Analysis of Transverse Vibration Acceleration of a High-Speed Elevator with Random Parameters under Random Excitation

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1 Introduction

In modern society, there are more and more high-rise buildings. As an essential means of transport in high-rise buildings, elevators have become faster, and the proportion of high-speed elevator (the speed ≥ 2.5 m/s) has increased year by year. The transverse vibration acceleration that is generated by the random excitation and random parameters has become a major factor affecting the ride comfort of the elevator. In recent years many scholars have studied transverse vibration of elevator cars. Feng et al. [1] established a dynamic model of the transverse vibration of an elevator car based on the rigid body dynamics theory, and she derived the differential equations based on Newton’s laws of motion and the Euler equations. Chang et al. [2] established a four degree-of-freedom elevator system to study the excitation characteristics and the car dynamic response, and developed an active mass driver based on H₂ direct output feedback control algorithm. Herrera et al. [3] considered the behavior of passengers in the car and established a model to analyze the influence of the car dynamic characteristics under different loading conditions. However, for objective random excitation and random parameters of an elevator car, most literature did not consider or approximate the deterministic parameter. In fact, random parameters not only affect the system of each mode of eigenvalues and eigenvectors, but also have an effect on the numerical characteristics of the response together with random excitation. So the study of dynamic response of the random parameter structure under random excitation is important for suppression of a vibration of elevator car, reliability sensitivity analysis, and safety assessment.

Xu et al. [4] analyzed the stochastic dynamic characteristics of beams under the stochastic material properties by the random factor method. However, the authors did not do this research combining the random excitation. Marcin et al. [5] solved the dynamic response of the truss structure by using the Taylor expansion stochastic finite element method. The stochastic finite element method needs to set up all kinds of random parameters corresponding to the stochastic finite element characteristic matrix, and it causes much inconvenience to its...
computer program design. Lasota et al. [6] obtained the digital characteristics of responses of the rotor shaft system by using the polynomial chaos method. Although the polynomial chaos method can quickly obtain the corresponding numerical characteristics, it can not very well solve problems combining the correlation between parameters. Therefore, it is necessary to find a convenient calculation method to makes it easy to design the calculation program. In this paper, a random perturbation method was used to derive the dynamic equation of the system response under random excitation and random parameters, and then the sensitivity expression of response was derived. The standard deviation of the acceleration response of the system was solved by establishing the displacement response covariance matrix and random parameters covariance matrix and combining with the pseudo excitation method.

2 High-speed elevator car system dynamics model

In order to solve the transverse acceleration response of a high speed elevator car system with random parameters under random excitation, a suitable model of the car’s dynamic model was established and the differential equation of the car’s vibration was derived in the high-speed elevator, in order to improve the ride comfort, there are a certain number of damping blocks between the car frame and the car, so they are an elastic connection. [7] In Fig. 1, a car vibration model is presented. The car frame is in contact with the guide rails by four guide wheel-guide shoe systems. The guide wheel-guide shoe system and damping block are simplified into a spring damping system [8]. The stiffness and damping of four guide wheel-guide shoe systems are \( k_i \) and \( c_i \), and the stiffness and damping of four damping blocks are \( k_c \) and \( c_c \). This system has four degrees of freedom, including the car frame’s transverse translation and rotation around the center of mass, and the car’s transverse translation and rotation around the center of mass. \( OXY \) is the coordinate system taking the system center \( O \) of the equilibrium position as the origin. \( l_{xi} (i=1,...,8) \) is the \( Y \)-coordinate of the car frame stress points in the coordinate system, and \( l_{yi} (i=5,...,8) \) is the \( Y \)-coordinate of the car stress points in the coordinate system. \( l_i (i=1,...,4) \) are the random geometrical parameters, where \( l_i = \| \mathbf{e}_i \| = \| \mathbf{e}_i \| \), \( l_i = \| \mathbf{e}_i \| = \| \mathbf{e}_i \| \), and \( l_i = \| \mathbf{e}_i \| = \| \mathbf{e}_i \| \). The car frame mass \( m_x \), moment of inertia \( j_x \) car mass \( m_y \) and moment of inertia \( j_y \) are the random mass parameters.

According to Newton’s second law and the rigid body dynamics formula, the four degrees of freedom system’s differential equations of motion can be expressed as:

\[
\mathbf{M} \ddot{\mathbf{X}} + \mathbf{C} \dot{\mathbf{X}} + \mathbf{K} \mathbf{X} = \mathbf{F}(t), \tag{1}
\]

In which

\[
\mathbf{M} = \text{diag}(m_x, j_x, m_y, j_y),
\]

\( \mathbf{F}(t) \) is random excitation due to the rail irregularity degree, and it can be expressed as:

\[
\begin{bmatrix}
\mathbf{X} \\
\dot{\mathbf{X}} \\
\mathbf{X}
\end{bmatrix} =
\begin{bmatrix}
\mathbf{X} \\
\dot{\mathbf{X}} \\
\mathbf{X}
\end{bmatrix}
\]

Fig. 1 Model of vibrations for elevator cabin systems

\[
\begin{pmatrix}
-4(c_1 + c_2) & 2c_1 (l_i - l_j) + 2c_2 (l_i - l_k) \\
2c_1 (l_i - l_j) + 2c_2 (l_i - l_k) & -4c_1 (l_i - l_j) \\
-2c_1 (l_i - l_j) - 2c_2 (l_i - l_k) & -2c_2 (l_i - l_j)
\end{pmatrix}
\]

\[
\begin{pmatrix}
4c_1 & -2c_1 (l_i - l_j) \\
-2c_1 (l_i - l_j) & 4c_2 \\
-2c_2 (l_i - l_j) & -2c_2 (l_i - l_j)
\end{pmatrix}
\]

\[
\begin{pmatrix}
4(k_1 + k_2) & -2k_1 (l_i - l_j) - 2k_2 (l_i - l_k) \\
-2k_1 (l_i - l_j) - 2k_2 (l_i - l_k) & 4k_2 \\
-2k_1 (l_i - l_j) - 2k_2 (l_i - l_k) & -2k_1 (l_i - l_j) - 2k_2 (l_i - l_k)
\end{pmatrix}
\]

\[
\begin{pmatrix}
2k_2 (l_i - l_j) \\
-2k_2 (l_i - l_j) \\
-2k_2 (l_i - l_j)
\end{pmatrix}
\]

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\end{pmatrix}
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\[
\begin{pmatrix}
2k_2 (l_i - l_j) \\
-2k_2 (l_i - l_j) \\
-2k_2 (l_i - l_j)
\end{pmatrix}
\]
It can be seen that the random parts of response $X_i$, $X_{i1}$, and $X_{i2}$ consist of two parts from formula (9). $X_{i1}$ represents the displacement response due to random excitation, and $X_{i2}$ represents the displacement response due to random parameters, and they satisfy the following equation:

$$X_i = X_{i1} + X_{i2}. \tag{10}$$

Equation (9) can be divided into two equations:

$$M_d \ddot{X}_{i1} + C_d X_{i1} + K_d X_{i1} = F_i(t), \tag{11}$$

$$M_d \ddot{X}_{i2} + C_d \dot{X}_{i2} + K_d X_{i2} = -\left(M_d \ddot{X}_d + C_d \dot{X}_d + K_d X_d\right). \tag{12}$$

Structural dynamic response sensitivity analysis is often used to assess the degree of influence of the changes in structural parameters on the response. The dynamic performance of the structure can be improved according to this sensitivity. It is an important part of structural dynamic optimization design. In order to facilitate solving Eq. (12) and derive the system dynamic response’s sensitivity sector $S(\ddot{X})$, $S(\dot{X})$, and $S(X)$, when the parameters’ random parts $b_{ni}$ are much smaller than the deterministic parts $b_{di}$, $M_d$, $C_d$, $K_d$, $X_d$, $\ddot{X}_d$, and $X_{i2}$ are expanded into the Taylor series in the vicinity of $b_{di}$ $(i = 1, 2, ..., m)$, and substituted into Eq. (12), and comparing the $b_{ni}$ coefficients, the following equation is obtained:

$$M_d S(\ddot{X}) + C_d S(\dot{X}) + K_d S(X) = -\left(\frac{\partial^2 M}{\partial b_d^2} \ddot{X}_d + \frac{\partial C}{\partial b_d} \dot{X}_d + \frac{\partial K}{\partial b_d} X_d\right). \tag{13}$$

In which $S(\ddot{X}) = \frac{\partial^2 X_d}{\partial b_d^2}$, $S(\dot{X}) = \frac{\partial \dot{X}_d}{\partial b_d}$, $S(X) = \frac{\partial X_d}{\partial b_d}$. The system dynamic response’s sensitivity vector $S(\ddot{X})$, $S(\dot{X})$, and $S(X)$ are obtained by solving Eq. (13), and the random parts of the system response can be obtained by substituting them into the Taylor expansion of $\ddot{X}_d$, $\dot{X}_d$, and $X_d$.

$$\ddot{X}_d = \sum_{i=1}^{m} S_i(\ddot{X}) b_{di}, \tag{14}$$

$$\dot{X}_d = \sum_{i=1}^{m} S_i(\dot{X}) b_{di}, \tag{15}$$

$$X_d = \sum_{i=1}^{m} S_i(X) b_{di}. \tag{16}$$

4 Analysis of means and standard deviation of a high-speed elevator with random parameters

The mean of the displacement response perturbation items is zero, which has the following expression:

$$E(X) = E(X_d) + E(\varepsilon X_i) = X_d. \tag{17}$$
Similarly
\[
E(\dot{X}) = E(\dot{X}_a) + E(\epsilon \dot{X}_r) = \dot{X}_a,
\]
\[
E(\ddot{X}) = E(\ddot{X}_a) + E(\epsilon \ddot{X}_r) = \ddot{X}_a.
\]

The respond standard deviation due to random excitation \( F(t) \) can be calculated by using the pseudo excitation method [13-17]. Assuming \( F(t) \) is the ergodic random process, then \( X_a, \dot{X}_a, \) and \( \ddot{X}_a \) also are the ergodic random process. According to the random vibration theory, when the linear structure is subjected to multi-point stationary random excitations of which the Auto-spectral Density matrix is \( \mathbf{S}_{\omega \omega}(\omega) \), the Auto-spectral Density of response \( \ddot{X}_a \) is
\[
\mathbf{S}_{\ddot{X}_a, \ddot{X}_a}(\omega) = \omega^4 \mathbf{H}(\omega) \mathbf{S}_{\omega \omega}(\omega) \mathbf{H}(\omega)^*.
\]

\( H(\omega) \) is frequency response. The frequency response connecting with the n-th mode is
\[
H_n(\omega) = \left[ 1 - \left( \frac{\omega}{\omega_n} \right)^2 + 2 \xi \frac{\omega}{\omega_n} i \right]^{-1} \quad (n=1,2,\ldots,q).
\]

A pseudo excitation \( \tilde{F}_c \) is constructed, which is \( \left( \mathbf{S}_{\omega \omega}(\omega) \right)^{1/2} e^{i\omega t} \), then the corresponding virtual response \( \ddot{X}_a \) is
\[
\ddot{X}_a = \omega^4 \left( \mathbf{S}_{\omega \omega}(\omega) \right)^{1/2} H(\omega) e^{i\omega t}.
\]

The Auto-spectral Density expression of the response is derived:
\[
\mathbf{S}_{\ddot{X}_a, \ddot{X}_a}(\omega) = \mathbf{S}_{\ddot{X}_a, \ddot{X}_a, \Omega}(\omega) = \omega^4 \mathbf{H}(\omega) \mathbf{S}_{\omega \omega}(\omega) \mathbf{H}(\omega)^*.
\]

The acceleration response standard deviation is
\[
\sigma_{\ddot{X}_a} = \sqrt{\frac{1}{2\pi} \int_{-\pi}^{\pi} \mathbf{S}_{\ddot{X}_a, \ddot{X}_a}(\omega) \, d\omega}.
\]

\( \mathbf{S}_{\ddot{X}_a, \ddot{X}_a}(\omega) \) is the column vector and its elements are formed of \( \mathbf{S}_{\ddot{X}_a, \ddot{X}_a}(\omega) \)'s diagonal elements.

For the standard deviation of random excitation \( X_{\omega} \) due to random parameters, first the displacement response covariance matrix \( \mathbf{N}_b \), random parameters covariance matrix \( \mathbf{N}_b \), and displacement response sensitivity matrix \( \frac{\partial \ddot{X}_a}{\partial b} \) are defined:
\[
\mathbf{N}_b = \begin{bmatrix}
\text{Var}(\dot{X}(1)) & \text{Cov}(\dot{X}(2),\dot{X}(1)) & \cdots & \text{Cov}(\dot{X}(k),\dot{X}(1)) \\
\text{Cov}(\dot{X}(1),\dot{X}(2)) & \text{Var}(\dot{X}(2)) & \cdots & \text{Cov}(\dot{X}(k),\dot{X}(2)) \\
\vdots & \vdots & \ddots & \vdots \\
\text{Cov}(\dot{X}(1),\dot{X}(k)) & \text{Cov}(\dot{X}(2),\dot{X}(k)) & \cdots & \text{Var}(\dot{X}(k))
\end{bmatrix},
\]
\[
\mathbf{N}_b = \begin{bmatrix}
\text{Var}(b_1) & \text{Cov}(b_1,b_2) & \cdots & \text{Cov}(b_1,b_k) \\
\text{Cov}(b_2,b_1) & \text{Var}(b_2) & \cdots & \text{Cov}(b_2,b_k) \\
\vdots & \vdots & \ddots & \vdots \\
\text{Cov}(b_k,b_1) & \text{Cov}(b_k,b_2) & \cdots & \text{Var}(b_k)
\end{bmatrix}.
\]

\[
\frac{\partial \ddot{X}_a}{\partial b} = \begin{bmatrix}
\frac{\partial \ddot{X}_a}{\partial b_1} \\
\frac{\partial \ddot{X}_a}{\partial b_2} \\
\vdots \\
\frac{\partial \ddot{X}_a}{\partial b_k}
\end{bmatrix}.
\]

The system architecture random parameters and their correlation coefficients are given, the standard deviation of the response can be obtained. This makes it easy to apply it to engineering practice. Simultaneously, in calculating the response sensitivity, only the required degree of freedom is chosen, so as to avoid a large amount of computation.

5 Case analysis

A high-speed elevator, with a speed of 5m/s, was simplified into the model as shown in Fig. 1. The means and standard deviations of random parameters are shown in Table 1. It was assumed that the random parameters are independent and subject to normal distribution, and their coefficient of variation CV=0.05.

A common pulse excitation was exerted on all guide wheels as the excitation’s deterministic part \( F_x \). A single rail length was 5m. The excitation is shown in Fig. 2.

5.1 The calculation of the high-speed elevator car system transverse acceleration response

Solving the response expression Eq. (8) by using Wilson-\( \theta \) method [18, 19], the acceleration response \( \ddot{X}_{\omega} \) is obtained. Then the observation point acceleration \( \ddot{X}_d \) in the x direction is obtained.
by letting the acceleration response $\ddot{X}_d$ pre-multiply the transformation matrix $T$, as shown in Fig. 3. The White Gaussian Noise (its standard deviation $\sigma = 20$N), letting the whole excitation’s coefficient of variation $CV=0.05$) is defined of which the Power Spectral Density is $S(\omega) = \frac{n_f}{2} = 400$ W/Hz. The transverse acceleration caused by the randomness of the excitation is obtained by solving Eq. (11), and pre-multiplying the transformation matrix $T$ and superimposing over $\ddot{x}_d$. Then the observation point’s transverse acceleration $\ddot{X}_{d+r}$ with the deterministic parameters under the random excitation is obtained, as shown in Fig. 4. The transverse acceleration $\ddot{X}_d$ caused by the randomness of parameters is obtained by solving Eq. (13) and substituting it into Eq. (14), and pre-multiplying the transformation matrix $T$ and superimposing over $\ddot{x}_{d+r}$. Then the observation point’s transverse acceleration $\ddot{X}_{d+r+\eta}$ with random parameters under the random excitation is obtained, as shown in Fig. 5.

With the help of MATLAB, $\ddot{x}_d$, $\ddot{x}_{d+r}$, and $\ddot{x}_{d+r+\eta}$ are transformed into a Fourier series, and the images of $\ddot{x}_d$, $\ddot{x}_{d+r}$, and $\ddot{x}_{d+r+\eta}$ in the frequency domains are shown in Fig. 6-Fig. 8.

### Table 1: The parameters’ means and standard deviation of the elevator cabin system

<table>
<thead>
<tr>
<th>Variable $b_i$</th>
<th>Mean $D_{b_i}$</th>
<th>Standard Deviation $\sigma_{b_i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_1$ $m_i$/kg</td>
<td>750</td>
<td>37.5</td>
</tr>
<tr>
<td>$b_2$ $j_i$/kg·m²</td>
<td>3000</td>
<td>150</td>
</tr>
<tr>
<td>$b_3$ $m_i$/kg</td>
<td>1200</td>
<td>60</td>
</tr>
<tr>
<td>$b_4$ $j_i$/kg·m²</td>
<td>1300</td>
<td>65</td>
</tr>
<tr>
<td>$b_5$ $k_i$/Nm²</td>
<td>10000</td>
<td>500</td>
</tr>
<tr>
<td>$b_6$ $k_i$/Nm²</td>
<td>20000</td>
<td>1000</td>
</tr>
</tbody>
</table>

Fig. 2 The common pulse excitation caused by guide rails

Fig. 3 Deterministic part of acceleration response of the observation point

Fig. 4 Car system acceleration response with deterministic parameters under random excitation
Comparing Fig. 3-5, when the randomness of excitation and parameters are not considered, the image of the observation point’s transverse acceleration shows a smoother curve in the time domain. After considering the randomness of excitation and parameters, the acceleration image shows an irregular jagged curve, and the maximum acceleration increased 21.4%. It can be seen that the randomness of the guide rails’ excitation and parameters have an impact on the transverse acceleration of the car, and the comfort is reduced. Comparing Fig. 6-8, in the frequency domain image, the maximum amplitude component is generally concentrated in the low-frequency range of 5Hz. The randomness of the guide rails’ excitation and parameters will only increase the amplitude, while it has little influence on the maximum amplitude frequency.

5.2 Analysis of Acceleration Response Sensitivity

The acceleration response’s sensitivity vector \( S(\hat{x}) \) is obtained by solving Eq. (13), and the acceleration response’s sensitivity vector of the observation point is obtained by pre-multiplying the transformation matrix \( T \). The root mean square \( s_i(\hat{x})_{rmss} \) of each response sensitivity is calculated, and the results are shown in the Table 2.
As can be seen from the table, the response sensitivities of geometrical parameters $I_1$, $I_2$, $I_3$, and $I_4$ are much larger than other parameters, and should be treated as random parameters. In addition, the response sensitivities of other parameters are low and can be used as the deterministic parameters.

### 5.3 Analysis of the Observation Point's Mean and Standard Deviation

The acceleration response of 6s-7s in which the amplitude is large was selected as the research object. The deterministic response $\ddot{X}_{\text{d}}$ was regarded as the acceleration response mean $\ddot{X}$. The standard deviation $\sigma_{\ddot{X}_{\text{d}}}$ caused by the randomness of the guide rails’ excitation is obtained by solving Eq. (23). The standard deviation $\sigma_{\ddot{X}_t}$ caused by the randomness of parameters is obtained by solving Eq. (30). There are substituted into Eq. (31), and the standard deviation $\sigma_{\ddot{X}}$ of the acceleration response is obtained. The coefficient of variation CV is calculated. The results are shown in Table 3.

### References


