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RESEARCH ARTICLE

The LQR Control Active of Smart Plate Based on the Finite Element Method

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Abstract
This paper presents a numerical study pertaining to on the active vibration control (AVC) of the 3-D rectangle simply supported plate bonded of the piezoelectric sensor/ actuator pairs. AVC is a large area of interest either in all sections of industry or in research. One way to control the vibration of dynamic systems is by using piezoelectric materials. A finite element method (FEM) analysis is used to model the dynamic behavior of the system. The frequencies of the isotropic pate and a smart structure are verified by the comparison between the analytical calculations and simulation. A LQR controller is designed based on the independent mode space control techniques to stifle the vibration of the system. The change in the sizes of the patches was a clear impact on the control results, and also in the values of the voltage in actuator. The results were established by simulating in ANSYS and MATLAB.

Keywords
smart plate, piezoelectric, FEM, AVC, vibration, LQR

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1 Introduction

Active vibration control (AVC) technology has been developed for more than 30 years. Combining the vibration theory with the control theory, it has been widely applied in the fields of civil engineering, aeronautics, astronautics, mechanical engineering and vehicle, etc. [7]. AVC using smart materials is being increasingly used for flexible structures in the aerospace industry. Over the last decade the usage of piezoceramics as actuators and sensors has considerably increased and they provide an effective means of high quality actuation and sensing mechanism [17].

Modeling smart structures often require a coupled modeling between the host structure and the piezoelectric sensors and actuators. They can be modeled as either lumped or distributed parameter systems, and usually these systems have complicated shapes and structural patterns that make the development and solution of descriptive partial differential equations burdensome, if not impossible. Alternatively, various discretization techniques, such as finite element (FE) modeling, modal analysis, and lumped parameters, allow the approximation of the partial differential equations by a finite set of ordinary differential equations. Since the 1970s, many FE models have been proposed for the analysis of smart piezoelectric structural systems [16].

With a multiple-input and multiple-output (MIMO) control system, linear quadratic control methods are the preferred choice and can be used effectively for Multimode vibration suppression, and the Linear Quadratic Regulator (LQR) control approach is well suited for the requirements of damping out the effect of disturbances as quickly as possible and maintaining stability robustness [8]. In this study a LQR controller is designed based on the independent mode space control techniques to suppress the first three ranks modes vibration of the system.

The paper is organized as follows. In Section 2 the FE of smart plate is formulated, Sections 3 and 4 are presented the AVC procedure and LQR problem successively; the illustrative example is described in detail in Section 5. Finally, conclusions are drawn in Section 6.

2 Finite element modeling of the smart plate

Modeling of piezoelectric smart structures by the finite element method has been presented [8, 12, 15]. The global matrix equations governing a smart structure system can be written as:

$$\begin{cases} [M_{uu}]\ddot{u} + [C]\dot{u} + [K_{uu}]u + [K_{u\varnothing}]\Phi = \{F\} \\ [K_{u\varnothing}{}^{T}]u + [K_{\varnothing\varnothing}]\Phi = \{Q\} \end{cases} \tag{1}$$

Where

Kinematically constant mass matrix

$$[M_{uu}] = \int \rho N_u^T N_u dV \tag{2}$$

Elastic stiffness matrix

$$[K_{uu}] = \int B_u^T C^E B_u dV \tag{3}$$

Proportional damping matrix

$$[C] = \alpha[M_{uu}] + \beta[K_{uu}] \tag{4}$$

Piezoelectric coupling matrix

$$[K_{u\varnothing}] = \int B_u^T h^T B_{\varnothing} dV \tag{5}$$

Dielectric stiffness matrix

$$[K_{\varnothing\varnothing}] = -\int B_{\varnothing}^T b^s B_{\varnothing} dV \tag{6}$$

Mechanical force

$$\{F\} = \int_V N_b^T f_b dV + \int_{S_1} N_{S_1}^T f_S dS_1 + N_u^T f_c \tag{7}$$

Electrical charge

$$\{Q\} = \int_{S_2} N_{S_2}^T q_S dS_2 - N_{\varnothing}^T q_c \tag{8}$$

Where

α and β : The Rayleigh's coefficients

f_b : The vector of body applied to the volume V

f_S : The surface force applied to the surface S_1

f_c : The concentrated force

q_s : The surface charge at surface S_2

q_c : The point charge

S_1 : Is the area where mechanical forces are applied,

S_2 : Is the area where electrical charges are applied.

3 Active control vibration

The equations of active control vibration of a smart plate in modal coordinate can be written as [3]:

$$\ddot{\alpha}_i + 2\zeta_i\omega_i\dot{\alpha}_i + \omega_i^2\alpha_i = \sum_{l=1}^{N_a} b_{il}u_i \qquad i = 1,\ldots,N \tag{9}$$

$$Y_j = \sum_{l=1}^{N} C_{jl}\alpha_l \qquad l = 1,\ldots,N_a \tag{10}$$

Where:

N the first eigenmodes are considered.

α_i, $\dot{\alpha}_i$ and $\ddot{\alpha}_i$ represent modal displacement, velocity and acceleration.

ω_i and ζ_i are the natural frequency and damping ratio of i th mode.

$b_{il}u_i$ is the i th modal constituent of the control force due to the electric potential u_i applied to the l th actuator.

A continuous time state space representation of the system is given by:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{11}$$

$$y = Cx(t) \tag{12}$$

Where [A], [B] and [C] denote the system matrix, the input matrix and the system output matrix, respectively. They can be obtained as.

$$[A] = \begin{bmatrix} [0] & [\omega_i] \\ -[\omega_i] & [2\zeta_i\omega_i] \end{bmatrix} \tag{13}$$

$$[B^T] = \begin{bmatrix} 0 & [b_{ij}] \end{bmatrix} \tag{14}$$

$$[C] = \begin{bmatrix} [c_{ij}] & [0] \end{bmatrix} \tag{15}$$

b_{ij} represents the action of the j th actuator to the i th eigenmode and equals to

$$b_{ij} = (2h + h_p)\int_S \left(e_{31}\frac{\partial^2\psi}{\partial x^2} + e_{32}\frac{\partial^2\psi}{\partial y^2} \right) dS \tag{16}$$

c_{ij} is the sensing constant of the j-th sensor due to the motion of the l-th mode and equals to

$$c_{ij} = \frac{1}{\varepsilon_{33}}(h + h_p/2)\int_S \left(e_{31}\frac{\partial^2\psi}{\partial x^2} + e_{32}\frac{\partial^2\psi}{\partial y^2} \right) dS \tag{17}$$

b_{ij} and c_{ij} depend respectively of the l-th actuator location and j-th sensor location.

(ω_i, ψ) represents the i-th couple of eigenvalue / eigenmode .

ζ_i : is a damping ratio of the of the i-th.

e_{31} and e_{32} are the piezoelectric coefficient.

Assuming that the state equation is controllable, it consists in using a control law:

$$\{\Phi\} = -[G]\{x\} \tag{18}$$

Where G is the state feedback gain matrix.

To design such a LQR compensator, first, we consider the minimization of the quadratic cost function as follows:

$$J_\phi = 1/2\int_0^\infty \left[\{x\}^T [Q]\{x\} + \{\Phi\}^T [R]\{\Phi\} \right] dt \tag{19}$$

Where

Q Is a positive semidefinite matrix and R is a positive matrix

The selection of Q and R is vital in the control design process. Q and R are the free parameters of design and stipulate the relative importance of the control result and the control effort. A large Q puts higher demands on control result, and a large R puts more limits on control effort [11].

The optimal solution is:

$$[G]=[R]^{T}[B]^{T}[K] \tag{20}$$

Where $[K]$ satisfies the Riccati equation:

$$[A]^{T}[K]+[K][A]-[K][B][R]^{-1}[B]^{T}[K]+[Q]=0 \tag{21}$$

4 Linear Quadratic Regulator (LQR) problem

The state feedback approach can provide a complete model of the global response of the system under control. They are particularly applicable to the control of the first few modes of a structure. The state feedback (Fig. 1) approach provides the best performance that can be achieved under an ideal feedback control system (full information and no uncertainty) [4, 6].

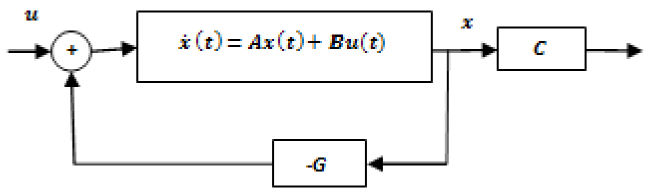

Fig. 1 The principle of the state feedback

In MATLAB, the command **lqr** is used to calculate the optimal gain matrix G.

$$\textbf{Syntaxe:}\quad [\mathbf{G},\mathbf{K},\mathbf{e}]=\mathbf{lqr}(\mathbf{A},\mathbf{B},\mathbf{Q},\mathbf{R})$$

Where e is the closed-loop eigenvalues.

$$e=eig(A-BG) \tag{22}$$

5 Example illustrative

The geometric model shown in Fig. 2 includes the plate equipped with two pairs rectangular PZT sensor/actuator pairs (Lp x lp x hp), attached to the top and bottom surfaces of the plate.

All simulations featured in this paper assume α = 0 and β = 0.01 damping constants. The time step Δt for Transient analysis is taken as $1/(20f_h)$, where f_h is the higher frequency.

Consider an initial displacement field applied to the plate equal to 1 mm.

Table 1 contains the material property data for the plate, and piezoelectric patches.

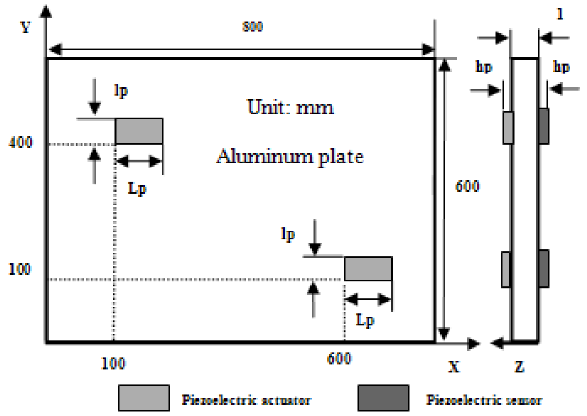

Fig. 2 The geomitrical model of the plate equipped with PZT patches

Table 1 Material properties of plate and piezoelectric

Properties	Units	Plate	PZT
E (Young's modulus)	Pa	207e9	69e9
ρ (Density)	kg/m^3	7800	7700
υ (Poisson)	---	0.3	0.3
e_{31}, e_{32} (Piezoelectric strain)	C/m^2	---	-12.351
ε (Piezo dielectric)	F/m	---	1.6e-8

5.1 First application: modelling of the plate (Lp=100, lp=50, hp=0. 1 mm) by ANSYS apdl

To build an ANSYS finite element model of a piezoelectric smart structure, the plate and the PZT materials have been modeled by the SOLID5 element, which has 8 nodes. The processing of the geometry and finite element mesh generation is provided by ANSYS processing analysis. A coupling electromechanical is created by the CP command and the appropriate voltage potential is assigned (Fig. 3).

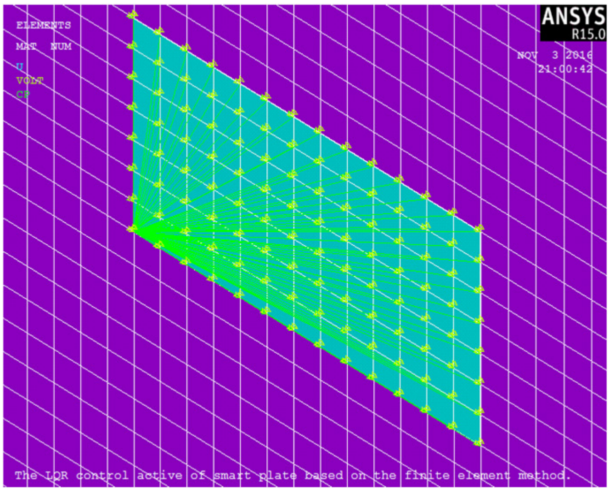

Fig. 3 The electromechanical coupling between the plate and the piezoelectric patches

The current structure is meshed by 104x84 eight-node solid elements, with 104 elements in width direction and 84 elements in the width direction. And each sensor and actuator is meshed with 91 identical elements (Fig. 4). The simulation denotes the mechanical response of the plate equipped with the piezoelectric actuators without control.

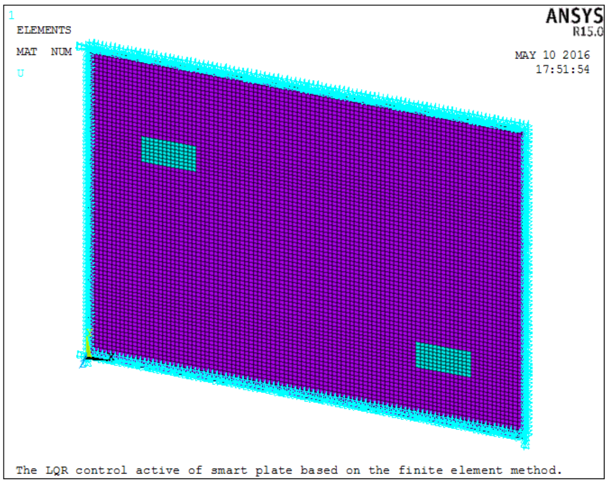

Fig. 4 FEM of the simply supported plate and boundary conditions

As the geometrical properties of piezoelectric are small compared to those of the elastic plate, piezoelectric patches can be neglected in the computation of eigenmodes.

Table 2 shows the first six natural frequencies of the smart plate. For simply supported thin isotropic plates; Leissa [9], presented the exact natural frequencies mathematically from the following closed form:

$$\omega_{rn} = \pi^2 \left[\left(\frac{r}{L} \right)^2 + \left(\frac{n}{l} \right)^2 \right] \sqrt{\frac{D}{ph}} \tag{23}$$

Where r and n are the number of half waves in the x and y directions.

And

$$D = \frac{Eh^3}{12(1-v^2)} \tag{24}$$

D Is the flexural rigidity of the plate.

(L, l, h) Are dimensions of a thin rectangle plate.

Fig. 5 shows the first six vibration modes of the smart plate.

5.2 Second application for active vibration control

In this section, we consider the active control of the previous plate.

The FEM results were exported to the MATLAB software in order, to determine the cost function and the state space representation of the system. This model was obtained by system identification commands from MATLAB software using the frequency response of the smart plate.

In this study, a linear quadratic optimal controller is considered to control the first three modes of the flexible plate. The dynamic response is calculated using the first three modes. As a result, the size of Size of system matrix [A] is 6x6 (13). In addition the size of the input matrix [B] is 2x6, the matrix [B] depend of the number of actuators (14) which two in our case.

Here, the control is started after an elapse of 0.5 s in order to compare the controlled and uncontrolled responses. Fig. 6 shows the displacement response.

The Bode plot of the open-loop and closed-loop system are shown in Fig. 7, when the control is open-loop are also shown for comparison.

As can be seen, in case of closed-loop the controller successfully damps the vibration of the plate. The actuator voltages are lower than the breakdown voltage of piezoelectric materials. Actuator voltages are shown in Fig. 8 and Fig. 9.

5.3 Third application: active control for different sizes of area piezoelectric patches

In this application, we consider the active control of the previous plate with various ad sizes of the patches.

The value of b_{ij} is given according to the size of the patches (16), and any change in value of these dimensions will change in the value of b_{ij}. And therefore, we will have different values for the voltages in each and actuators. The effectiveness of the control related to the values of b_{ij}.

The response displacements of the smart plate shown in Fig. 10 are different according to the size of the patch area.

The actuator voltages for cases 1, 2 and 3 are shown in Fig. 11 and Fig. 12.

The variation of surface values of the patches is very important to obtain good control. The cases studied in this article are verified this approach.

Table 2 The first six natural frequencies of the plate and the smart plate

	Natural frequencies (Hz)					
Mode (r,n)	(1,1)	(2,1)	(1,2)	(3,1)	(2,2)	(3,2)
Aluminum plate [10] (analytical)	10.628	22.107	31.035	41.238	42.513	61.644
Aluminum plate (ANSYS)	10.636	22.139	31.082	41.331	42.640	61.923
Smart plate (ANSYS)	10.499	21.769	30.632	40.637	41.857	60.656

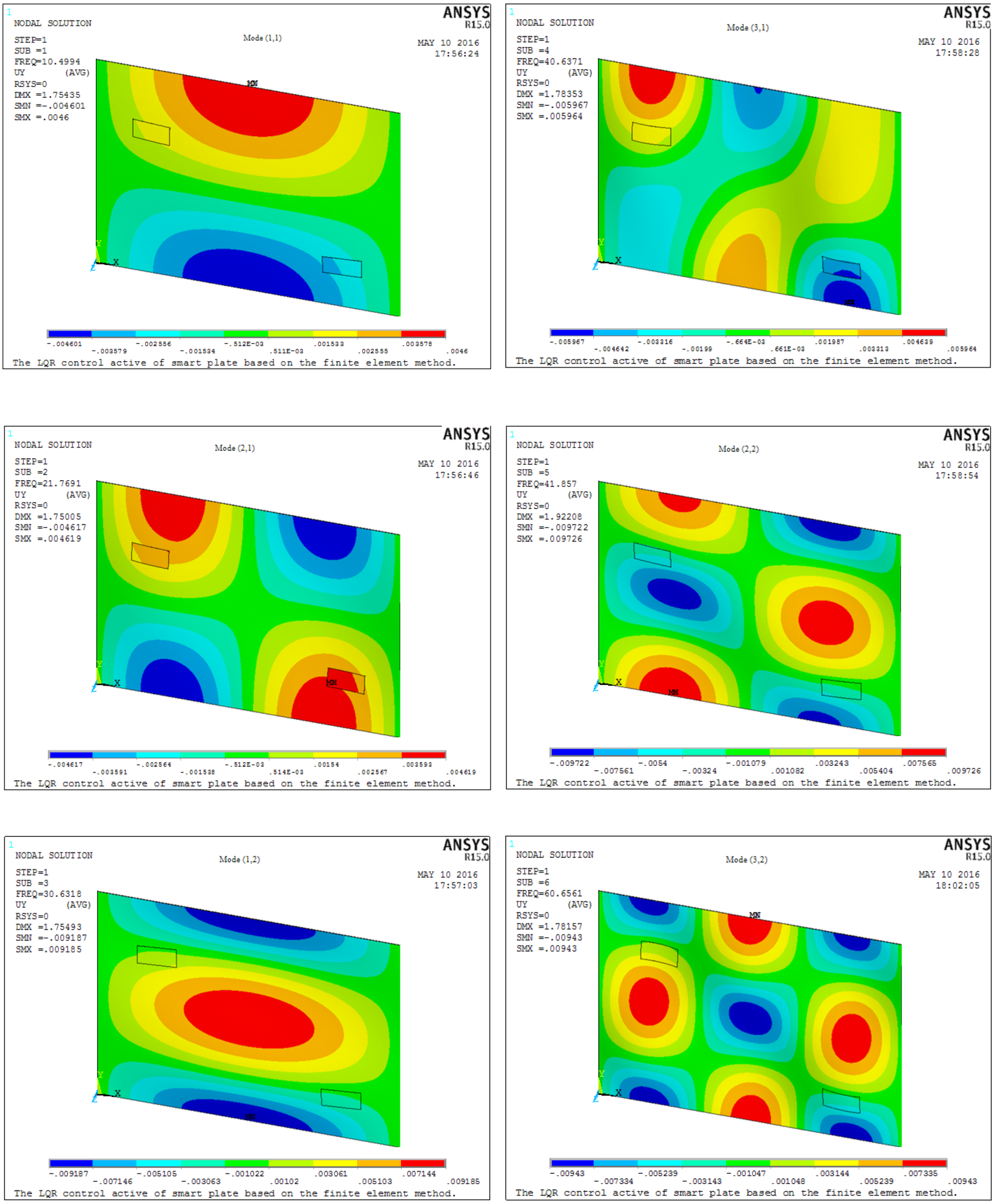

Fig. 5 The first six vibration modes of the simply supported smart plate.

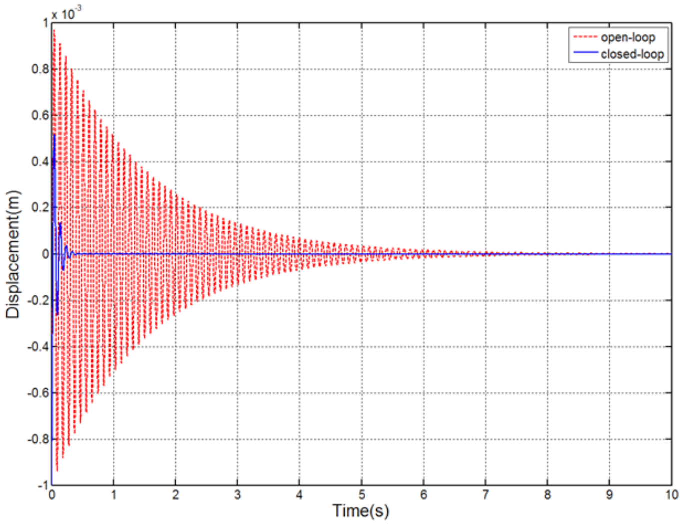

Fig. 6 Comparison of open-loop (no control) and the closed-loop (with control) displacement response of a simply supported smart plate with LQR controller.

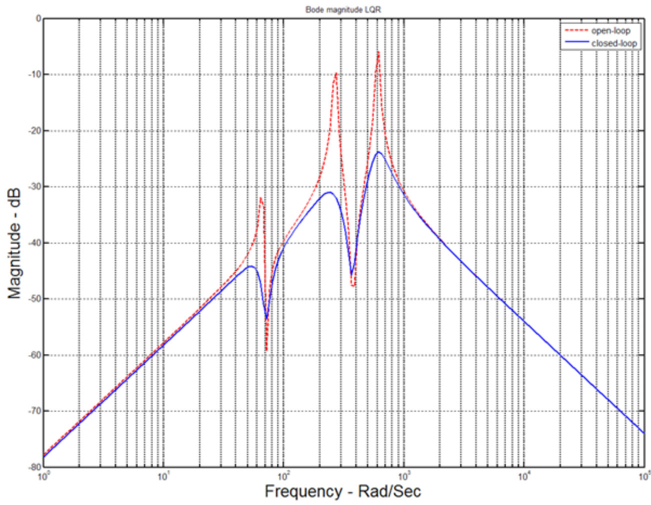

Fig. 7 Comparison of the Owen-loop and the closed-loop frequency response of a simply supported smart plate with LQR controller.

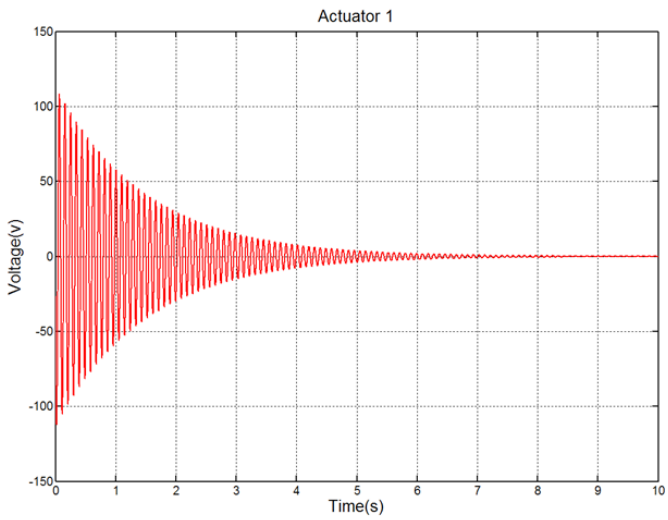

Fig. 8 Time history of actuator 1 voltage

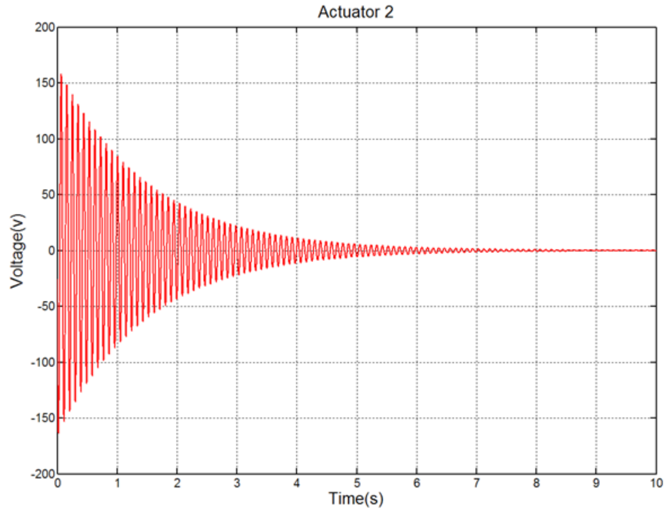

Fig. 9 Time history of actuator 2 voltage

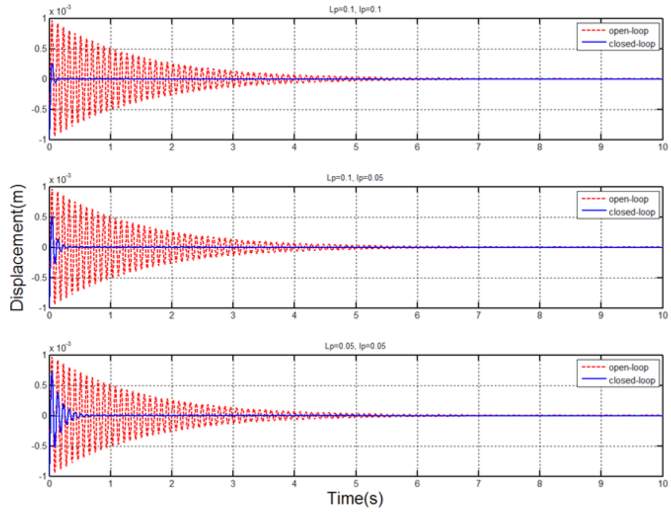

Fig. 10 Comparison of open-loop (no control) and the closed-loop (with control) displacement response of a simply supported smart plate with LQR controller for cases 1, 2 and 3.

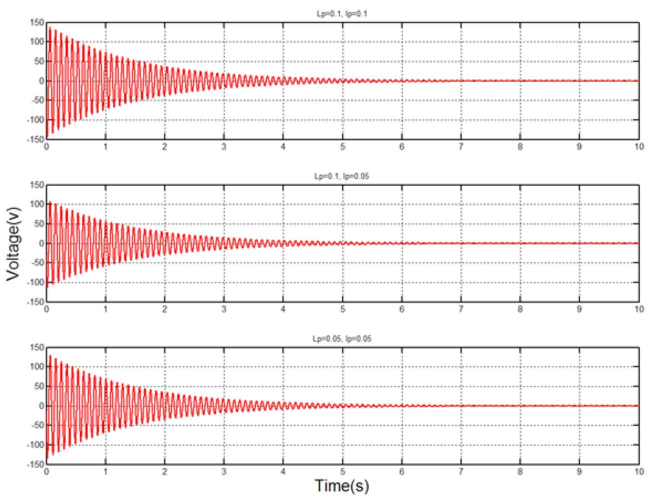

Fig. 11 Time history of actuator 1 voltage for cases 1, 2 and 3

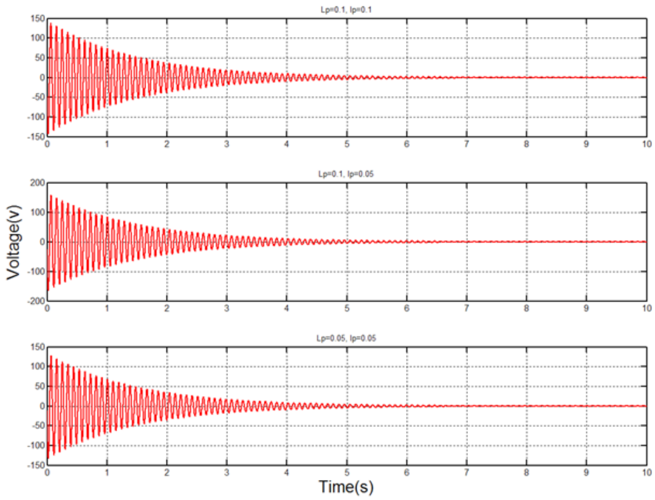

Fig. 12 Time history of actuator 2 voltage for cases 1, 2 and 3

6 Conclusion

This paper is concerned with the numerical modeling of discrete piezoelectric sensors and actuators for active modal control of a flexible simply supported rectangular plate, excited and sensed by rectangular actuators and sensors bonded symmetrically to both sides of the plate. Finite element model of the smart plate is constructed by an ANSYS APDL program.

A LQR controller is considered based on the independent mode control techniques to suppress the first three modes vibration of the system. Simulation results and the curves of open-loop and close-loop for different sizes of the piezoelectric patches are given by MATLAB demonstrate the effectiveness of the method in this paper.

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