Periodica Polytechnica
Mechanical Engineering

60(1), pp. 58-65, 2016
DOI: 10.3311/PPme.8636

RESEARCH ARTICLE

Downhill Motion of the Skater-Skateboard System

Balázs Várszegi[1*], Dénes Takács[2]

Received 04 October 2015; accepted 03 December 2015

Abstract
The lateral instabilities of vehicles are well-known phenomena, see for example, the shimmy motions of bikes, motorbikes or the steered wheels of cars. Another interesting phenomenon is the snaking motion of the skateboard-skater system that is analyzed in this study. A mechanical model of the downhill skateboarding is constructed in order to consider the effect of the slope angle on the stability. The equations of motion are obtained with the help of the Gibbs-Appell method. The linear stability analysis of the rectilinear motion is carried out analytically using the Routh-Hurwitz criterion. The influence of different realistic parameters of the skater and the board are investigated. A critical position of the skater on the board also is determined.

Keywords
skateboard, Gibbs-Appell, stability, downhill

[1] Department of Applied Mechanics, Faculty of Mechanical Engineering, Budapest University of Technology and Economics
H-1111 Budapest, Műegyetem rkp. 3, Hungary

[2] MTA-BME Research Group on Dynamics of Machines and Vehicles, H-1111 Budapest, Műegyetem rkp. 3, Hungary

[*] Corresponding author, e-mail: varszegi@mm.bme.hu

1 Introduction

The history of skateboards goes back to the early 20's of the former century, when the first skateboard was constructed from wooden board and metal wheels. Although, the first commercially produced skateboard appeared in the 50's, the most important dates of the skateboard history relate to the 70's. On the one hand, polyurethane wheels were introduced, which allowed higher speeds than the metal ones. On the other hand, the first scientific papers [1] and [2] on skateboard dynamics also date back to that time.

The early publications of Hubbard on the skateboard explained the behaviour at low speed, namely, he proved that the special wheel suspensions of the board have a positive effect on the linear stability of the rectilinear motion. Thus, the skateboard became a very fascinating example of non-holonomic systems, and the investigation of the motion of the skateboard is a popular field among researchers even nowadays. Methods developed for non-holonomic systems can be tested and compared by means of the analyses of the skateboard dynamics. For example, in the paper of Ispolov and Smolnikov [3], the Gibbs – Appell method was applied in order to determine the equations of motion. Moreover, another interesting behaviour of the skateboard can be discovered by means of nonlinear analyses, see, for example, the studies of Kremnev and Kuleshov [4, 5] for the low speed motion.

The investigations of the high speed dynamics became relevant after the appearances of the first polyurethane wheels and the so-called longboards. In the recent publications (e.g.: [6] and [7]), researchers focused on the loss of stability at high speeds, where the effect of the human control system is also considered. Some of these studies have identified that the reflex delay of the skaters can have a key role although none of these studies operate with complex control scheme like McRuer's approach for drivers in [8]. The presence of the reflex delay also gives explanation for instabilities in case of many other applications, like the simple human balancing (Stepan [9]) or the balancing on a balance board (Chagdes et al. [10]). Since the control strategy of the human control in case of skateboarding is unknown, the analysis of the interaction between the skateboard dynamics and the human control can be an interesting research topic for future works.

In this study, the down hilling motion of the skateboard is in focus. In practice, longboards are usually used for such scenarios since this type of boards has better behaviour at high speed. The positive effect on the stability of the longer boards is also confirmed in theoretical studies e.g. [7]. But, there is no mechanical model for the case when the skateboard runs on a slope, however, the effect of the slope of the ground can be important in point of view of stability, too. Moreover, during downhill skateboarding, the slope of the road is non-negligible, for example, the world speed record was set on with 18% grade [11]. In the wider literature, one can find related articles, in which the downhill skateboarding is analysed from different viewpoints. For example, the aerodynamics was investigated by Hart et al. in [12].

In our study, we focus on the effect of the slope of the road while we use a simplified drag force model and we also neglect the human control. The standing position of the skater is one of the intricate parameter of our investigation.

2 Mechanical model

The mechanical model in question (see Fig. 1) is based on [2] and [4], where a similar model is investigated in case of the movement on a horizontal plane. Here, we investigate the motion of the skateboard on a slope, but we simplified the geometry (i.e. we neglect the mass of the board and the mass moments of inertia of the board and the skater) in order to have less parameters.

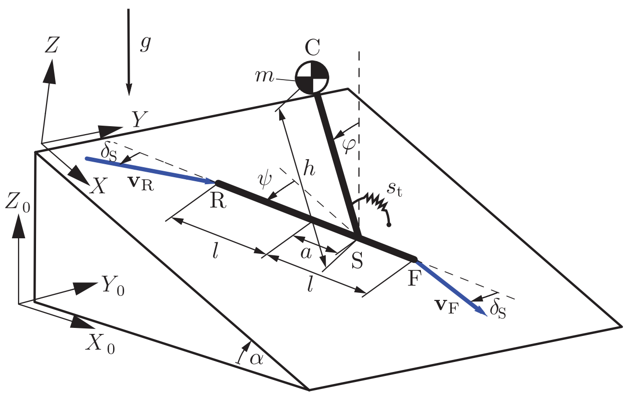

Fig. 1 Mechanical model of a skateboard

2.1 Description of the model

The model consists of two massless rods. One of them models the skateboard itself between the front (F) and rear (R) points. The other massless rod (between points S and C) with a lumped mass m at the end point C represents the skater. Hereby, the connection between the skater and the board is assumed to be rigid. The so formed rigid body has no mass moment of inertia with respect to its centre of gravity C, which makes the derivation of equations of motion simpler. Namely, the motion of a lumped mass has to be described only, although the skateboard moves in the three dimensional gravitational field on the slope that is characterized by the inclination angle α. For convenience, the coordinate system (X,Y,Z) attached to the slope is used for the description of the motion.

In this paper, we do not consider the loss of the contacts between the wheels and the ground. Due to the fact that the longitudinal axes of the board is always parallel to the plane of the slope, one can choose four generalized coordinates to describe the motion: x and y are the coordinates of the point S; ψ describes the direction of the longitudinal axes of the skateboard; and finally, φ is the inclination angle of the skater's body from the normal direction of the slope. Due to the rigid connection between the skater and skateboard, the deflection angle of the board is equal to φ.

The geometrical parameters of our model are the following: the height of the skater is $2h$; the length of the board is $2l$; parameter a characterizes the skater's location on the board ($a > 0$ means skater stands in front of the centre of the board); m represents the mass of the skater; while parameter g stands for the gravitational acceleration.

We consider that the free motion of the skater-board system is obstructed by a torsional spring, which models the stiffness s_t of the wheel suspension system. We also consider a drag force, which models the braking effect of the aerodynamic forces. The drag force formula can be given as

$$F_D = \frac{1}{2}\rho C_D A_D V^2, \tag{1}$$

where ρ is the density of the fluid, C_D is the dimensionless drag coefficient, A_D is the normal cross section area and V is the speed of the body relative to the fluid. Here, we use a simplified model, namely, we assume that the direction of the drag force is opposite to the longitudinal speed $\dot{x}\cos\psi + \dot{y}\sin\psi$ of the board, and instead of using the formula its magnitude is calculated by

$$F_D = \frac{1}{2}k\left(\dot{x}\cos\psi + \dot{y}\sin\psi\right)^2, \tag{2}$$

Where the coefficient k of the drag force is introduced to describe the combined effect of the parameters ρ, C_D and A_D. Moreover, we assume that the drag force acts at the centre of gravity C. A sophisticated aerodynamic analysis was presented in [12], where the geometry of the skater's helmet and his position were also the part of the numerical investigation of the flow around the skater. But our goal is different, we focus on the stability of the rectilinear motion, so the simple analytical expression of Eq. (2) is an appropriate choice.

2.2 Kinematic constraints

Regarding the rolling wheels of the skateboard, kinematic constraining equations can be given. Namely, the directions of wheel axes depend on the deflection angle φ through the steering angle δ_S (see Fig. 1). The connection between this steering angle and the deflection φ can be expressed:

$$\sin\varphi\tan\kappa = \tan\delta_S, \tag{3}$$

where κ is the complementary angle of the so-called rake angle in the skateboard wheel suspension (for the derivation of this relation please see [4] or [6]). Based on this, two scalar kinematic constraining equations can be constructed for the velocities $\mathbf{v}_F$ and $\mathbf{v}_R$ of points F and R:

$$\begin{aligned}&(\cos\psi\sin\varphi\tan\kappa-\sin\psi)\dot{x}\\&\quad+(\sin\psi\sin\varphi\tan\kappa+\cos\psi)\dot{y}+(l-a)\dot{\psi}=0,\\&(\cos\psi\sin\varphi\tan\kappa+\sin\psi)\dot{x}\\&\quad+(\sin\psi\sin\varphi\tan\kappa-\cos\psi)\dot{y}+(l+a)\dot{\psi}=0.\end{aligned}\tag{4}$$

These constraints are linear in terms of the generalized velocities, hence, the Gibbs – Appell method can be straightforwardly applied (see in [13]).

2.3 Equations of motion

In order to apply the Gibbs – Appell equations, pseudo velocities have to be chosen, by which the kinematic constraints can be eliminated. In our case, two pseudo velocities are required since the difference of the numbers of generalized coordinates and the kinematic constraints are two. These pseudo velocities can be produced as linear combinations of the generalized velocities. An appropriate choice can be the longitudinal speed of the board and the angular velocity of the skateboard around its longitudinal axis:

$$\sigma_1 = \dot{x}\cos\psi + \dot{y}\sin\psi \quad \text{and} \quad \sigma_2 = \dot{\varphi}. \tag{5}$$

Now, the generalized velocities can be expressed with the generalized coordinates and the pseudo velocities by using the kinematic constraints:

$$\begin{aligned}\dot{x}&=\left(\cos\psi+\frac{a}{l}\tan\kappa\sin\varphi\sin\psi\right)\sigma_1,\\\dot{y}&=\left(\sin\psi-\frac{a}{l}\tan\kappa\sin\varphi\cos\psi\right)\sigma_1,\\\dot{\psi}&=-\frac{1}{l}\tan\kappa\sin\varphi\sigma_1,\\\dot{\varphi}&=\sigma_2.\end{aligned}\tag{6}$$

According to the Gibbs – Appell method, the so-called energy of acceleration ($\mathcal{A}$) is needed. In this model, where we have one lumped mass only, this quantity can be computed as:

$$\mathcal{A}=\frac{1}{2}m\mathbf{a}_C\cdot\mathbf{a}_C, \tag{7}$$

where $\mathbf{a}_C$ refers to the acceleration of the lumped mass. One can obtain:

$$\begin{aligned}\mathcal{A}=&\frac{1}{2}m\left(1+\tan\kappa\sin^2\varphi\left(\tan\kappa\left(\frac{a^2}{l^2}+\frac{h^2}{l^2}\sin^2\varphi\right)\text{-}2\frac{h}{l}\right)\right)\dot{\sigma}_1^2\\&+\frac{1}{2}mh^2\dot{\sigma}_2^2+m\frac{ah}{l}\tan\kappa\sin\varphi\cos\varphi\dot{\sigma}_1\dot{\sigma}_2-m\tan\kappa\frac{ah}{l}\sin^2\varphi\sigma_2^2\dot{\sigma}_1\\&+\frac{1}{2}m\tan\kappa\left(\tan\kappa\left(\frac{a^2}{l^2}+3\frac{h^2}{l^2}\sin^2\varphi\right)\text{-}3\frac{h}{l}\right)\sin(2\varphi)\sigma_1\sigma_2\dot{\sigma}_1\\&+\frac{1}{2}m\frac{h}{l}\tan\kappa\sigma_1\left(2a\sigma_2\cos^2\varphi+\sigma_1\sin(2\varphi)\left(1\text{-}\frac{h}{l}\tan\kappa\sin^2\varphi\right)\right)\dot{\sigma}_2+\ldots\end{aligned}\tag{8}$$

The parts of the energy of acceleration, which do not depend on the pseudo accelerations ($\dot{\sigma}_1$ and $\dot{\sigma}_2$), are not necessary to calculate since the equation of motion can be obtained with the help of

$$\frac{\partial\mathcal{A}}{\partial\dot{\sigma}_i}=\Gamma_i. \tag{9}$$

The right hand side of this formula is the pseudo force Γ_i related to the ith pseudo velocity σ_i, which can be determined from the virtual power of the active forces, i.e. the gravitational force, the torque produced by the spring and the force of the drag force:

$$\delta P=\mathbf{F}_g\cdot\delta\mathbf{v}_C+\mathbf{M}_{st}\cdot\delta\boldsymbol{\omega}+\mathbf{F}_D\cdot\delta\mathbf{v}_C, \tag{10}$$

where δ refers to the virtual quantities. We obtain:

$$\begin{aligned}\delta P=&\left(mgh(\cos\alpha\sin\varphi+\sin\alpha\cos\varphi\sin\psi)-s_t\varphi\right)\delta\sigma_2\\&+mg\sin\alpha\left(\frac{a}{l}\tan\kappa\sin\varphi\sin\psi+\cos\psi\left(1-\frac{h}{l}\tan\kappa\sin^2\varphi\right)\right)\delta\sigma_1\\&-\frac{1}{2}k\sigma_1^2\delta\sigma_1+\frac{1}{2}\frac{kh}{l}\tan\kappa\sin^2\varphi\sigma_1^2\delta\sigma_1,\end{aligned}\tag{11}$$

wherefrom Γ_i can be identified as the coefficient of $\delta\sigma_i$.

Based on the Gibbs – Appell Eq. (9), the equations of motion can be expressed as:

$$\begin{aligned}\dot{\sigma}_1&=\frac{A}{2mhB}+\frac{C}{4mlB}\sigma_1^2+\frac{D}{2B}\sigma_1\sigma_2+\frac{E}{B}\sigma_2^2,\\\dot{\sigma}_2&=\frac{F}{mh^2B}+\frac{G}{2mhl^2B}\sigma_1^2+\frac{H}{hlB}\sigma_1\sigma_2+\frac{K}{B}\sigma_2^2,\end{aligned}\tag{12}$$

where the parameters are:

$$A = 2mghl\sin\alpha\tan\kappa\sin^2\varphi\left(a\sin\varphi\sin\psi - h\cos\psi\right) + 2mghl\left(l\sin\alpha\cos\psi - a\cos\alpha\cos\varphi\tan\kappa\sin^2\varphi\right) + als_t\tan\kappa\sin\left(2\varphi\right)\varphi,$$
$$B = \left(a^2 + h^2\right)\tan^2\kappa\sin^4\varphi - 2hl\tan\kappa\sin^2\varphi + l^2,$$
$$C = am\tan^2\kappa\left(l\sin^2\left(2\varphi\right) - 4h\tan\kappa\sin^4\varphi\cos^2\varphi\right) - 2kl^3 + 2kl^2h\tan\kappa\sin^2\varphi,$$
$$D = \tan\kappa\sin\left(2\varphi\right)\left(3hl - \left(a^2 + 3h^2\right)\tan\kappa\sin^2\right),$$
$$E = ahl\tan\kappa\sin^2\varphi,$$
$$\begin{aligned} F = &-s_t\varphi\left(\tan\kappa\sin^2\varphi\left(\tan\kappa\left(a^2 + h^2\sin^2\varphi\right) - 2hl\right) + l^2\right) + mgh^2\sin\alpha\tan^2\kappa\sin^3\varphi\cos\varphi\left(a\cos\psi + h\sin\varphi\sin\psi\right) \\ &+ mgh\cos\alpha\sin\varphi\left(\tan\kappa\sin^2\varphi\left(\tan\kappa\left(a^2 + h^2\sin^2\varphi\right) - 2hl\right) + l^2\right) \\ &+ mghl\sin\alpha\left(l\cos\varphi\sin\psi - \tan\kappa\sin\varphi\left(a\cos\varphi\cos\psi + h\sin(2\varphi)\sin\psi\right)\right),\end{aligned}$$
$$\begin{aligned} G = &\, mh\tan^3\kappa\sin\left(2\varphi\right)\sin^4\varphi\left(\tan\kappa\left(a^2 + h^2\sin^2\varphi\right) - 3hl\right) + ml\tan^2\kappa\sin(2\varphi)\sin^2\varphi\left(3hl - a^2\tan\kappa\right) \\ &+ l^3\tan\kappa\sin\left(2\varphi\right)\left(\frac{ak}{2} - m - \frac{akh}{2l}\tan\kappa\sin^2\varphi\right),\end{aligned}$$
$$H = a\tan\kappa\cos^2\varphi\left(h\tan\kappa\sin^2\varphi - l\right)\left(2h\tan\kappa\sin^2\varphi + l\right),$$
$$K = -a^2\tan^2\kappa\sin^3\varphi\cos\varphi. \tag{13}$$

Equations in (12) are connected to the expression of the generalized velocities in (6). Thus, our system can be described in a 6 dimensional state-phase.

Let us note here, that x and y are cycles coordinates, so Eq. (12) and the last two equations of (6) describe the motion uniquely. Only these four equations can be used for the stability analysis, consequently, the zero characteristic exponents will not emerge in our analysis. The zero characteristic exponents are not relevant in practical point of view while they can limit the use of classical stability criteria.

3 Stability analysis

The rectilinear motion of the mechanical model corresponds to the case when the skateboard moves parallel to the X direction along the slope. We are interested in the linear stability of this rectilinear motion with special respect to the effects of the standing position and the grade.

3.1 Linearized equation of motion

The values of the state variables in case of the rectilinear motion with constant longitudinal speed V are the following: both the pseudo velocities are constant: $\sigma_1 \equiv V$, $\sigma_2 \equiv 0$, the positions of the board on the slope are $x \equiv Vt$ and $y \equiv y_0$, while the angle of the board on the slope and the deflection angle of the skater are zeros, i.e. $\psi \equiv 0$ and $\varphi \equiv 0$.

Substituting these values into the equations of motion, we obtain a condition from the first equation of (12):

$$0 = -\frac{kV^2}{2m} + g\sin\alpha, \tag{14}$$

wherefrom the longitudinal speed can be calculated as a function of the mass, the inclination angle of the slope and the coefficient of the drag force:

$$V = \sqrt{\frac{2mg\sin\alpha}{k}}. \tag{15}$$

It can be shown, that this is the maximal reachable speed of the skater.

The equation of motion linearized around the rectilinear motion can be determined assuming small perturbation in σ_1, σ_2, ψ and φ. The linear system can be written as

$$\dot{\mathbf{X}}(t) = \mathbf{J}\cdot\mathbf{X}(t), \tag{16}$$

where

$$\mathbf{J} = \begin{pmatrix} J_{1,1} & 0 & 0 & 0 \\ 0 & J_{2,2} & J_{2,3} & J_{2,4} \\ 0 & 0 & 0 & J_{3,4} \\ 0 & 1 & 0 & 0 \end{pmatrix},$$
$$J_{1,1} = -\sqrt{\frac{2gk\sin\alpha}{m}}, \quad J_{2,2} = J_{1,1}\frac{ma}{hkl}\tan\kappa, \quad J_{2,3} = \frac{g}{h}\sin\alpha,$$
$$J_{2,4} = -\frac{s_t}{mh^2} + \frac{g}{h}\left(\cos\alpha - \frac{2m\sin\alpha\tan\kappa}{kl}\right), \quad J_{3,4} = J_{1,1}\frac{m}{l}\tan\kappa \tag{17}$$

and

$$\mathbf{X}^{\mathrm{T}}(t) = \left[\sigma_1(t) \quad \sigma_2(t) \quad \psi(t) \quad \varphi(t)\right]. \tag{18}$$

3.2 Linear stability of the rectilinear motion

The stability analysis of the linear ordinary differential equation system (16) can be carried out with the help of the Routh – Hurwitz criterion (for details see [13]). The so-called Hurwitz matrix can be constructed from the coefficients of the characteristic function:

$$D(\lambda)=\sqrt{\frac{2g\sin\alpha}{mk}}\left(3\frac{g}{h}\frac{m}{l}\sin\alpha\tan\kappa-k\frac{g}{h}\cos\alpha+k\frac{s_t}{h^2m}\right)\lambda$$
$$+\left(\frac{2g\sin\alpha}{hl}\tan\kappa\left(a+\frac{m}{k}\right)+\frac{s_t}{mh^2}-\frac{g}{h}\cos\alpha\right)\lambda^2$$
$$+\sqrt{\frac{2gk\sin\alpha}{m}}\left(1+\frac{am}{hkl}\tan\kappa\right)\lambda^3+\lambda^4+2\frac{g^2}{hl}\sin^2\alpha\tan\kappa. \tag{19}$$

The Hurwitz matrix can be composed as

$$\mathbf{H}=\begin{pmatrix} a_3 & a_1 & 0 \\ a_4 & a_2 & a_0 \\ 0 & a_3 & a_1 \end{pmatrix} \tag{20}$$

where a_k refers to the coefficient of λ^k. The ith sub-determinant of the Hurwitz matrix can be denoted by Δ_i and the investigated equilibrium, in our case the rectilinear motion, is asymptotically stable if and only if all of the sub-determinants Δ_i are greater than zero. This criterion can be rephrased according to the Linéard – Chipart conditions (see in[13]), which state that all the roots of the characteristic equation $D(\lambda) = 0$ have negative real parts if all of the coefficients of the characteristic polynomial have the same sign and the condition

$$\Delta_3 > 0. \tag{21}$$

is also fulfilled. These conditions in our case can be expressed as:

$$\begin{gathered}
\sqrt{\frac{2g\sin\alpha}{mk}}\left(3\frac{g}{h}\frac{m}{l}\sin\alpha\tan\kappa-k\frac{g}{h}\cos\alpha+k\frac{s_t}{h^2m}\right)>0\\
k+\frac{am}{hl}\tan\kappa>0\\
\frac{2g\sin\alpha}{hl}\tan\kappa\left(a+\frac{m}{k}\right)+\frac{s_t}{mh^2}-\frac{g}{h}\cos\alpha>0\\
a>\frac{mghl\cos\alpha-ls_t-2gh^2lk\sin\alpha}{2mgh\sin\alpha\tan\kappa}-\frac{3m}{2k}\\
a>\frac{mgh^2kl\sin\alpha}{mgh(2m\sin\alpha\tan\kappa-kl\cos\alpha)+kls_t}
\end{gathered} \tag{22}$$

The following assumptions can be taken into account: all of the parameters are real and only the parameter a can be negative. Moreover the angles α and κ are less than $\pi/2$. Furthermore, during the derivation of the last two inequality for a in expression (22) from inequality (21), we have used another assumption, namely the spring stiffness is larger, than a critical value s_t^{cr}:

$$s_t^{cr}=mgh\left(\cos\alpha-2\frac{m}{kl}\tan\kappa\sin\alpha\right). \tag{23}$$

If the spring stiffness is less than this value the rectilinear motion is linearly unstable. Or from different point of view, a critical grade (grade^{cr} = $\tan\alpha^{cr}$) can be expressed also for a given spring stiffness based on

$$\alpha^{cr}=2\arctan\left(\sqrt{\frac{mgh-s_t}{mgh+s_t}+\left(2\frac{m^2gh\tan\kappa}{kl(mgh+s_t)}\right)^2}-2\frac{m^2gh\tan\kappa}{kl(mgh+s_t)}\right). \tag{24}$$

This means, that if the grade is less than this critical value, then the rectilinear motion is unstable. Although this statement is contrary to our physical sense, the result is in good agreement with the speed dependence of the stability of skateboards (see in [2]). Namely, the higher the speed, the more stable the skateboard. And in our model, the grade determines the speed of the stationary rectilinear motion.

Considering the critical spring stiffness and the positivity of the parameters, the first two conditions of (22) are automatically fulfilled, and the remaining ones can be rearranged as conditions for the positon of the skater on the board. The relevant condition is provided by the last expression of (22):

$$a>\frac{mgh^2kl\sin\alpha}{mgh(2m\sin\alpha\tan\kappa-kl\cos\alpha)+kls_t}. \tag{25}$$

Thus, we have a lower boundary for the position, in which the critical value is always greater than zero, except the case $\alpha = 0$. Since the slope angle determines the stationary speed for a given coefficient k of the drag force, the stability can be investigated by means of stability charts in the space of α and a. The structure of the stability charts can be qualitatively different depending on the spring stiffness, namely, two different main cases can be considered. First, when the spring is stiff enough ($s_t > mgh$) to stabilize the skater-skateboard system even at zero grade angle (i.e. at zero speed), the stability boundary is shown in Fig. 2a. The asymptotically stable domain is shaded. The local extremum of the stability boundary can be calculated in closed form, namely:

$$\alpha^*=\arccos\left(\frac{mgh}{s_t}\right). \tag{26}$$

In the other case, when the spring is not stiff enough ($s_t > mgh$) at zero grade angle, the stability boundary is plotted in Fig. 2b.

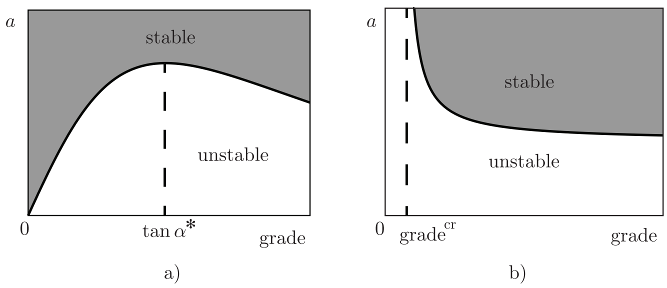

Fig. 2 Structure of the stability chart a) the spring is stiff enough even at zero grade angle, b) spring is not stiff enough at zero grade angle

Whereas the boundary of the stable domain is expressed analytically (see (25)), the stability limit at infinite grade, which corresponds to the slope angle $\alpha = \pi/2$, can be computed too:

$$a^{\mathrm{cr}}\Big|_{\alpha\to\frac{\pi}{2}} = \frac{mgh^2kl}{2m^2gh\tan\kappa + kls_{\mathrm{t}}}. \quad (27)$$

4 Case study

In this section we are going to investigate a skater - skateboard system with the realistic parameters of Table 1.

Table 1 Parameters of a skateboard

h [m]	m [kg]	g [m/s^2]	s_t [Nm/rad]	l [m]	κ [°]
0.85	75	9.81	100	0.3937	63

Our analysis also requires a realistic value for the coefficient of the drag force. To determine it, we based our calculation on the data of a word speed record.

4.1 Estimation of the coefficient of the drag force

The greatest skateboard speed achieved in standing position is 129.94 km/h (see in [11]). It was achieved by a Canadian skater in 2012 at Les Éboulements (Quebec, Canada), where some parts of the road has 18% grade. Based on these data, k can be computed numerically by the expression (15) assuming m = 90 kg. So, the drag coefficient is k = 0.335 Ns^{2}m^{-2}. This value can be considered as the possible smallest drag coefficient. Obviously, in normal cases, this value is not realistic, so for further investigation we consider it as k = 0.52 Ns^{2}m^{-2}. It was chosen based on [14], where the dimensionless drag coefficient was measured for clothed humans, both from sided and facing directions in case of small, average and large subjects. For the investigation of skateboarding, the sided setup is relevant, and we used the value of the average subjects ($C_{\mathrm{D}} = 1.11$ and $A_{\mathrm{D}} = 0.38$ m^2). Furthermore, we considered the density of the air as 1.225 kg/m^3 and we can get this value based Eq. (1).

4.2 The effects of the parameters on the stability

In the following, we are going to focus on the effect of the parameters, namely the effect of the height h, the mass m, the length $2l$ of the board and the stiffness s_{t} of the suspension system.

The effects of these parameters are illustrated in stability charts in the plane of the grade and the relative standing position a/l. The thick solid lines, in Fig. 3, Fig. 4, Fig. 5 and Fig. 6, belong to the original parameters (see Table 1), while the dashed lines belong to the modified ones.

First, let us look the influence of the skater's parameters. Although, the skater can tune the parameter h easily by means of changing his/her pose on the board, this parameter has minor effect on the stability boundary. See Fig. 3, where the stability chart is very similar for extremely different heights ($2h$) of the skater. But we can say that the stable domain is larger when the skater is smaller. This result has good agreement with practical observations, namely, skater often couches down instinctively in order to ride faster and safer. Let us note, that the height does not have effect on the critical grade, but it allows the rider to stand closer to the center of the board.

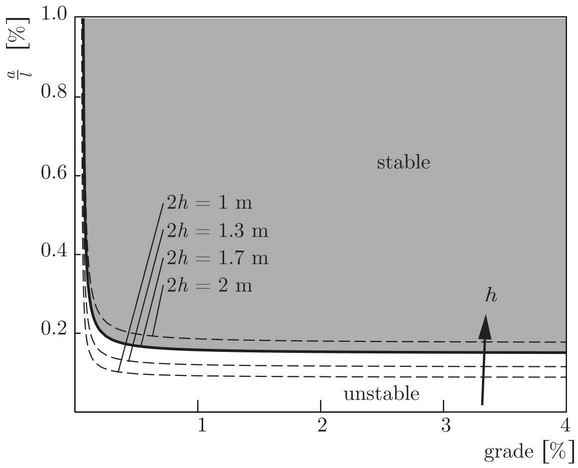

Fig. 3 Effect of the rider's height on the stability of the rectilinear motion

One can see in Fig. 4 that the effect of the mass is inverse. The higher the mass, the larger the stable domain. Moreover the critical grade become smaller. This result has good agreement with the results of previous studies ([2] and [4]), namely, the speed makes the skateboarding to be more stable. If the mass is higher, the speed of the stationary motion is higher too (see expression (15)).

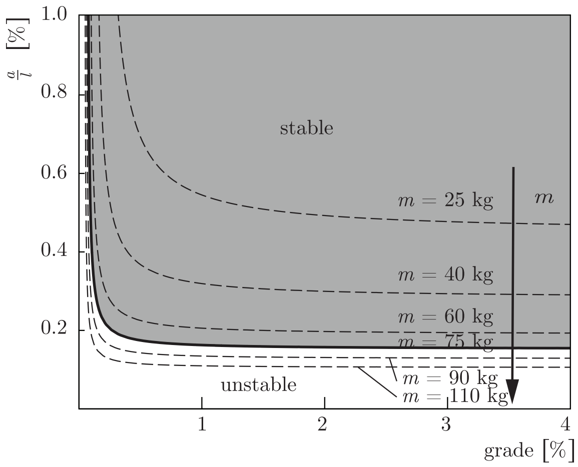

Fig. 4 Effect of the rider's mass on the stability of the rectilinear motion

Now, we can see that it is more difficult to ride on a skateboard on a slope if the skater is a child because the mass has stronger effect on the size of the stable domain than the height.

Let us focus on the parameters of the skateboard. As it was mentioned before, longboards are recommended in practice for higher speeds, thus, one of the main questions of this study is about the effect of the length of the skateboard. Figure 5 shows the stability boundary for different lengths. One can see, that the length of the board has effect only at low grades and it is not significant even there. This means, that longer boards do not have any advantage in point of view of the linear stability at higher speeds.

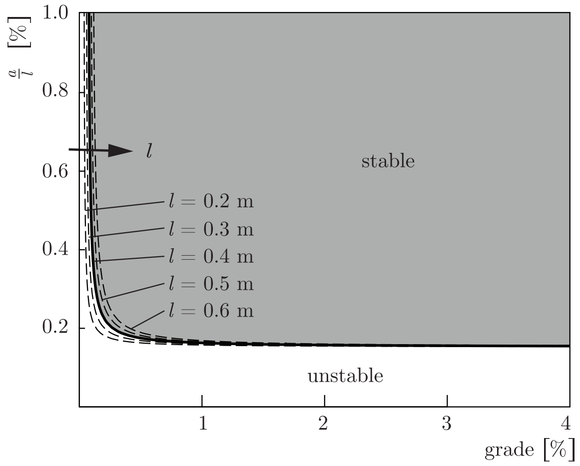

Fig. 5 Effect of the size of the board on the stability of the rectilinear motion

The most radical effect on the stability chart is generated by the stiffness parameter of the board. As it was shown in Fig. 2, the structure of the stability chart changes at the critical stiffness value *mgh*. In Fig. 6, this can be observed too. If the spring stiffness is greater than the critical value ($s_t = 250.155$ Nm/rad) then the rectilinear motion can be stable at very low speed. Or from another point of view, the expression for the critical spring stiffness gives positive value for very small slope angles in case of realistic parameters.

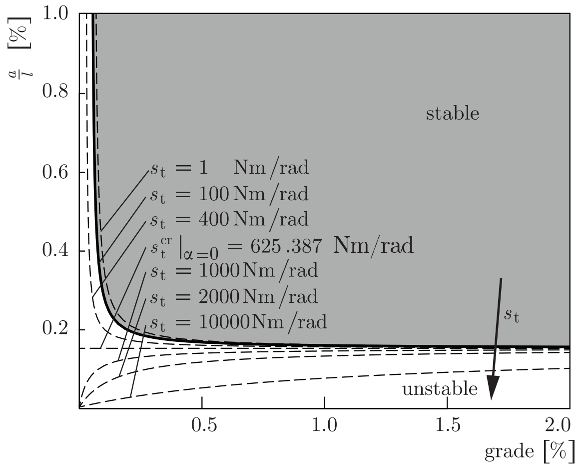

Fig. 6 Effect of the stiffness of the board suspension system on the stability of the rectilinear motion

5 Conclusion

The mechanical model used here can be regarded as an extension of the simplest model of the skateboard-skater system. The equations of motion were obtained for the downhill motion by means of the Gibbs Appell method and the linear stability of the rectilinear motion was analysed with the help of the Routh – Hurwitz criterion.

As it was already discovered in the first paper [1] of the topic, the position of the skater has key role and the rectilinear motion can be stable if the skater stands before the centre point of the board. Here, it was shown that there is a criterion for the standing position, which is even stricter, namely a non-negative lower limit for the parameter *a* exist. This stricter criterion is valid if the torsional spring stiffness of the suspension system is not stiff enough to stabilize the system at zero grade, what is the typical case in practice.

In order to obtain a more complex picture, the effects of other parameters were also investigated. It was shown that the effect of the height of the skater is negligible relative to the effect of the mass of the skater. Let us note here, that in our model the mass has also effect on the speed of the corresponding stationary motion but this effect is not highlighted in the constructed stability charts. For example, a kid (with lower mass) can reach lower maximal speed on the same slope than an adult if the same drag force coefficient is considered. But larger grade can lead to such a speed that can stabilize the rectilinear motion.

The parameters of the board were investigated separately. It was found, that the centre of gravity must be more ahead in case of longboards. Nevertheless, longer boards can be beneficial with respect to other aspects that are not considered in our model. For example, the actuation of the board by the human control system can be carried out easier, or the position on the board can be modified more unlimitedly. This latter aspect can also be relevant from aerodynamic viewpoint, i.e., the skater can balance against the aerodynamic drag force comfortably.

The stiffness of the suspension system has a qualitative effect on the stability of the rectilinear motion. But, this effect is important at small grade only.

Acknowledgement

This research was partly supported by the Janos Bolyai Research Scholarship of the Hungarian Academy of Science and by the Hungarian National Science Foundation under grant no. OTKA PD105442.

References

[1] Hubbard, M. "Lateral dynamics and stability of the skateboard." *Journal of Applied Mechanics*. 46 (4). pp. 931-936. 1979. DOI: 10.1115/1.3424680

[2] Hubbard, M. "Human control of the skateboard." *Journal of Biomechanics*. 13 (9). pp. 745-754. 1980. DOI: 10.1016/0021-9290(80)90236-5

[3] Ispolov, Y., Smolnikov, B. A. "Skateboard dynamics." *Computer Methods in Applied Mechanics and Engineering*. 131 (3-4). pp. 327-333. 1996. DOI: 10.1016/0045-7825(95)00932-9

[4] Kremnev, A. V., Kuleshov, A. S. "Dynamics and simulation of the simplest model of skateboard." In: *Proceedings of Sixth EUROMECH Nonlinear Dynamics Conference*. ENOC-2008, Saint Petersburg, Russia, June, 30–July, 4 2008.

[5] Kremnev, A. V., Kuleshov, A. S. "*Nonlinear Dynamics and Stability of a Simplified Skateboard Model*." 2007.

[6] Varszegi, B., Takacs, D., Stepan, G., Hogan, S. J. "Balancing of the skateboard with reflex delay." In: *Proceedings of Eighth EUROMECH Nonlinear Dynamics Conference, ENOC2014*. 2014. ISBN: 978-3-200-03433-4

[7] Rosatello, M., Dion, J., Renaud, F., Garibaldi, L. "The skateboard speed wobble." In: *Proceedings of ASME 11th International Conference on Multibody Systems, Nonlinear Dynamics, and Control*. Boston. 2015.

[8] McRuer, D. T., Weir, D. H., Jex, H. R., Magdeleno, R. E., Allen, R. W. "Measurement of driver-vehicle multiloop response properties with a single disturbance input." *IEEE Transactions on Systems, Man and Cybernetics*. SMC-5 (5). pp. 490-497. 1975. DOI: 10.1109/TSMC.1975.5408371

[9] Stepan, G. "Delay effects in the human sensory system during balancing." *Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*. 367(1891). pp. 1195-1212. 2009. DOI: 10.1098/rsta.2008.0278

[10] Chagdes, J. R., Haddad, J. M., Rietdyk, S., Zelaznik, H. N., Raman, A. "Understanding the Role of Time-Delay on Maintaining Upright Stance on Rotational Balance Boards." In *Proceedings of ASME 11th International Conference on Multibody Systems, Nonlinear Dynamics, and Control*. Boston. 2015.

[11] Guiness World Records. [Online]. Available from: http://www.guinness-worldrecords.com/. [Accessed: 2nd June 2015]

[12] Hart, J. H., Allen, T., Holroyd, M. "Downhill skateboard aerodynamics." *Procedia Engineering*. 2(2). pp. 2523-2528. 2010. DOI: 10.1016/j.proeng.2010.04.026

[13] Gantmacher, F. "*Lectures in Analytical Mechanics*." MIR Publisher, Moscow, Russia. 1975. ISBN-13: 978-0846405511

[14] Penwarden, A. D., Grigg, P. F., Rayment, R. "Measurements of wind drag on people standing in a wind tunnel." *Building and Environment*. 13(2). pp. 75-84. 1978. DOI: 10.1016/0360-1323(78)90026-4