technique level, it is reasonable that concrete cracking could be controlled in the harm-allowable range [2]. Simulating concrete fracture behaviour is the main way to achieve aforesaid aims. A lot of researches on concrete fracture behaviour have developed many effectual concrete fracture models.

The cracking strength of concrete is determined by using fracture mechanics concepts [3, 4]. It is believed that the crack starts to propagate in concrete as the crack tip stress intensity factor reaches the fracture toughness value (that is always defined quantitatively in terms of critical stress intensity factor Kc or fracture energy Gc). A large number, of research efforts, has been made in studying the fracture toughness of normal strength concrete, with various mixing ratios of ingredients [5-7]. At first, the fracture mechanics started developing and was introduced to concrete beams to measure fracture toughness [8]. Later, many studies were carried out [9, 10] on the fracture of cement mortar and concrete to measure Kc and Gc. It is mentioned that LEFM (Linear Elastic Fracture Mechanics) could be applied to these systems. Also certain authors are given away from their studies [11] that fracture mechanics could be usefully applied for the failure investigation of concrete dams. Meanwhile many studies were carried out [12] on fracture toughness of self-compacting concrete. Recent publications [13, 14] have shown that fracture mechanics has now been established as a fundamental approach that can explain certain nonlinear aspects of concrete behaviour, help to prevent brittle failures of the structure and be an important aid in materials engineering.

This paper uses tow approaches to analyze the crack propagation of cracks three-point bending concrete beam and Brazilian disk with initial crack based on determination of stress intensity factor analytically starting from weight functions [15] and numerically using the finite element software platform Abaqus, and compares with the features in simulating soften behaviour for various concrete fracture models.

2 Linear elastic fracture mechanic:

Application of the fracture mechanics to concrete [16] has been intensively studied. It was recently realized that the fracture process zone was created ahead of the crack in concrete. As a result, the non-linear fracture mechanics [17] was introduced instead of the linear elastic fracture mechanics (LEFM). Although most efforts have been lately devoted to study on fracture damage behaviour of concrete [18, 19], linear elastic fracture mechanics seems to be still useful for studying the failure of concrete.

The theory of linear-elastic fracture mechanics (LEFM) is integrated using an analytical approach occurring that solid bodies containing cracks can be characterized by defining a state of stress near a crack tip and the energy balance coupled with fracture. Introducing the Westergaard's and Airy's complex function and will allow the development a significant stress analysis at the crack tip (Fig. 1).

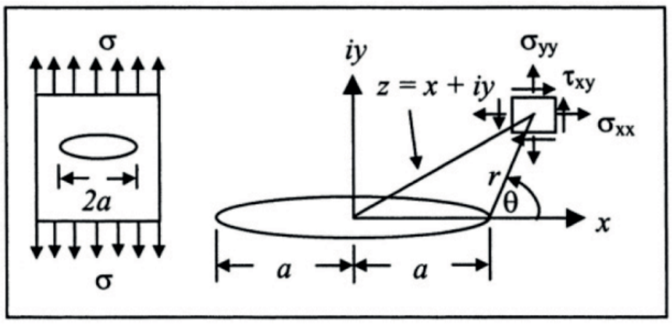

Fig. 1 The crack tip stress field in complex coordinates

The Airy's complex function and Westergaard's complex function are, respectively

$$\Phi = \mathrm{Re}\,\overline{Z}' + y\,\mathrm{Im}\,\overline{Z} \tag{1}$$

$$Z(z) = \mathrm{Re}\,Z + i\,\mathrm{Im}\,Z \tag{2}$$

Where

$\mathrm{Re}\,\overline{Z}'$ = Real part

$\mathrm{Im}\,\overline{Z}$ = Imaginary part

Z = analytic stress function

$i = \sqrt{-1}$

$i^2 = -1$

For instance, Irwin treated the singular stress field by introducing a quantity [20] known as the stress intensity factor, which is used as the controlling parameter for evaluating the critical state of a crack.

3 Stress intensity factor

The mechanical behaviour of a solid containing a crack of a specific geometry and size can be predicted by evaluating the stress intensity factors (K_I, K_{II}, and K_{III}) shown in Fig. 2.

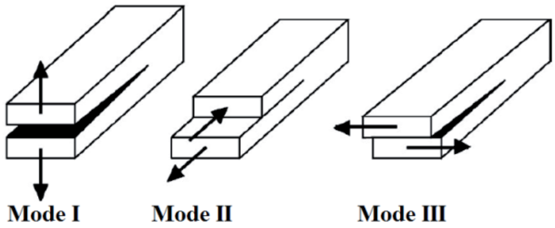

Fig. 2 Three basic modes of fracture propagation

If crack growth occurs along the crack plane perpendicular to the direction of the applied external loading mode, then the stress intensity factors are defined according to the American Society for Testing Materials (ASTM) E399 Standard Test Method as

$$K_I = \lim_{r\to 0}\left(\sigma_{yy}+\sqrt{2\pi r}\right) f_I(\theta) \tag{3}$$

$$K_{II} = \lim_{r\to 0}\left(\tau_{xy}+\sqrt{2\pi r}\right) f_{II}(\theta) \tag{4}$$

$$K_{III} = \lim_{r\to 0}\left(\tau_{yz}+\sqrt{2\pi r}\right) f_{III}(\theta) \tag{5}$$

Here, $f_I(\theta)$, $f_{II}(\theta)$ and $f_{III}(\theta)$ are trigonometric functions to be derived analytically.

4 Application

Determination of stress intensity factors is a critical task in fracture mechanics, numerical methods such as the finite element and boundary element methods [21] which have successful applications in various fields of engineering problems. It is shown that it is applicable to the description of not only different size specimens [22, 23], but also specimens with varying geometry.

4.1 Models studied

The numerical model used in our study, combines a criterion of the linear elastic fracture mechanics and a distributed model of cracking that leads to taking account the singularity of stress field at the peak of a crack and the influence of geometrical parameters on the development of cracking. This model must also keep on certain basic assumptions:

- The not fissured concrete is regarded as an isotropic homogeneous material with an elastic linear behaviour describing its mechanics.
- The micro crack zones and the nonlinear behaviour of the material at the point of the cracks are neglected.

4.2 Analytical and numerical analyses

It is a question of determining for the load patterns considered in this work, the values of the stress intensity factor K_I and K_{II} and which are expressed [24] starting from the following relation:

$$K_I = \sigma \cdot F \cdot \sqrt{\pi a} \tag{6}$$

$$K_{II} = \tau \cdot F \cdot \sqrt{\pi a} \tag{7}$$

Where:
a is the length or half-length of the crack,
σ and **τ** are respectively the normal and shear stresses applied.
F is the geometrical Functions.

Two cases are studied:

- Three-points bending test: The geometrical data and the principle of the test are schematized in Fig. 3:

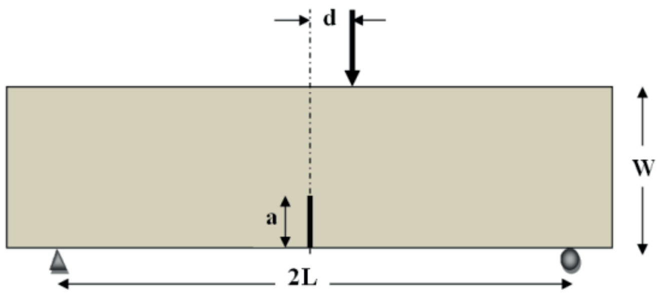

Fig. 3 Three-bending Beam Tests.

The specimen subjected to the bending test is a beam of concrete where its length is **2L** and width **w**, receiving a load **P** with an eccentricity **d**.

The beam is supposed to have an initial crack of length is **a**, located in the middle of the beam.

- Brazilian disc test: The geometrical data are shown in Fig. 4

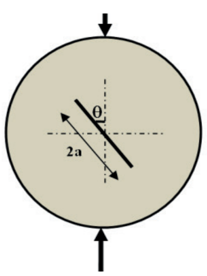

Fig. 4 Description of Brazilian disc.

The surface of the disc is supposed to be notched (half-length of the crack a). The crack forms an angle θ with the vertical axis of the disc.

In the numerical study, the values of stress intensity factors K_I and K_{II} are numerically obtained using finite element modelling in a two-dimensional medium by the ABAQUS software. Models used in the simulation are represented in Fig. 5 and 6.

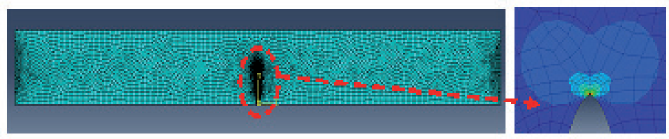

Fig. 5 Geometrical representation by ABAQUS of three points bending test .

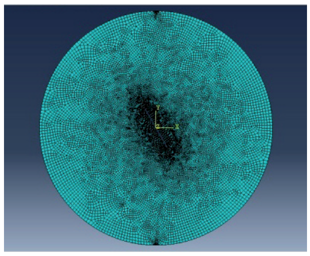

Fig. 6 Geometrical representation by ABAQUS of Brazilian disc test

The mechanical properties of material studied (concrete) are shown in Table 1.

Table 1 Summary of mechanical characteristics of concrete

E	Elastic modulus	35982 MPa
σ_C	Compressive strength	35 MPa
σ_T	Tensile strength	2.7 MPa
ν	Poisson's ratio	0.2

5 Discussion and results

Mode I:

- Three-points bending test:

Variation of the stress intensity factor K_I according to the length of the crack:

The graph above shows the variation of stress intensity factor K_I (compared numerically and analytically) according to the length of the crack; it is observed that the values of K_I increase when the report a/w increases.

Influence of the load's position on the stress intensity factor K_I:

Figure 8 represents the variation of the stress intensity factor K_I according to position of the load; it is noticed that the increase in the d/w leads to a reduction in the values of stress intensity factor K_I and when the d/w = 0 the values of K_I are maximum, which can be explained by the presence of high tensile stresses due to maximal bending moment.

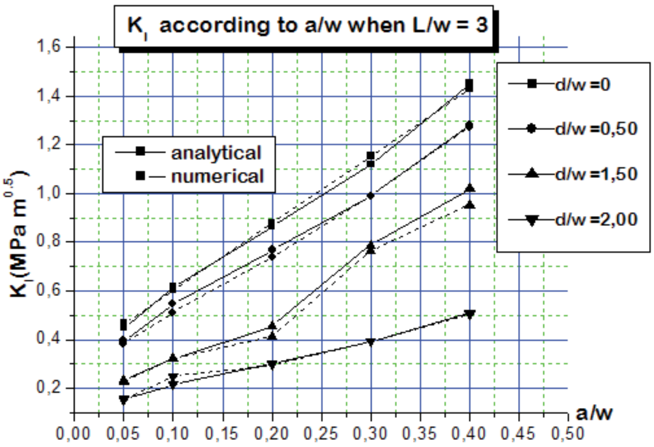

Fig. 7 Variation of the SIF K_I according to the report a/w

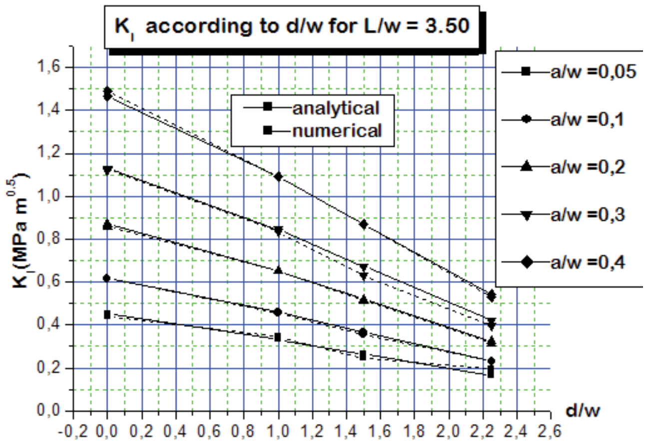

Fig. 8 Variation of the SIF K_I according to the load's position.

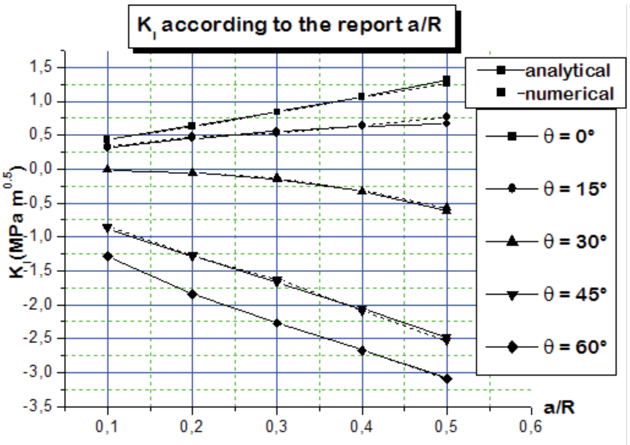

Fig. 9 Variation of the SIF K_I according to the report a/R.

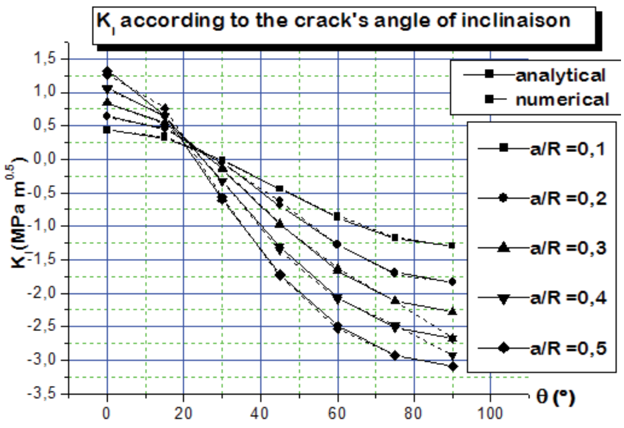

Fig. 10 Variation of the SIF K_I according to the angle of inclinaison of the crack.

- Brazilian disc test:

Influence of the crack's size on the stress intensity factor K_I:

Figure 9 represents the variation of the stress intensity factor K_I according to the report a/R (Brazilian disc). It is noted that for angles of inclination of the crack lower than 30°, the values of the stress intensity factor K_I grow when the report a/R grows. For values of the angles of inclination higher or equal to 30°, the values of the stress intensity factor K_I decrease and become negatives.

Influence angle of inclination of the crack on the stress intensity factor K_I:

Figure 10 represents the variation of the stress intensity factor according to the angle of inclination of the crack in the disc. It is noted that for values of the angle of inclination θ lower than 22°, the values of the stress intensity factor K_I are positives and become maximum for θ = 0. For values of the angle θ higher than 22°, the values of stress intensity factor K_I are negatives and are important in absolute values, in this case, the specimen is in compression and the crack remains closed.

Mode II

In three-points bending test, the beam is not subjected to shearing and the values of K_{II} are negligible, we will limit only with the case of Brazilian Disc test.

Influence of the crack's size on the stress intensity factor K_{II}:

Figure 11 represents the variation of the stress intensity factor K_{II} according to the size of the crack; we can note that values K_{II} vary proportionally with the report a/R, such as, when the length of the crack in the disc increases, the values of the stress intensity factor KII increase also what results in saying that there exist high shear stresses what facilitates the fracture in mode II and the propagation of the cracks is carried out quickly.

If the crack is not inclined or not formed at an angle of θ =0° or θ =90°, we can note that KII values are cancelled and the increase of the crack's size does not have any effect on the variation of stress intensity factor KII because of the absence of shear stresses in these two cases.

Influence angle of inclination of the crack on the stress intensity factor K_{II}:

Figure 12 represents the variation of the stress intensity factor K_{II} according to the angle of inclination of the crack in the disc; which we can observe that the values of the stress intensity factor K_{II} reach their maximum values when the angle of inclination θ of the crack is in the interval of 30° to 60° because shear stresses are highest in this zone. On the other hand if the angle of inclination θ tends towards 0° or tends towards 90° the values of the stress intensity factor K_{II} decrease until they will be cancelled.

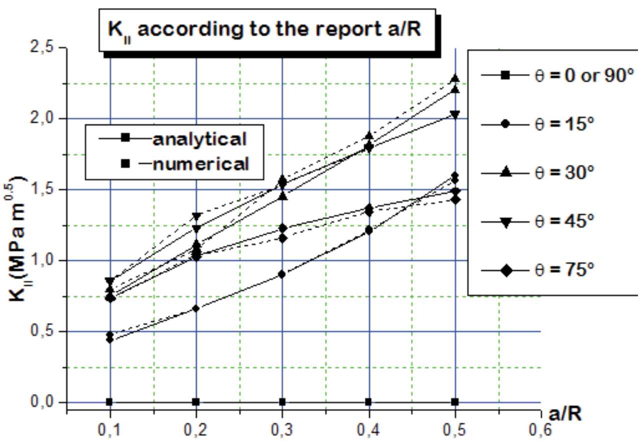

Fig. 11 Variation of the SIF K_{II} according to the report a/R.

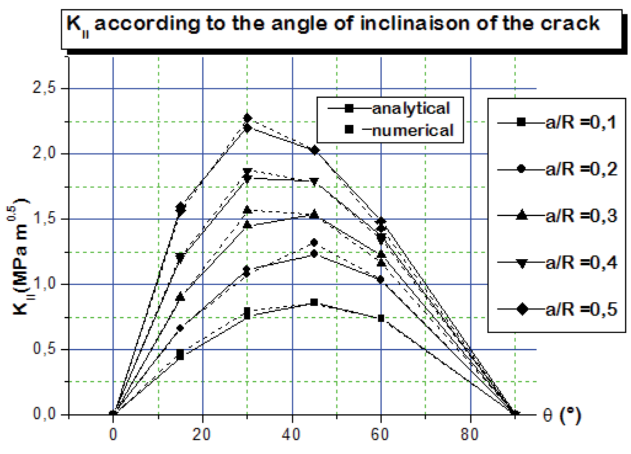

Fig. 12 Variation of the SIF K_I according to the angle of inclinaison of the crack.

6 Conclusion

Cracks propagation in concrete structures is a phenomenon which can be described by linear-elastic fracture mechanics concept starting from the criterion of the critical stress intensity factor. This method makes it possible to quantify the effects of the presence of a crack and influence of this one on the behaviour with the fracture of the structure.

In this study, we compared two methods to determinate the stress intensity factor that the results obtained are effectively comparables.

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