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Application of Learning Curves in Operations Management Decisions

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Abstract
In the time of industry 4.0 and big data, methods which are based on the collection and the processing of a large amount of data in order to support managerial decisions have outstanding significance. The learning curve theory pertains to these methods. The purpose of this paper is to explore some application possibilities of the classical learning curve in manufacturing and service operations. The learning effect assumes that as the quantity of units manufactured increases, the time needed to produce an individual unit decreases. The function describing this phenomenon is the learning curve. Various learning curves have been developed and applied in the area of production economics and much research studies the significance of the learning effect in management decisions. This study summarizes the main learning curve models and demonstrates how learning can be considered in three classical areas of operations management. First, the calculation of economic manufacturing quantity in the presence of learning is studied. Next, the effect of learning in break-even analysis and assembly line balancing is explored. The results show that with the consideration of the learning effect, calculations become more complex and require greater efforts, but the application of the learning curve concept can provide valuable insight both at operational and strategic levels.

Keywords
learning curve, operations management, break-even analysis, economic production quantity, assembly line balancing

1 Introduction

Learning curves assume that performance improves as a task is repetitively performed. The application of the learning curve concept provides several benefits for firms. For example, learning curves can be applied to estimate the time needed to complete production runs when learning takes place, to set more accurate labor standards, to predict production output or to estimate the cost reduction in production costs due to the learning effect.

The first application of the learning curve phenomenon was reported by Wright (1936). The use of this concept began to gain importance during World War II when an accurate prediction of the time and cost of producing military ships and combat aircraft was needed (Yelle, 1979). Data collected during the war were used by some researchers to study Wright's learning curve in the aerospace industry (Alchian, 1950; Asher, 1956). After the war, private companies also started to use the learning curve and an increasing amount of research has come to light on the subject.

Since then, an extensive number of research studies have reported the use of learning curves in industrial applications and research settings. The organizational learning characteristics (Argote and Epple, 1990), scheduling problems (Biskup, 1999; Mosheiov, 2001), statistical process control (Yang et al., 2009), construction processes (Hinze and Olbina, 2009), online ordering systems (Kull et al., 2007), and manual order picking procedures (Grosse and Glock, 2013) are some examples of the application of learning curves in the area of operations management. Learning models extended for forgetting and relearning in the production process also pertain to this field (Davidovitch et al., 2008; Jaber and Bonney, 2003).

The objective of our paper is to review the relevant literature related to the development, improvement and application of learning curves, and to demonstrate the possible insight, which its application can provide in three special areas of production and operations management. The remainder of this paper is structured as follows. Section 2

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introduces the main learning curve models used in practice. Section 3 presents the calculation of economic manufacturing quantity (EMQ) in the presence of the learning effect. Section 4 discusses cost-volume-profit (CVP) analysis and profit and cost planning with learning considerations. Section 5 presents the effect of learning on the operation of simple assembly lines. Finally, Section 6 provides some general conclusions.

2 Learning curve models

There is a widespread literature on the different types of learning processes. General surveys concerning the learning models and their applications have been published for example by Yelle (1979), Anzanello and Fogliatto (2011) or Grosse et al. (2015). In this section, the most frequently used learning curve models are introduced.

2.1 Log-linear models

The *Wright model* is considered by the literature as the basic curve. Wright (1936) analyzed assembly processes in the aircraft industry, where he observed that as the quantity of units manufactured doubles, the time required to produce an individual unit decreases at a uniform rate. Wright's learning curve is formulated as follows,

$$Y(Q) = aQ^b, \tag{1}$$

where $Y(Q)$ is the cumulative average time (or cost) per unit required to produce Q units, a is the cost or time required to produce the first unit, Q is the cumulative number of units, and b is the slope of the learning curve. This learning curve is often referred to as the *cumulative average model*. Crawford (1944) defines $Y(Q)$ as the unit time (or cost) for the Q^{th} unit. For this reason, the Crawford approach is often referred to as the *incremental unit time* (or cost) *model*.

The range of b, in theory, runs from $-\infty$ to 0, but according to Keachie and Fontana (1966), b is never smaller than −0.6. The typical industrial values lie between −0.415 and −0.074, corresponding to 75-95 % learning rates (L). The relation of b and L is as follows,

$$b = log\, L / log\, 2. \tag{2}$$

If the learning rate is 80 %, then b equals to −0.322. Note that the smaller the learning rate is, the higher the progress ratio. It means that as the learning rate decreases in percentages, the unit manufacturing time or cost decreases as well.

Wright's model can be applied to describe the reduction in both time and costs, but it ignores a number of factors (Lolli et al., 2016). First of all, Wright's model is unrealistic if the cumulative volume of production tends to infinity, because it does not include any stabilization value (plateau effect). It suggests that total cost approaches zero as the volume of production approaches infinity, which is impossible for both time and costs. Secondly, Wright's learning curve is based on the assumption of defect-free conditions, that is, the operations are repeated frequently without disruption. Next, the negative value of b implies that only learning, that is the decrease of time, is possible, while forgetting, that is the increase of time, is excluded. Finally, Wright's model is not concerned with the experience from performing the same task previously.

Many versions of the learning curve have been proposed to overcome the shortcomings of the basic model. The models summarized in Table 1 are the most widely used curves in practice.

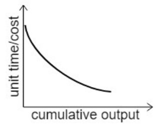
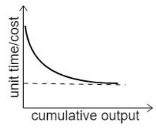
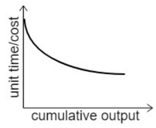
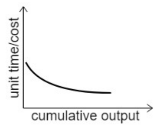
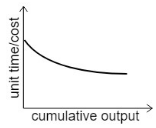
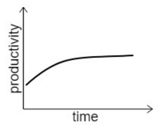
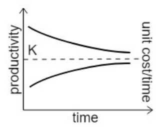
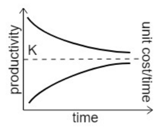
The *plateau model* included in Table 1 completes Wright's model with a constant C in order to overcome the problem of zero time/cost at large quantities. The constant C refers to the phenomenon of plateauing, which means that the learning effect is finite (Baloff, 1971). The unit cost/time can only decrease to Q_s (steady state level), which is followed by a steady state, where the unit cost/time is considered as constant.

De Jong's *model* makes a distinction between manual and machine controlled parts of the processes. While the manual operations are compressible, this is not true for the machine controlled operations. Production time is divided into two parts. One becomes shorter due to the learning effect, while the other remains constant. De Jong (1957) added a factor M to Wright's model, which represents the proportion of the "incompressible" component. The value of $M (0 \leq M \leq 1)$ depends on the degree of automation. When $M = 0$, the operation is completely manual, the model is equal to the basic Wright model, thus, Eq. (1) can be applied. According to Baloff (1971), plateauing is much more likely to occur in machine-intensive than in labor-intensive industries, due to the higher proportion of machine-paced labor.

The *Stanford-B model* completes Wright's basic model with prior work experience (Badiru, 1992). A parameter B is added to the function in order to express the accumulated knowledge. This parameter shifts the learning curve downwards.

The *S-curve model* is the combination of the Stanford-B model and the DeJong's model, and it uses parameters M and B as well (Carlson, 1973). This model is named after

Table 1 Summary of learning curve models (Grosse et al., 2015:p.404)

Model	Formula	Typical learning curve
Wright's model	$Y(Q) = aQ^b$	
Plateau model	$Y(Q) = \begin{cases} aQ^b & \text{if} \quad Q < Q_s \\ C & \text{if} \quad Q \geq Q_s \end{cases}$	
DeJong's model	$Y(Q) = a\left[M + (1-M)Q^b\right]$	
Stanford-B model	$Y(Q) = a(Q+B)^b$	
S-curve model	$Y(Q) = a\left[M + (1-M)(Q+B)^b\right]$	
Group learning curve	$Z(T) = \sum_{i=1}^{n} Y_i(T) + \sum_{i=1}^{n}\sum_{j=1}^{n} X_{ij}(T)$	
2-parameter hyperbolic model	$Y_t = K\left(\frac{t}{t+L}\right)$	
3- parameter hyperbolic model	$Y_t = K\left(\frac{t+p}{t+p+L}\right)$	

the shape of the learning curve when graphed on a double log paper.

2.2 Exponential models

The *group learning curve* is based on the assumption that individual skills are enhanced by the prior experience of others, when one works in a group, and therefore learning takes place sooner. In this model, $Z(T)$ represents the number of units produced by the group over time T; $Y_i(T)$ is the amount produced by individual i over time T; and $X_{ij}(T)$ is the amount produced by individual i over time T as a result of the knowledge transfer by individual j.

2.3 Hyperbolic models

The last two learning curves in Table 1 belong to the class of hyperbolic models. Mazur and Hastie (1978) proposed a learning curve model relating the number of conforming units to the total number of units produced. They also included the worker's prior experience in executing the task. Nembhard and Uzumeri (2000) improved the original parameters in these models and created the 2- and 3-parameter hyperbolic learning curve models. Hyperbolic learning curves, which have become particularly popular in recent years, can be widely used to display both the increase and the decrease of unit time.

In the 2-parameter hyperbolic model, $Y(t)$ is the number of units produced over time t, L denotes the learning rate, and K is the maximum output over time t without learning/forgetting. If $L=0$, no learning takes place, and $Y(t) = K$ for any t. If learning takes place, $L<0$, and K is multiplied by a number which is higher than 1. The increase in the production time per unit due to fatigue is expressed by $L>0$. In that case, K is multiplied by a number which is less than 1.

The 3-parameter hyperbolic model incorporates the prior experience of the workforce with parameter $p\,(p\geq 0)$.

As presented above, researchers have defined the form of the learning curve in many different ways. Although there has been much research conducted on the selection of the model type, Wright's basic learning curve is still the most widely used model and it is found to fit empirical data quite well (Jaber, 2006).

3 Calculation of the Economic Manufacturing Quantity with the learning effect

This section reviews the main literature and its results related to the calculation of Economic Manufacturing Quantity (EMQ) in the presence of the learning effect. The EMQ model determines the optimal manufacturing lot size in batch production, assuming that the production rate is constant. It means that the produced quantities are constant in each production period; furthermore, the set-up and unit variable manufacturing costs are constant. These assumptions, however, are not valid under certain circumstances, especially when working with a new workforce, with new products, with new technology, or with long production runs over an extended product lifecycle (Jaber and Bonney, 1999; Cheng, 1994; Keachie and Fontana, 1966).

Plenty of EMQ models which take into consideration the effect of learning can be found in the literature. Keachie and Fontana (1966) were among the first who applied the learning curve in EMQ calculations. They limited their study to the simple case when demand is known, there is no cost of shortage, and set-up costs are independent of the produced quantity. They assumed that the manufacturing quantity is so large that the learning effect could be encountered. According to their model, there is worker learning in processing times, but there is no worker learning in setups. Consequently, the set-up and the holding costs are not influenced by learning, only the unit manufacturing cost.

Keachie and Fontana (1966) calculated the cumulative average unit cost by Eq. (3) in the case of a lot size equal to n.

$$\overline{Y(Q)}_n = 1\Big/ n\sum_{Q=1}^{n} Y(Q) = a\Big/ n\sum_{Q=1}^{n} Q^b \,. \tag{3}$$

Approximating the value of the summa by integral, the average unit cost is calculated as follows,

$$\overline{Y(Q)}_n = 1/n\sum_{Q=1}^{n} Y(Q) = a/n\sum_{Q=1}^{n} Q^b \,. \tag{4}$$

Fig. 1 shows that at the beginning of each manufacturing period, the learning curve starts with the first unit costing a. This amount decreases by a constant slope (b) in a log scale in every period. It can be seen that longer manufacturing periods have a significant advantage on profit because of the cost decrease. Longer reorder intervals lead to smaller average unit cost – if n is higher, n^b is smaller ($b < 0$) – which results in smaller manufacturing and total costs.

In the Keachie-Fontana model, the total cost function is formulated as follows,

$$TC(Q) = AD/Q + (Q/2)vr + \left[a/(b+1)\right]D/Q^{-b} \,, \tag{5}$$

where A denotes set-up cost, D/Q is the number of set-ups (D – demand), v is the unit purchasing cost and r is the inventory holding rate. The first part of the equation refers to the set-up costs, the middle part refers to the holding costs and the last part represents the manufacturing costs influenced by learning. The optimal lot size (Q^*) can be calculated by taking the derivative of Eq. (5) and equating it with zero.

Steedman (1970) examined the results from the presented method and proved that the optimal lot size Q^* is always larger than the lot size calculated by the classical EMQ model (Q_0). It was also observed that the optimal lot size decreases as the negative b parameter increases, which means that the higher the learning rate, the less it is worth applying large lot sizes.

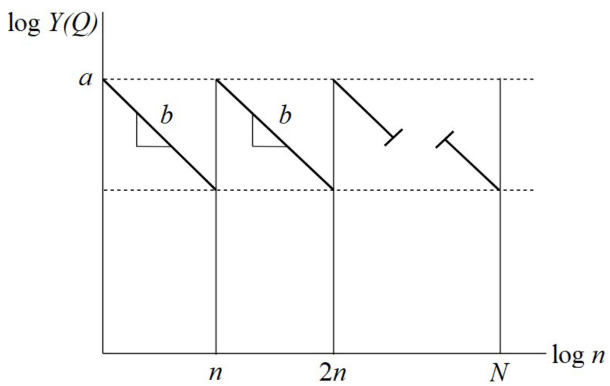


Fig. 1 Effect of learning on optimal lot size (Keachie and Fontana, 1966:p.105)

Wortham and Mayyasi (1972) recommended the application of the classical square-root formula for the calculation of the optimal lot size, but instead of a constant holding cost, a lower average cost should be used.

Muth and Spremann (1983) suggested some additions to the findings above. First, they claimed that the manufacturing cost consists of two components; one of them is affected by the learning curve, the other is linear. Next, they assumed that the Q^*/Q_0 ratio is a function of two parameters: the progress rate and the cost ratio. Finally, they proposed a simple approximation formula for Q^*.

Chand (1989) analyzed the effect of learning on the lot sizes and on the setup frequency. He assumed learning in the case of the setup time and the process quality, but he ignored learning in the case of the processing time. He concludes that learning in setups increases the setup frequency and reduces the total cost. The effect can be significant for companies where the production rate and the cost of error are high. He also concludes that the effect of learning in process quality is not significant. These results support the theory of zero inventory systems and the just-in-time approach.

Cheng (1994) compared the optimal solutions obtained with equal and unequal manufacturing sizes. His results indicate that the application of the classical EMQ model simplifies the process and provides close approximations to the optimal solutions.

Applying algorithms which incorporate the learning effect in EMQ calculations requires extra computational efforts compared to the classical EMQ formula. The main conclusion is, however, that the classical model can serve as a very good approximation even in the case of learning.

4 Profit and cost planning with learning considerations

In Section 4 the effect of learning on cost-volume-profit analysis and production cost planning is presented.

4.1 CVP analysis with the learning curve

The cost-volume-profit (CVP) analysis is a useful tool for planning and monitoring different managerial decisions and their effects. It is used to determine how changes in costs, production quantity, selling price, product mix and other related factors influence a company's operating and net profit (Jaedicke and Robichek, 1964; Adar et al., 1977; Magee, 1975).

The traditional CVP analysis applies several assumptions, including, for example, that the unit selling price, the unit variable cost and the total fixed costs are constant. Consequently, the classical CVP analysis applies linear cost and revenue functions. When production involves labor intensive new products or new technology, the learning effect becomes a factor in the analysis, and linearity cannot be assumed any more.

Fig. 2 shows the traditional CVP diagram with linear functions. Quantity produced below the break-even point – the point at which total costs equal total revenue – results in net loss, while quantity produced above this point results in net profit. Fig. 3 shows how the CVP diagram changes when learning is introduced into the analysis. The variable cost function is degressive, because the slope of this function (unit variable cost) decreases as a consequence of learning.

As a result of the degressive cost function, break-even output becomes lower than in the case of the traditional model. Since the break-even point in the non-linear case is reached sooner, the expected profit is larger.

McIntyre (1977) developed a model for CVP analysis, which incorporates a nonlinear cost function in order to express the learning effect. In his paper, he examined the effects of learning on the break-even equation and he performed a sensitivity analysis on the estimated profit and break-even quantities when estimation errors occur.

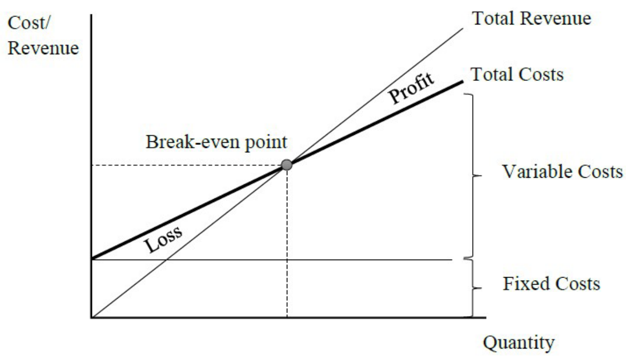


Fig. 2 Traditional CVP diagram

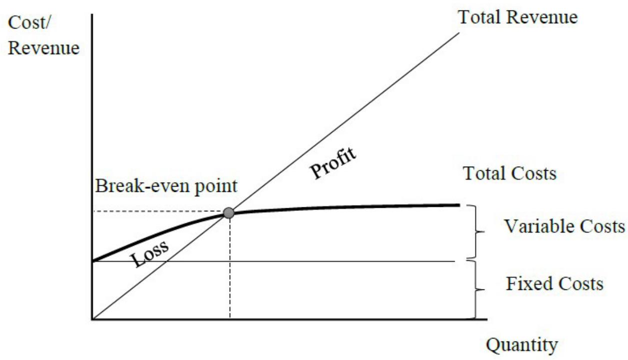


Fig. 3 CVP diagram with nonlinear cost function

The analysis uses the cumulative average time model to consider learning. It is formulated as the basic Wright learning curve, that is,

$$Y(Q) = aQ^b, \tag{6}$$

where $Y(Q)$ refers to the cumulative average unit production time if Q units have been produced.

The basic profit equation can be written as follows,

$$\pi = pQ - cQY(Q) - f, \tag{7}$$

where p is the unit selling price reduced by all variable costs other than labor costs, c is the unit labor costs and f is the fixed costs per period. Substituting Eq. (6) in Eq. (7) and assuming parallel production runs, we get the following multiprocess profit function,

$$\pi = pQ - nca(Q/n)^{b+1} - f, \tag{8}$$

where Q units are produced by n labor teams, and each team produces Q/n units.

McIntyre (1977) extended Eq. (8) for the plateau model and determined the steady-state production conditions. He concluded that it is possible to estimate the number of units required to reach the steady-state production (Q_s) with the following formula,

$$Q_s = \left[t_s/(b+1)a\right]^{1/b}, \tag{9}$$

where t_s represents the steady-state marginal time and is calculated as follows,

$$t_s = (b+1)aQ_s^{\ b}. \tag{10}$$

Using the estimates of t_s and Q_s, the cumulative profit is formulated as follows,

$$\pi = pQ - ncT_s - ct_s(Q - nQ_s) - f, \tag{11}$$

where T_s is the total production time for the first Q_s units and it is calculated as follows,

$$T_s = aQ_s^{\ b+1}. \tag{12}$$

Equation (11) shows that the cumulative profit for $Q \geq nQ_s$ can be expressed as the difference of the total revenue and the total costs, where total costs consist of the total labor cost of the first nQ_s units, the total labor cost of all units over the first nQ_s units and the fixed costs.

The general solution of McIntyre (1977) to break-even (Q_B) under these conditions can be calculated as follows,

$$Q_B = \left[cn(T_s - t_sQ_s) + f\right]/(p - ct_s). \tag{13}$$

McIntyre's results extend the application possibility of CVP analysis for cases where learning influences the unit variable cost.

4.2 Competitiveness and learning

An important consequence of learning is the change of competitiveness. This section reviews some relevant research results dealing with the connection of learning and competitiveness.

Harvey (1976) was among the first who examined the effects of learning on production start-ups. In his study, the cumulative average model and the incremental unit model were used. He extended these models for the analysis of profit and cash flow planning in start-ups. Harvey (1976) divided total cost into three major components, direct material costs, direct labor costs and fixed costs of production facilities (rent, heat, light, etc.). After this division, he created the formula of profit and net operating cash flow.

Majd and Pindyck (1989) examined the effects of the learning curve on production leveling and pricing. They showed how a firm’s current production decision can be determined in order to be consistent with financial objectives. They noted that learning affects production in such a way that part of the production cost converts into an investment in reduced future costs.

Spence (1981) analyzed competitive interaction and showed how the presence of a learning curve can be taken into account when setting price and output levels. He declared that the presence of the learning curve in an industry can create entry barriers and protection against cost reduction based competition.

Morse (1972) studied the effect of learning on production costs. First, he described the production process assuming two products, a physical product and an intangible product. The intangible product refers to the learning phenomenon, which helps to produce additional units with lower production costs. Next, he illustrated a learning curve (L-C) cost allocation model, which can be used to project the production costs of a product to its entire anticipated life cycle. After the comparison of the actual cost model and the L-C cost allocation model, the study concluded that the second model considers the cost reducing value of the production know-how or the intangible product.

Lolli et al. (2016) introduced a new accounting model for production cost in a single-product case. They proposed a cost curve which incorporates both the learning and the forgetting phenomenon during production and reworking operations. The total cost function in their model is composed of four parts: production cost, failure cost, prevention cost and appraisal cost. They noted that the cost model is suitable only for firms which have the ability of high level data collection.

Petrakis et al. (1997) created a model to examine the relationship between perfect competition and the learning effect. They showed that these two concepts are compatible. Their results also showed that in the presence of learning, firms must enter an industry early or they will never be competitive. Furthermore, some firms who entered earlier may be forced to exit the market despite their experience.

While the study of Petrakis et al. (1997) excluded the possibility of learning spillovers and considered only firm-specific learning, Lieberman (1987) examined learning when diffusion is possible. Lieberman (1987) used a game-theoretic model in order to explore how the learning curve affects competitive strategy. He concluded that information diffusion plays a very important role in the process of developing competitiveness with the help of the learning effect. The study noted that firms can develop their competitiveness even with late entry, as entry barriers become more eroded as the diffusion of learning increases.

Based on the reviewed literature we can conclude that learning curves affect many aspects of a firm's competitiveness. Consequently, the learning effect should not be missed when important managerial decisions have to be made about pricing policy, cost or profit planning.

5 Assembly line balancing with learning curves

The assembly line balancing (ALB) problem was first studied by Salveson (1955). Assembly lines are flow-oriented production systems. The objective of ALB is to balance workloads of workstations, while meeting the required production rate and to satisfy technological conditions. An assembly line consists of several consecutive workstations, where operators perform the same tasks repetitively. Each task is associated with a processing time, called task time. The parts are launched down the line and are moved on from station to station until they are finished. The difference between the completion time of two consecutive parts is called the cycle time. Due to technological or organizational requirements, tasks assigned to workstations are determined by precedence constraints. General input information of an assembly line can be structured into a precedence graph, where each node represents a task and the node weights refer to the task times.

Classical ALB models assume that task times are independent of the produced quantity. As a consequence of the repeated performance of several identical tasks at a workstation, however, learning in most cases is present. In the case of the presence of learning, task times and station times decrease.

The literature on the effect of learning on ALB is not very extended. Globerson and Shtub (1984) examined the possibilities of the minimization of makespan in assembly lines with long cycle times. They developed an 11-stage long model to identify the optimum equilibrium point (referring to the number of repetitions), in which the line utilization reaches its maximum value.

Cohen and Dar-El (1998) formulated several nonlinear mathematical programming models which determine the optimal number of workstations in an assembly in the presence of learning. They discussed two approaches, a cost minimization problem and a profit maximization problem, and they developed a formula for finding the optimal number of stations.

Cohen et al. (2006) defined the optimal upper envelope of the learning curves of the stations of an assembly line. They formulated a non-linear mathematical programming model for work allocation under homogeneous learning and they proved that balancing in not an optimal policy. They showed that the decrease of the ratio of the makespan of the optimal allocation of work and the makespan of the balanced allocation are higher if the number of stations is higher, the learning rate is lower and the lot size is higher.

Cohen et al. (2008) analyzed decreasing, increasing and constant pattern stations' learning in order to find the optimum allocation of work to the stations of an assembly line. They presented optimal solutions to all three cases.

Toksarı et al. (2008) illustrated the operation of a simple assembly line balancing problem and a U-type line balancing problem with the learning effect. They proposed heuristics for the minimization of the total flow time and tested the proposed algorithm.

Koltai et al. (2015) presented an algorithm to determine the throughput time of a simple assembly line in the presence of learning. They demonstrated that while classical ALB models consider constant cycle time, in the case of learning, cycle time can change for two main reasons. First, cycle time decreases exponentially according to the station time function with learning. Second, the bottleneck may shift from station to station, causing further changes in the cycle time.

Let $s_j(Q)$ denote the station time of station j at the Qth performance of the operations. Fig. 4 shows the station time functions of station l and k, where k is a preceding station of station l. At the intersection of the two functions ($Q_l(k,l)$), the bottleneck shifts from one station to another station. At this quantity station l leaves the bottleneck and station k enters the bottleneck.

The formula for calculating the throughput time in a simple assembly line in the presence of learning can easily be calculated, if the bottleneck shifts are known. The algorithm is based on the selection of the relevant station time function intersections (see Koltai et al., 2015).

Koltai and Kalló (2017) analyzed the sensitivity of the throughput time with respect to the learning rate in simple assembly lines. They concluded that if the learning rate decreases, the production quantity belonging to bottleneck shifts increases. It means that if learning is underestimated, bottleneck shifts may occur more frequently than expected. Second, throughput time is very sensitive to learning rate changes, especially in cases with small quantities or high learning rates.

We may conclude that if learning is present in simple assembly lines, the revision of the traditional assembly line balancing methods is required. Balancing does not necessarily provide an optimal solution to capacity related problems.

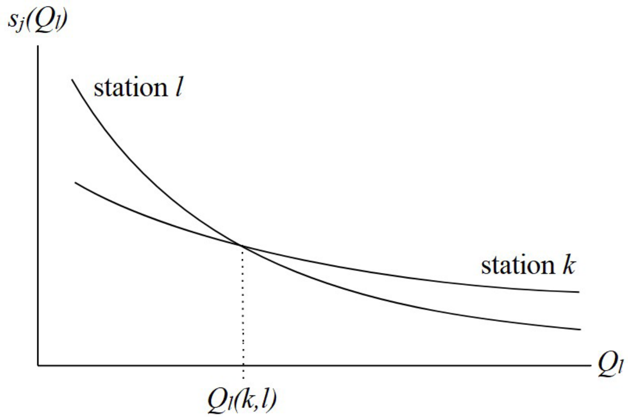


Fig. 4 Illustration of bottleneck shift (Koltai et al., 2015:p.316)

6 Conclusion

Nowadays, we are living in the world of industry 4.0 and big data, which causes various changes in production and service processes. It is shown, for example in Belvedere et al. (2018) or Cavata et al. (2018), that industry 4.0 technologies have positive impact on a firm's performance and the use of such technologies can lead to a significant increase in productivity.

In the time of these new manufacturing approaches, methods which are based on the collection and processing large amounts of data in order to support managerial decisions have outstanding relevance. The learning curve theory pertains to these methods, because detailed information about operation times is required, advanced statistical methods are used, and the results are embedded in complex mathematical models. The advanced information technology environment of today makes the application of learning theory more feasible than ever.

In this paper relevant literature related to the application of learning in three major areas of production and operations management was explored.

The consideration of the learning effect in the traditional area of inventory management showed that the classical EMQ formula becomes very complicated and its result can be considered a very good approximation for the learning curve case.

The consideration of the learning effect in traditional break-even analysis showed that if learning is ignored, financial possibilities are considerably underestimated; furthermore, opportunities of competitiveness are overlooked.

Finally, in the case of the presence of learning in assembly lines the traditional balancing principles are not relevant anymore, and a new approach is required to determine the optimal operation.

As a summary it can be concluded that if learning is present, its effect should be analyzed. Several traditional operations management models can be completed with the consideration of learning; furthermore, new areas of its application can be found. The advanced information technology environment requires as well as facilitates the application of the learning curve theory more than ever.

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