Periodica Polytechnica
Mechanical Engineering

60(3), pp. 137-141, 2016
DOI: 10.3311/PPme.8429

RESEARCH ARTICLE

Friction Coefficient Variation with Sliding Velocity in Copper with Copper Contact

Ali Anvari[1*]

Received 22 July 2015; accepted 25 February 2016

Abstract
In this paper, by applying analytical method and using experimental method data, the effect of sliding velocity on friction coefficient for contact of copper with copper is investigated. Some equations for obtaining friction coefficient as a function of sliding velocity for contact of copper with copper by applying second analytical method and using experimental method data are achieved. The results have shown, for each duration of rubbing, the existence of an exclusive equation for friction coefficient. The friction coefficient functions that are achieved by second analytical method are shown as a curve that indicates specific friction coefficient for each sliding velocity. Friction coefficient can be used to obtain friction force and stress in contact region of materials. In order to have a suitable design, the amounts of stress and friction force in contact region are required.

Keywords
friction coefficient function, sliding velocity, contact region, copper

[1] Department of Mechanical and Aerospace Engineering, University of Missouri-Columbia, Columbia, MO, U.S.A.

[*] Corresponding author, e-mail: alianvari330@yahoo.com

1 Introduction

It seems that friction is an undeniable phenomenon in contacting materials. So in this paper, by obtaining friction coefficient equations as a function of sliding velocity, the effect of sliding velocity on friction coefficient is investigated. Friction coefficient is used to obtain friction force and stress in contact region of materials. Friction force and stress are necessary to be considered for designing contacting materials. In order to obtain friction models, until now, many approaches are developed.

The detailed analysis of the modeled friction dynamics and its properties provided by Armstrong-Helouvry and Chen [1]. Al-Bender and Swevers mentioned several advanced, empirically motivated friction models widespread in the system and control community [2]. The novel two-state dynamic friction model with elasto-plasticity was presented by Runderman and Bertman and further evaluated on two different experimental systems [3, 4]. An enhanced friction modeling for steady-state rolling tires is submitted by Rene van der Steen [5]. Surface topography modeling for reduced friction is provided by Sedlacek et al. [6]. The modified Maxwell-slip model of presliding friction is presented by Runderman and Bertman [7]. The effect of normal load and sliding velocity on friction coefficient is submitted by Chowdhury et al. by applying experimental method [8]. Also, Numerical and experimental study of frictional behavior in bending under tension test is provided by Hirpa et al. [9]. Moreover, frictional contact FE analysis in a railway wheel-rail contact, is presented by Zwierczyk and Varadi [10].

Experimental and analytical methods for obtaining friction coefficient in contacting materials are available, but it seems developing new equations or models for friction coefficient as a function of sliding velocity for different materials are still required.

In this paper, by applying two kinds of analytical method and using experimental method data, the effect of sliding velocity on friction coefficient for contact of copper with copper is investigated. Some equations or models of friction coefficient as a function of sliding velocity at each duration of rubbing (DOR) are achieved. Finally, the results of both analytical methods are compared.

2 Experimental method

2.1 Experimental results

Table 1 indicates the results of experimental method data [8]. As it is shown in the Table 1, friction coefficient in contact of copper with copper at different sliding velocities and different duration of rubbings are shown. The numerical values of minimum and maximum friction coefficient for contact of copper with copper that are indicated in Table 1, are respectively, $\mu_s = 0$ and $\mu_m = 0.153$. These data are used in both analytical methods to obtain the numerical value of friction coefficient while the sliding velocity is changing [5]. In Table 1, for each DOR, three friction coefficients at three sliding velocities are indicated. It is obvious that for three distinct points, there is a possibility of existing one exclusive curve. So, for each three points (x = sliding velocity and y = friction coefficient), there is a special curve. In this paper, by using these three points for each DOR, one friction coefficient function is achieved. These functions are shown in Table 5.

Table 1 Friction coefficient at different sliding velocities by using experimental method data (normal load: 15 N, relative humidity: 70 %) [8].

Material pairs	Copper-copper		
Sliding velocity, (*m*/*s*)	1	2	3
Time, (min)	Friction coefficient		
0.0	0.000	0.000	0.000
0.5	0.090	0.112	0.137
1.0	0.095	0.118	0.142
2.0	0.100	0.125	0.149
3.0	0.105	0.131	0.153
4.0	0.110	0.134	0.153
5.0	0.113	0.134	0.153

2.2 Calculation

In this section, by applying Table 1, the following method is used to obtain the average friction coefficient at different DOR.

$$\mu_{Average} = \frac{(\mu_{\text{at 0 min DOR}}) + (\mu_{\text{at 0.5 min DOR}}) + (\mu_{at1 \text{ min DOR}}) + (\mu_{at2 \text{ min DOR}}) + (\mu_{at3 \text{ min DOR}}) + (\mu_{at4 \text{ min DOR}}) + (\mu_{at5 \text{ min DOR}})}{7}.$$

The results are shown in Table 2.

Table 2 Average friction coefficient at 0, 0.5, 1, 2, 3, 4, and 5 min durations of rubbing at different sliding velocities by using experimental method data (normal load: 15 *N*, relative humidity: 70 %).

Material pairs	Copper-copper		
Sliding velocity, (*m*/*s*)	1	2	3
	μ_{Average} Average friction coefficient at 0, 0.5, 1, 2, 3, 4, and 5 min duration of rubbing, (-)		
	0.0876	0.1077	0.1267

2.3 Experimental condition

Table 3 shows the experimental condition used to obtain the results in Table 1 [8].

Table 3 Experimental condition using a pin on disc apparatus [8].

No.	Parameters	operating Conditions
1.	Normal load	15 *N*
2.	Sliding velocity	1, 2, 3 *m*/*s*
3.	Relative humidity	70 (±5)%
4.	Duration of rubbing	10 minutes
5.	Surface condition	Dry
6.	Material pair (disc – pin)	Copper-copper

3 First analytical method

In this part of paper, the equations for obtaining the first friction coefficient model as a function of sliding velocity is introduced. Then, by using material properties and experimental method data related to copper, the mentioned equations are solved [5, 8]. In Figure 1, the model used to obtain the first analytical method is illustrated.

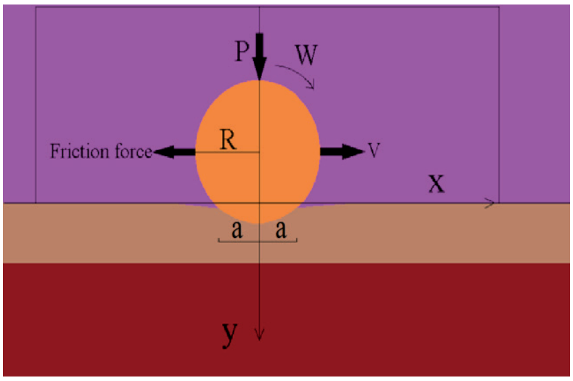

Fig. 1 A cylinder sliding on the substrate in contact region with radius, *R*, velocity in *x* direction, *V*, angular velocity, *W*, normal load, *P*, and half of the contact length, a.

In Eq. (1), μ_s is the friction coefficient, μ_m is the maximum friction coefficient, h_0 is a dimensionless parameter reflecting the length of speed region that friction coefficient is varying in it. v_s is the sliding velocity at μ_s, and $v_{s,\max}$ is the sliding velocity at μ_m [5].

$$\mu(v_S) = \mu_s + (\mu_m - \mu_s)\exp\left\{-h_0^2 \log^2\left(\frac{v_S}{v_{s,\max}}\right)\right\}. \quad (1)$$

By applying numerical values of mentioned unknown quantities in the Eq. (1), numerical value of friction coefficient as a function of sliding velocity can be obtained. It is noticeably;

sliding velocity is equal to zero in stick region that materials are stuck together, so the Eq. (1) changes to Eq. (2).

$$\mu(v_S)=\mu_s. \tag{2}$$

For obtaining Eq. (1), at first, obtaining v_s and $v_{s,\max}$ is necessary. In order to obtain v_s and $v_{s,\max}$, Eqs. (6) and (7) are used [11]. By substituting Eqs. (4) and (5) into the Eq. (3), the Eqs. (6) and (7) are obtained.

$$\frac{v_S}{V}=\zeta-\frac{\partial u_x}{\partial x}, \tag{3}$$

$$\frac{\partial u_x}{\partial x}=-2k\mu\frac{(k+1)(1-v)hx}{\left[1-(k+1)v\right]\mathrm{a}^2}. \tag{4}$$

In Eq. (3), V is the velocity in x direction. ζ is the creep ratio between contacting materials that is defined in Eq. (5) [12]. In Eqs. (3) and (4), $\partial u_x/\partial x$ is the strain in x direction. In Eq. (4), h is the indentation of cylinder into the substrate, v is the Poisson ratio, a, is half of the contact length in contact of cylinder with substrate, μ is the friction coefficient, and k is a parameter reflecting the material properties of the substrate. While $k \to 1$, the substrate is elastic and while $k \to 0$, the substrate is homogeneous [13].

$$\zeta=\frac{RW-V}{V}, \tag{5}$$

$$v_S=RW-V\left(1-2k\mu_s\frac{(k+1)(1-v)hx}{\left[1-(k+1)v\right]\mathrm{a}^2}\right), \tag{6}$$

$$v_{s,\max}=RW-V\left(1-2k\mu_m\frac{(k+1)(1-v)hx}{\left[1-(k+1)v\right]\mathrm{a}^2}\right). \tag{7}$$

In above equations, W is the angular velocity of the cylinder that is sliding on the substrate, and R is the cylinder radius. In the following equations, β, I_k, and J_k are the parameters reflecting the material properties that are used to obtain a, in Eq. (11) [13]. By substituting the Eqs. (6) and (7) sequentially instead of v_s and $v_{s,\max}$ into the Eq. (1), the Eq. (13) is obtained.

$$\beta=\beta(v,k)=\sqrt{(k+1)\left(1-\frac{kv}{1-v}\right)}, \tag{8}$$

$$I_k=\frac{\pi\Gamma(3+\kappa)}{2^{(k+2)}(k+2)\Gamma\left(\frac{3+k+\beta}{2}\right)\Gamma\left(\frac{3+k-\beta}{2}\right)}, \tag{9}$$

$$J_k=\frac{\beta}{(k+1)}I_k. \tag{10}$$

In Eq. (11), P is the normal load, Γ is a function that defines the substrate stiffness ratio, and E_0 is the substrate elastic modulus [11, 13].

$$\mathrm{a}^{(2+k)}=\frac{(1-v^2)2RP\beta(k+2)\sin(\beta\pi/2)}{E_0(1-k^2)\pi}\,\Gamma\left(\frac{3+k+\beta}{2}\right)\Gamma\left(\frac{3+k-\beta}{2}\right)\frac{\Gamma\left(\frac{3-k}{2}\right)}{\Gamma\left(\frac{3+k}{2}\right)}, \tag{11}$$

$$h=\frac{\mathrm{a}^2}{2Rk}, \tag{12}$$

$$\mu(v_S)=\mu_s+(\mu_m-\mu_s)\exp\left\{-h_0^2\log^2\left(\frac{RW-V\left(1-2k\mu_s\frac{(k+1)(1-v)hx}{\left[1-(k+1)v\right]\mathrm{a}^2}\right)}{RW-V\left(1-2k\mu_m\frac{(k+1)(1-v)hx}{\left[1-(k+1)v\right]\mathrm{a}^2}\right)}\right)\right\}. \tag{13}$$

By using the first analytical method and experimental method data (μ_s = 0 and μ_m = 0.153), with material properties of copper, the indicated results of variation of friction coefficient with sliding velocity are shown in Table 4 [5].

Table 4 The variation of the friction coefficient with sliding velocity by applying the first analytical method and by using experimental method data for copper material properties.
(P = 15 (*N*), relative humidity: 70 %, v = 0.355, E_0 = 1/17e11 (N/m^2), I_k = 0.3489, J_k = 0.3241, β = 1.0219, k = 0.1, h = 0.0053 (*m*), a = 0.0178 (*m*), Γ = 1, R = 0.3 (*m*), h_0 = 1).

μ_s, (-)	μ_m, (-)	V, (m/s)	W, (rad/s)	V_s, at middle of contact zone, (m/s)	$\mu(v_s)_{\text{average}}$, (-)
0	0.153	27.8	92.6	0.000	0.0000
0	0.153	27.7	92.6	0.078	0.0840
0	0.153	27.0	926	0.778	0.1385
0	0.153	26.0	92.6	1.778	0.1505
0	0.153	25.0	92.6	2.778	0.1521
0	0.153	24.0	92.6	3.778	0.1525
0	0.153	23.0	92.6	4.778	0.1527
0	0.153	22.0	92.6	5.778	0.1528
0	0.153	21.0	92.6	6.778	0.1529
0	0.153	20.0	92.6	7.778	0.1529
0	0.153	19.0	92.6	8.778	0.1529
0	0.153	18.0	92.6	9.778	0.1529
0	0.153	17.5	92.6	10.278	0.1530
0	0.153	17.0	92.6	10.778	0.1530
0	0.153	10.0	92.6	17.778	0.1530

4 Second analytical method

In this part of the presented paper, obtaining a friction coefficient function by applying second analytical method and using experimental method data in Table 1 at different sliding velocities (1, 2, and 3 *m*/*s*) and 0.5 min DOR for contact of copper with copper is explained.

4.1 Calculation

By applying Eq. (14), an equation for friction coefficient as a function of sliding velocity can be obtained.

$$\mu = b\left(v_s\right)^2 + cv_s + d. \tag{14}$$

In Eq. (14), *b*, *c*, and *d*, are the unknown quantities and v_s is the sliding velocity. By applying the data in Table 1, the numerical values of *b*, *c*, and *d* can be obtained. At 0.5 min DOR in Table 1, there are

$$\text{point no. 1,}\quad v_s = 1,\quad \mu\left(v_s\right) = 0.09,$$

$$\text{point no. 2,}\quad v_s = 2,\quad \mu\left(v_s\right) = 0.112,$$

$$\text{point no. 3,}\quad v_s = 3,\quad \mu\left(v_s\right) = 0.137.$$

As a result of the points number 1, 2, and 3 (three boundary conditions), Eqs. (15), (16), and (17) are obtained.

$$0.09 = b(1)^2 + c(1) + d, \tag{15}$$

$$0.112 = b(2)^2 + c(2) + d, \tag{16}$$

$$0.137 = b(3)^2 + c(3) + d, \tag{17}$$

In Eqs. (15), (16), and (17), *b*, *c*, and *d* are unknown quantities that are mentioned in Eq. (14). By solving the three Eqs. (15), (16), and (17), the numerical values of *b*, *c*, and *d*, are obtained. By substituting the numerical values of *b*, *c*, and *d* into the Eq. (14), the Eq. (18) is obtained.

$$\mu\left(v_s\right) = 0.0015\left(v_s\right)^2 + 0.0175\left(v_s\right) + 0.071. \tag{18}$$

Equation (18) is a friction coefficient function for 0.5 min DOR in contact of copper with copper. The above solution is an example of applying second analytical method. In Table 5, by applying the presented analytical solution and using data in Table 1, friction coefficient functions for 0.5, 1, 2, 3, 4, 5, and average minutes duration of rubbing are indicated.

5 Results and discussions

In Table 4, the results of friction coefficient for first analytical method by using material properties of copper are shown. As it is obvious in Table 4, with increasing the sliding velocity from zero to the numerical value of almost v_s = 3.7447 (*m*/*s*), the friction coefficient increases. For the numerical values more than v_s = 3.7447 (*m*/*s*), the friction coefficient increases in a very low amount and is almost constant. But in Table 5, that is achieved by using the experimental method data and the second analytical method, for the numerical values more than v_s = 3.7447 (*m*/*s*), the numerical value of friction coefficient is not constant.

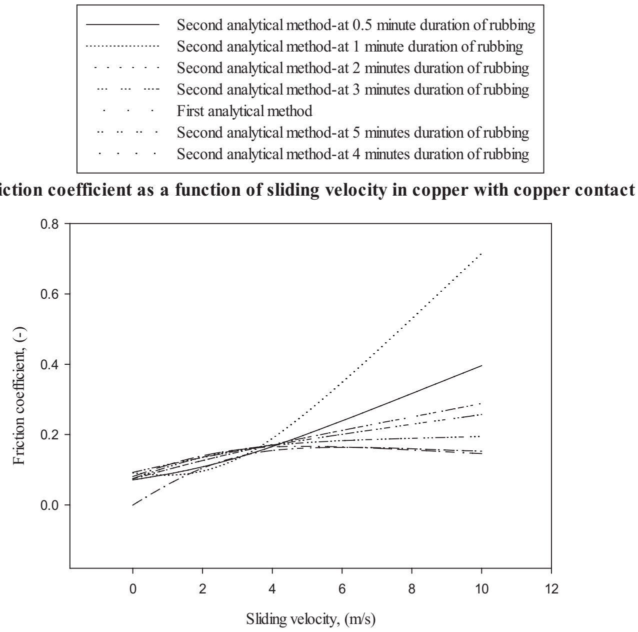

Fig. 2 The comparison of the friction coefficient variation with sliding velocity between first and second analytical method.

Table 5 Friction coefficient equations as a function of sliding velocity at different durations of rubbing by using experimental method data and applying the second analytical method (material pairs: copper-copper, normal load: 15 *N*, relative humidity: 70 %)

Eq no.	Material pairs	Copper-copper
(18)	Friction coefficient function at 0.5 min DOR	$0.0015v_s^2 + 0.0175v_s + 0.071$
(19)	Friction coefficient function at 1 min DOR	$0.0065v_s^2 - 0.0025v_s + 0.091$
(20)	Friction coefficient function at 2 min DOR	$-0.0005v_s^2 + 0.0265v_s + 0.074$
(21)	Friction coefficient function at 3 min DOR	$-0.002v_s^2 + 0.032v_s + 0.075$
(22)	Friction coefficient function at 4 min DOR	$-0.0025v_s^2 + 0.0315v_s + 0.081$
(23)	Friction coefficient function at 5 min DOR	$-0.0005v_s^2 + 0.0215v_s + 0.093$
(24)	Average friction coefficient function at 0, 0.5, 1, 2, 3, 4, and 5 min DOR	$-0.00055v_s^2 + 0.02175v_s + 0.0664$

Table 6 Comparison of friction coefficient variation with sliding velocity between first and second analytical method.

v_s, sliding velocity, (m/s)	$\mu_{average}$, Average friction coefficient obtained by first analytical method, (-)	$\mu_{Average}$, Average friction coefficient obtained by second analytical method, (-)
0.0000	0.0000	0.0664
0.0778	0.0840	0.0681
0.7778	0.1385	0.0830
1.7778	0.1501	0.1033
2.7778	0.1521	0.1226
3.7778	0.1525	0.1407

The results achieved by second analytical method have shown increase of friction coefficient with increasing the sliding velocity, but this increasing may change into decreasing in higher numerical values of sliding velocities and is not constant at all.

6 Conclusions

It is obvious that regarding to Table 4, while sliding velocity of contacting copper increases, friction coefficient as a function of sliding velocity tends to the numerical value of maximum friction coefficient (μ_m).

For each material pairs that are in contact, there is a friction coefficient as a function of sliding velocity. The results of friction coefficient equations have shown that functions are totally distinctive at different durations of rubbing. So, in the second analytical method, for different durations of rubbing of contact between specified materials, different functions of friction coefficient are required. It is expected that applying these results contribute to different concerned mechanical designs and processes. This fact is possible because friction coefficient can be used to obtain friction force and friction stress in contact region of materials.

For the future work, it is reasonable to apply more experiments to derive friction coefficients for other amounts of sliding velocities for different materials that are in contact. This is significant because it can contribute to find out that which method can express the best model for changing the friction coefficient with sliding velocity and to achieve a proper general equation to obtain the friction coefficient of different materials.

Acknowledgement

The author would like to appreciate the guidance of advisors. All the funding related to this research is provided by the presented author.

References

[1] Armstrong-Helouvry, B., Chen, Q. "The Z-properties chart." *IEEE Control Systems*. 28(5), pp. 79-89. 2008. DOI: 10.1109/mcs.2008.928939

[2] Al-Bender, F., Swevers, J. "Characterization of friction force dynamics." *IEEE Control Systems*. 28(6), pp. 64-81. 2008. DOI: 10.1109/mcs.2008.929279

[3] Runderman, M., Bertman, T. "Friction model for elasto-plasticity for advanced control applications." In: Proceeding of IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM2010), Montreal, Canada, July 6-9, 2010, pp. 914-919. DOI: 10.1109/aim.2010.5695765

[4] Runderman, M., Hoffman, F., Bertman, T. "Friction dynamics with elasto-plasticity in transient behavior." In: Proceeding of International Conference on Noise and Vibration Engineering (ISMA2010), Leuven, Belgium, 2010, pp. 1215-1258.

[5] Steen, R. V. D. "Enhanced friction modeling for steady-state rolling tires." Ph.D. thesis, Eindhoven University of Technology, Eindhoven, the Netherlands. 2010.

[6] Sedlacek, M., Vilhena, L., Podgornik, B., Vizintin, J. "Surface topography Modeling for reduced friction." *Strojniski vestnik – Journal of Mechanical Engineering*. 57(9), pp. 647-680. 2011. DOI: 10.5545/sv-jme.2010.140

[7] Runderman, M., Bertman, T. "Modified Maxwell-slip model of presliding friction." Proceedings of the 18th IFAC World Congress, Milano, Italy, 2011. pp. 10764-10769. DOI: 10.3182/20110828-6-it-1002.00309

[8] Chowdhury, M. A., Nuruzzaman, D. M., Mia, A. H., Rahaman, M. L. "Friction coefficient of different material pairs under different normal loads and sliding velocities." *Tribology in Industry*. 34(1), pp. 19-23. 2012.

[9] Hirpa, G., Lemu, T., Trzepiecinski, T. "Numerical and experimental study of Frictional behavior in bending under tension test." *Strojniski vestnik – Journal of Mechanical Engineering*. 59(1), pp. 41-49. 2013. DOI: 10.5545/sv-jme.2012.383

[10] Zwierczyk, P. T., Váradi, K. "Frictional contact FE analysis in railway wheel-rail contact." *Periodica Polytechnica Mechanical Engineering*. 58(2), pp. 93-99. 2014. DOI: 10.3311/PPme.7229

[11] Guler, M. A., Adibnazari, S., Alinia, Y. "Tractive Rolling Contact Mechanics of Graded Coatings." *International Journal of Solids and Structures*. 49(6), pp. 929-945. 2011. DOI: 10.1016/j.ijsolstr.2011.12.005

[12] Mazilu, T. "Some aspects about Driving wheel/rail contact in steady state interaction." In: 10th International Conference on Tribology, Bucharest, Romania, 2007. pp. 33-38.

[13] Giannakopoulos, A. E., Pallot, P. "Two-Dimensional Contact Analysis of Elastic Graded Materials." *Journal of the Mechanics and Physics of Solids*. 48(8), pp. 1597-1631. 2000. DOI: 10.1016/s0022-5096(99)00068-x