https://doi.org/10.3311/PPso.14313

Periodica Polytechnica Social and Management Sciences, 29(2), pp. 136–144, 2021

Shortcomings of NPV Calculation: Does One Error Annul the Other?

Nikolett Sereg[1*]

[1] Department of Finance, Faculty of Economic and Social Sciences, Budapest University of Technology and Economics, H-1117 Budapest, Magyar Tudósok krt. 2, Hungary
[*] Corresponding author, e-mail: sereg@finance.bme.hu

Received: 06 May 2019, Accepted: 21 June 2020, Published online: 04 February 2021

Abstract

The research aims at analyzing the aggregate effect of possible errors related to the "textbook" method of present value calculation. Two main errors could stem from end-of-period convention and calculation according to expected lifespan. The magnitude of such errors depends on the cash flow pattern and the probability distribution of asset life, therefore the combination that may be regarded as the most typical in practice has been chosen as the subject of the examination, i.e., the continuous exponential cash flow pattern with exponentially distributed life. Based on the result of previous studies examining the errors separately, it seems possible that the two errors lead to a more accurate approximation – considering the absolute value of the relative error – compared to making only one of the errors. After the examination, I conclude that in the most typical cases of practice, it is not worth to take either the correct cash flow pattern or the life uncertainty into account beyond the "textbook" method.

Keywords

present value calculation, end-of-period convention, uncertainty of the asset life, relative error

1 Introduction

There are several approaches to value cash flows generated by a financial instrument, i.e. an asset. This research will put under the microscope the Discounted Cash Flow (DCF) valuation, which is based on present value calculation. The essence of the present value calculation is that the value of an asset can be determined by its future expected cash flows discounted to the present, considering the cost of capital as a discount rate, which reflects the riskiness of the asset. There are two main ways of the present value calculation: discounting discrete cash flows occurring at specified times with a discrete discount factor for a period with given length, or discounting cash flows in continuous time with a continuous discount factor for a given period (Eschenbach, 2011; Park and Sharp-Bette, 1990; Remer et al., 1984). In this paper, only the continuous case will be considered, but with consistent inputs both ways lead to equivalent results.

There is a well-known form of present value calculation, that is referred to further on as the "textbook" method. According to the "textbook" method, the following inputs are needed to calculate the present value of an asset: length of the valuation period (usually a year); the expected life of the valued asset (in the same unit as the valuation period); the estimated cash flows aggregated to the end of each period; and the value of the estimated discount factor. These are mostly simplifying assumptions, which make the calculation easier, but also less accurate. The inaccuracy of the "textbook" method, i.e., its error could stem from, for example: the end-of-period convention (i.e., cash flows are aggregated to the end of the period in which they occur); calculation according to expected lifespan (i.e., neglecting the uncertainty of the asset's economic life); estimation errors in the expected cash flows and the discount rate, etc. Following related research (e.g., Fleischer et al., 1998; Lohmann and Oakford, 1984), the relative error will be applied to measure the inaccuracy of a method. It gives a better result than the absolute error used by Chen and Manes (1986), since the relative error has no dimension (i.e. does not depend on the volume of the cash flows). The relative error was first used by Andor and Dülk (2013a) to characterize the deviation between the approximate and the accurate present value for uncertain asset life.

Cite this article as: Sereg, N. (2021) "Shortcomings of NPV Calculation: Does One Error Annul the Other?", Periodica Polytechnica Social and Management Sciences, 29(2), pp. 136–144. https://doi.org/10.3311/PPso.14313

It is also important, that the magnitude of the error depends on the cash flow pattern and the probability distribution of life of the asset, therefore the combination that may be regarded as the most typical in practice, i.e., continuous exponential cash flow pattern with exponentially distributed life is the subject of the examination. The possible correlations between the cash flows and the lifespan will not be considered in this research, but Van Horne (1972) dealt with its numerical approach in discrete time.

The relative error is defined as the deviation of the ratio of the estimated and the theoretically accurate present value from one; in this case, its value depends on three parameters: the growth rate, the discount rate and the expected life of the asset. As a preliminary point, it is important to point out, that some restrictions must be made regarding these parameters in order to get mathematically appropriate and interpretable results, the details of which will be discussed later.

Based on previous studies examining the errors separately, it can be established for the above-mentioned pattern-life combination that, for non-negative discount rates exceeding the growth rate, calculation according to expected life leads to overestimation (Andor and Dülk, 2013a), while in general for non-negative discount rates the end-of-period convention leads to underestimation (Andor and Dülk, 2013b) of the present value of the asset. The idea behind this research is that making the two errors at the same time (i.e., in the case of the "textbook" method) may result in a more accurate approximation – considering the absolute value of the relative error – compared to making only one of the errors.

For all these reasons, the research aims at analyzing the aggregate effect of errors from the end-of-period convention and calculation according to expected life. In addition to the "textbook" method, two other methods will be examined, which individually dissolve its simplifying assumptions, i.e., the correct cash flow pattern method and the probability weighted cash flow method. After determining the basic analytical equations of the present values and the relative error of each method, I compare the absolute value of these relative errors to each other to decide whether the well-known "textbook" method of present value calculation gives a reasonably accurate estimation considering its simplicity or we should calculate the present value of an asset in a more complex way to get a better result.

Based on the results of the comparison, I conclude that in the most typical cases of practice, it is not worth to take either the correct cash flow pattern or the life uncertainty into account beyond the "textbook" method.

2 Methods and their relative errors

The main question is, should we calculate the present value of an asset in a more complex way and possibly get a better approximation, or does the "textbook" method give a reasonably accurate result, considering its simplicity? To decide this, we should compare the "textbook" method to other methods where some of its simplifying assumptions have been resolved. The inaccuracy of a method can be measured with the relative error (ε) which is defined as the deviation of the ratio of the estimated ($\hat{P}$) and the theoretically accurate present value (P_a) from one, such as:

$$\varepsilon = \hat{P}/P_a - 1. \tag{1}$$

When comparing the methods, the absolute value of the relative error will be mostly considered, because only the magnitude of the error, i.e., the absolute value of the relative difference compared to the theoretically accurate present value is important, its sign is not. The magnitude of error depends on the cash flow pattern and the probability distribution of the life of the asset.

Fig. 1 illustrates the continuous exponential cash flow pattern, $\left[F(t) = Ce^{j}t\right]$ with exponentially distributed lifespan, $\left[f(T) = \lambda e^{-\lambda T}\right]$ which is in the focus of this research.

The theoretically accurate present value (P_a) can be calculated analytically in case of the assumed cash flow pattern and lifespan combination (e.g., Zinn et al., 1977) as:

$$P_a = \int_0^\infty E\left[F(t)\right]e^{-rt}\mathrm{d}t = C/\left(-j+\lambda+r\right), \tag{2}$$

where C is a constant cash flow parameter, j is the continuous growth rate, r is the continuous discount factor, t is the time and λ is the parameter of the exponential distribution (i.e.,

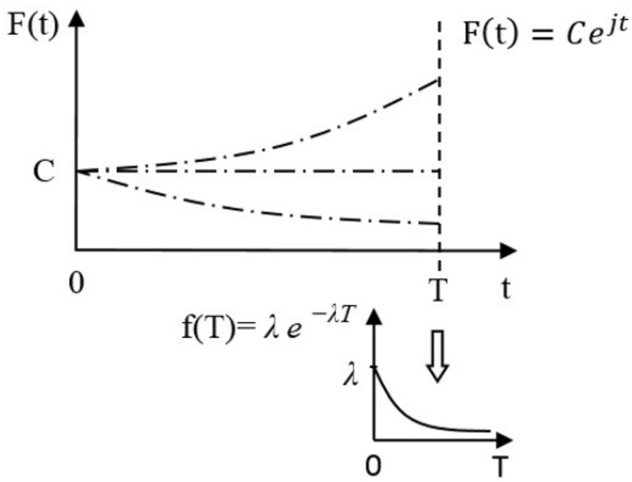

Fig. 1 Continuous exponential cash flow pattern (dash-dotted lines illustrate the possibility of a positive, negative or zero growth) with exponentially distributed lifespan (t[year])

$1/E(T)$ where $E(T)$ is the expected life of the asset). The calculation of the theoretically accurate present value leads to a conditional expression, which means that there is a convergence criterion, $j < (\lambda + r)$ and it must hold further on, or else the improper integral would be divergent, i.e., the present value would be infinite, in other terms it would not exist. I will only examine the interval of the parameters where this inequality holds, because the theoretically accurate present value is the basis of the relative error, which is used to the comparison of the methods, therefore it must exist.

The estimated present values of the methods are also needed to be able to calculate the relative errors. In the following, I will show the basic analytical equation to calculate the present values, starting with the "textbook" method and then the other two methods, which individually dissolve the two main simplifying assumptions of the "textbook" method, such as the end-of-period convention (i.e., cash flows are aggregated to the end of the period in which they occur) and the calculation according to expected life-span (i.e., neglecting the uncertainty of the asset's economic life). Further on, the second one is called correct cash flow pattern method and the third one is called probability weighted cash flow method.

The analytical estimation of the present value of the "textbook" method ($\hat{P}_t$) is the following:

$$\hat{P}_t = \sum_{t=1}^{E(T)} \left(\int_{t-1}^{t} Ce^{jt} \mathrm{d}t \right) e^{-rt}. \tag{3}$$

The second case is calculation according to the correct cash flow pattern ($\hat{P}_p$):

$$\hat{P}_p = \int_{0}^{E(T)} Ce^{jt} e^{-rt} \mathrm{d}t. \tag{4}$$

The third case, i.e., the probability weighted cash flow method ($\hat{P}_w$) is where the end-of-period cash flows are weighted by the period ends' probability of occurrence associated with the life uncertainty, thereby approximately taking into account second and higher moments of the distribution of the lifespan. It is worth noting, that this method has the same logic as the calculation of the theoretically accurate present value, and it can be easily implemented in Excel by simple multiplication.

$$\hat{P}_w = \sum_{t=1}^{E(T)} \left(\int_{t-1}^{t} Ce^{jt} \mathrm{d}t \right) \times e^{-\lambda t} \times e^{-rt} \tag{5}$$

The relative error depends on three parameters: the growth rate (j), the discount rate (r) and the expected life of the asset ($E(T) = 1/\lambda$). The examined interval of the growth rate is between 0 % and 20 %, the examined interval of the discount rate is between 0 % and 30 % and the examined expected lifespan is 5, 10, 20 and 40 years (sometimes only 5, 10 and 20 years will be presented graphically). Negative growth rate would be a possibility in the real world, but it is assumed that every corporation strives to generate increasing or at least non-decreasing cash flows from time to time. Andor and Dülk (2015) show in their study that a negative discount rate is also a possibility through negative betas in the Capital Asset Pricing Model, for example in case of energy efficiency projects, but I settle with covering the cases that are the most typical in practice. A negative discount rate can be a consequence of a negative risk-free rate too, which nowadays is quite common, but mostly the negative risk-free rate is increased by a larger positive risk premium, thus the discount rate is typically positive. I also maximize the examined interval of the discount rate at life-like 30 % (in related studies (e.g., Andor and Dülk, 2013b; Horvath, 1995; Lawrence, 2009) the maximum was mostly 50 %).

Some formula resulting from the above-mentioned analytical equations cannot be interpreted when $j = 0$ and $j = r$, so these subcases need to be examined in the case of every method in order to get a fully consistent comparison. In the following, I will analyze the relative error of the methods in different subgroups, first when the growth rate is not equal to the discount rate and not equal to zero, then when the growth rate is equal to zero and finally when the growth rate is equal to the non-zero discount rate.

Fig. 2 shows the plot of the relative errors in cases when the expected lifespan is 5, 10, 20, 40 years and when the growth rate is not equal to the discount rate and not equal to zero. There are shades of two colors, blue and red on Fig. 2; shades of blue are used when the method is underestimating the theoretically accurate present value, i.e., the relative error is negative and shades of red when the method is overestimating, i.e., the relative error is positive, respectively. The scale of the error can be seen at the bottom of the figure, it starts with the darkest blue, when underestimation is more than 50 % and it ends with the darkest red when overestimation is more than 25 %. However, the white areas on the plots refer to the violation of the convergence criterion, in this case, when $j < (\lambda + r)$.

Fig. 2 is mostly useful to graphically illustrate the nature of the relative errors over the lifespan. The correct cash flow pattern method, which measure the error of the calculation according to expected life was examined by Andor and Dülk (2013a) and they established

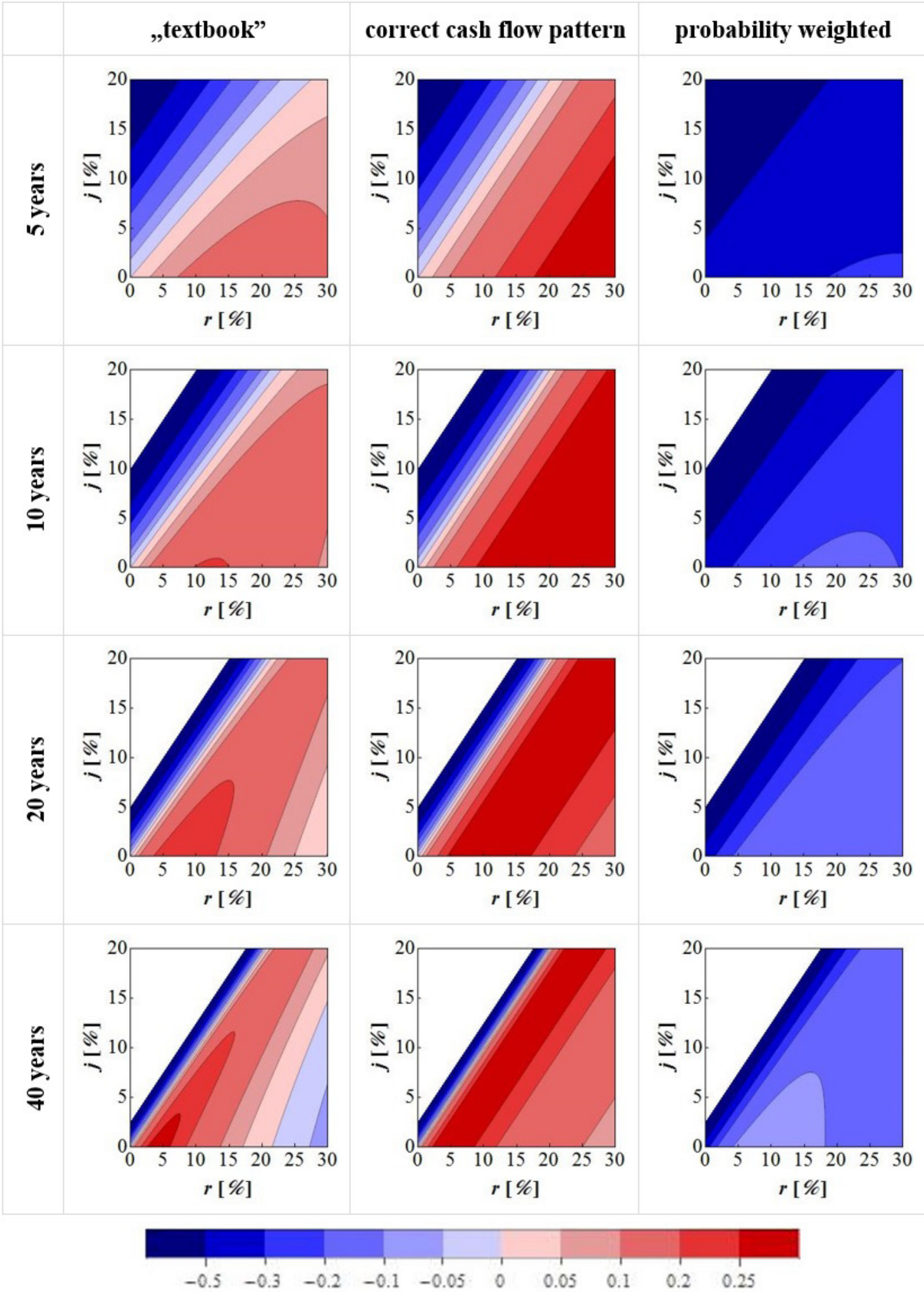

Fig. 2 Relative errors of the methods when the expected life is 5, 10, 20 and 40 years (colored intentionally)

that based on the Jensen-inequality, the method generates underestimation when the convergence criterion holds, and the growth rate is greater than the discount rate, while it generates overestimation when the growth rate is smaller than the discount rate. Jensen-inequality means, in the framework of probability calculation, that when an X variate and a φ convex function exists, then $\varphi(E[X]) \leq E[\varphi(X)]$ is true. They made their plot of this method using the absolute value of the error and with different range of the intervals, but recreation was necessary to illustrate the difference between the nature of relative errors in this paper.

The underestimation generated by the probability weighted cash flow method is decreasing with increasing lifespan (ceteris paribus), just as expected, since the increasing uncertainty in the asset's life is now so to speak considered in the present value calculation. This view is reflected in Andor and Dülk (2013a) studies, where they used Taylor's linear approximation to conclude that the difference between the theoretically accurate present value and the present value calculated according to expected life is increasing with the increasing variance of the expected life, which variance is equal to the square of the expected value in case of exponential distribution.

Naturally there is a different relative error when the growth rate is equal to zero. In this case the relative error depends only on the other two parameters, the discount rate and the expected life of the asset. Fig. 3 shows the absolute value of the relative errors when the growth rate is equal to zero. The scale at the bottom of the figure shows which type of line belongs to which expected life. I also recreated the plot which belongs to the correct cash flow pattern method examined by Andor and Dülk (2013a) to illustrate the difference between the errors. As it can be seen on Fig. 3, there is a local maximum in the case of each expected lifespan, and its level is greater when the expected life of the asset is longer in the case of the "textbook" and the correct cash flow pattern methods, while it is smaller in case of the probability weighted cash flow method. The local maximum on the examined interval is 27.12 % in the case of he "textbook" method and 29.84 % in the case of the correct cash flow pattern method, when the expected life is 40 years, while the probability weighted cash flow method has a local maximum when the expected life is 2 years and it is more than 50 %.

Finally, Fig. 4 shows the absolute value of the relative errors, when the growth rate is equal to the discount rate. The relative error of the "textbook" method only depends on the discount rate, which means that it looks the same in the case of each expected lifespan and it has a maximum value, 13.606 % at the end of the examined interval, when the discount rate is 30 %. In Fig. 4 it is marked with the black thick line. The relative error of probability weighted cash flow pattern method depends on the discount rate and the expected life, the magnitude of the error is seemingly high (the minimum of the error is more than 36.7 %) in the case of every expected lifespan and increasing when the discount rate is increasing. In Fig. 4, the different types of gray lines correspond to the various expected lifetime of the asset in the case of the probability weighted cash flow method. The correct

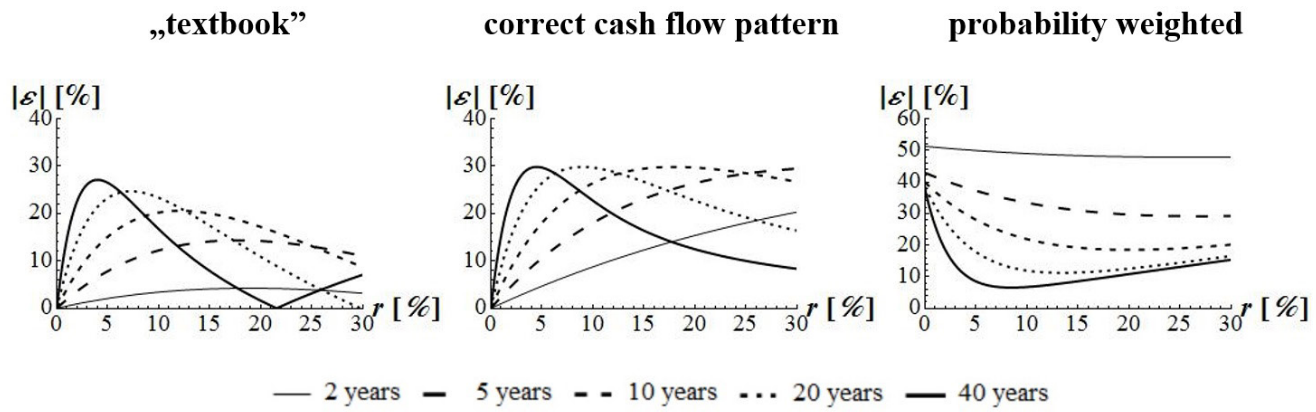

Fig. 3 Relative errors of the methods in absolute sense when the growth rate is equal to zero

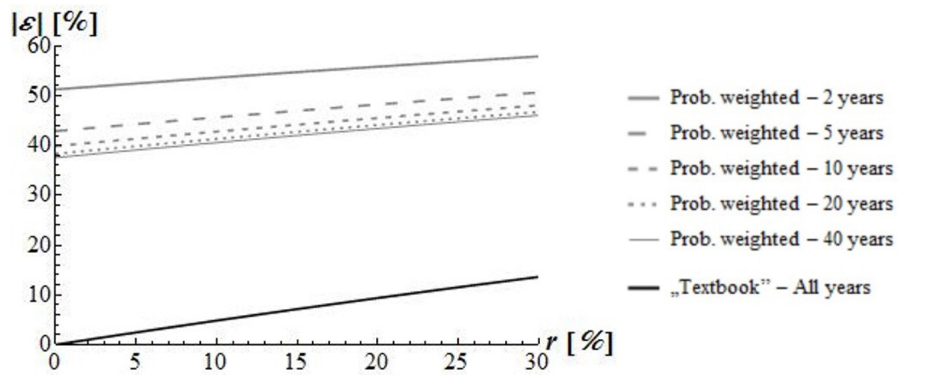

Fig. 4 Absolute value of the relative errors of the methods when the growth rate is equal to the discount rate

cash flow pattern method is equal to the theoretically accurate present value in this case; thus, it has no error as it was stated by Andor and Dülk (2013a) as well.

3 Comparison and results

The goal is to see how the relative errors behave compared to each other on the examined intervals of the parameters and decide whether the "textbook" method is sufficient or one of the other methods gives considerably better result. I define the deviation between two methods significant if the difference of the absolute values of the relative errors is more than 5 %. The chosen limit is mostly based on the typical level of tolerance in practice. Whenever the "textbook" method's relative error gives less than 5 % deviation compared to one of the other two methods, then it will be preferable on that interval of the parameters due to its simplicity. Fig. 5 shows region plots (i.e., preference diagrams, which show the range of the parameters where a method has the minimum of the absolute value of the relative error compared to the others) by 5, 10 and 20 years of expected life. The legend at the bottom of the figure indicates which marker pattern belongs to which method. As it can be seen on the figure, the correct cash flow pattern gives the minimum absolute value of the relative error when the growth rate is greater than the discount rate as it was stated by Andor and Dülk (2013a) as well, while the "textbook" method gives the minimum relative error in absolute sense when the growth rate is smaller than the discount rate and the expected life of the asset is less than 10 years; however, when the expected life is more than 10 years, the probability weighted cash flow pattern method starts to conquer greater and greater regions on the plots.

Hereafter based on the above, the difference between the "textbook" and the correct cash flow pattern method will be examined when the growth rate is greater than the discount rate and afterwards when the growth rate is equal to the discount rate, while the difference between the "textbook" and the probability weighted cash flow method will be examined when the growth rate is smaller than the discount rate and afterwards when the growth rate is equal to zero.

The "textbook" method will be compared first to the correct cash flow pattern method when the growth rate is greater than the discount rate. The difference between the two methods can be seen on Fig. 6, and as the scale at the bottom of the figure shows it is mostly less than 5 %. Similar scales will be used further on to characterize the magnitude of the errors; for clarification, the lightest gray level means that the absolute value of the relative error is less than 5 %, and the magnitude is increasing with the darkening shades of gray. It can be said in general, that when the convergence criterion holds ($j < (\lambda + r)$), and the growth rate is less than 10 % and greater than the discount rate, then the deviation between the two methods is less than 5 %, so in this case the "textbook" method gives a reasonably accurate result considering its simplicity. The region where the discount rate is greater than the growth rate has been marked as irrelevant, since in this region the "textbook" method is only competing with the probability weighted cash flow method and it will be discussed separately.

In the next case, when the growth rate is equal to the discount rate, the methods are easily comparable, since the correct cash flow pattern method gives the theoretically accurate present value, i.e., its relative error is zero. Thus, the "textbook" method's own relative error will be the difference between the two methods, which has been discussed based on Fig. 4. The relative error of the "textbook" method only depends on the value of the discount rate, and it has a maximum at the end of the examined interval, 13.606 %. It can be said that when the convergence criterion holds, and the discount rate is less than

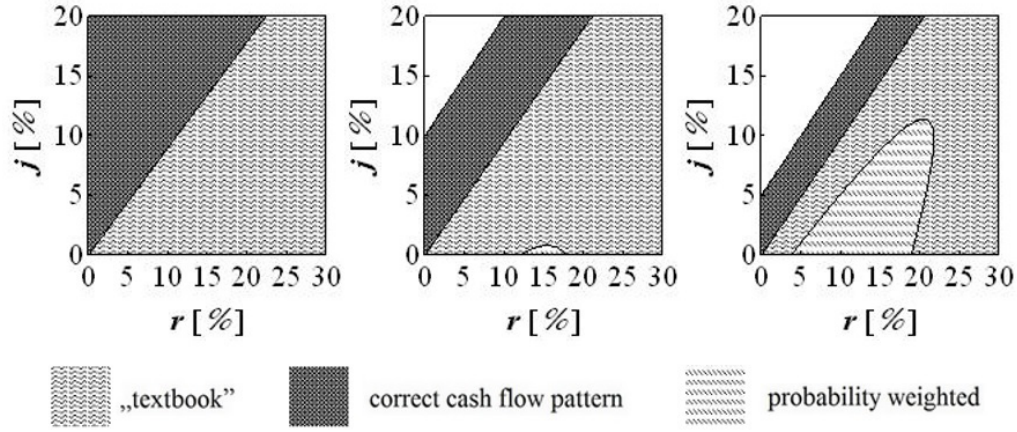

Fig. 5 Comparison of the minimum relative errors of the methods in absolute sense when the expected life is 5, 10 and 20 years respectively

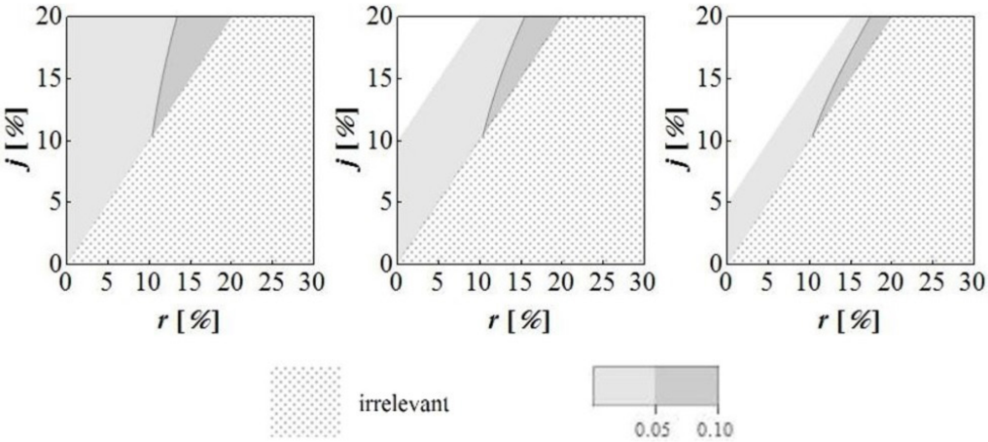

Fig. 6 The difference between the absolute value of relative errors of the "textbook" and correct cash flow pattern when the expected life is respectively 5, 10, 20 years

10.34 % then the difference between the two methods is less than 5 %, so the "textbook" method gives reasonably accurate results considering its simplicity. The convergence criterion holds when the expected life is longer than zero ($\lambda > 0$) in case of equal rates.

When the growth rate is smaller than the discount rate, competition is taking place between the "textbook" and the probability weighted cash flow method. Since the overestimation generated by the "textbook" method is increasing, while the underestimation generated by the probability weighted cash flow method is decreasing over the increasing lifespan, there is a neat line where the two methods change places in the ranking. The difference between the absolute value of relative error of the "textbook" and the probability weighted cash flow method remains within the admissible 5 % when the expected life is around 13 years. The left-hand side of Fig. 7 shows a similar region plot as in Fig. 5 when the expected life is 13 years and the right-hand side of the figure shows the plot as an evidence that the difference between the two relative errors in absolute sense mostly remains within 5 %. The greater part of the relevant region on the right-hand side of Fig. 7, which is marked with the label, "textbook" is preferred on the legend is out of the focus of the comparison because only the region where the probability weighted cash flow method seems better must be investigated. The darker gray area on the investigated part of the plot between 10 % and 15 % of the discount rate indicates that the transition between the two methods is taking place when the expected life is a bit less than 13 years. Generally, it can be said, that when the growth rate is smaller than the discount rate and the expected life is less than 13 years, then the "textbook" method gives a reasonably accurate result considering its simplicity.

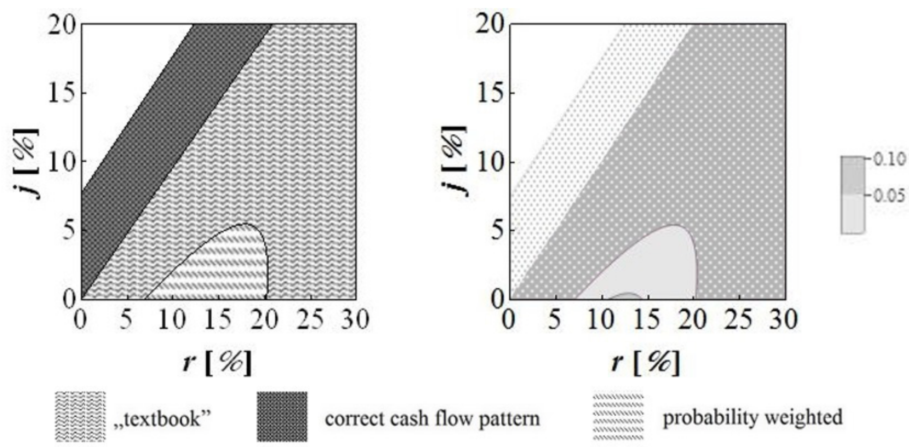

Fig. 7 The difference between the absolute value of relative errors of the "textbook" and probability weighted cash flow method when the expected life is 13 years

Finally, I consider the case when the growth rate is equal to zero. The relative error depends only on the other two parameters, the discount rate and the expected life of the asset. The left-hand side of Fig. 8 is a region plot, i.e., preference diagram and it shows that only the "textbook" and the probability weighted cash flow method compete in this case. The right-hand side of Fig. 8 shows the comparison of the relative errors. The comparison must only focus on the region where the probability weighted cash flow method seems to be better, the "textbook" method is basically preferred otherwise. As it can be seen on the figure, the transition between the two method is around 13 years again, i.e., if the expected life of the asset is less than 13 years than the deviation between the absolute value of relative error of the "textbook" and the probability weighted cash flow method is less than the admissible 5 %. Generally, it is true, that when the convergence criterion holds, and the growth rate is equal to zero, and the expected life of the asset is less than 13 years, then the "textbook" method gives reasonably accurate result considering its simplicity. The convergence criterion holds if the sum of the expected life and the discount rate is greater than zero ($\lambda + r > 0$), which is easily fulfilled because of the assumption of the non-negative discount rate and expected life.

4 Conclusions

I looked for an answer to the question of whether we should calculate the present value of an asset in a more complex way to get a better result, or the well-known "textbook" method of the present value calculation gives a reasonably accurate result considering its simplicity. I compared the "textbook" method with other two methods, which meant to individually dissolve the two main simplifying assumptions of the "textbook" method, such as the end-of-period convention (i.e., cash flows are aggregated to the end of the period in which they occur) in the case of the so called correct cash flow pattern method and the calculation according to expected life-span (i.e., neglecting the uncertainty of the asset's economic life) in the case of the so called probability weighted cash flow method. Continuous exponential cash flow pattern with exponentially distributed lifespan was in the focus of this research since the magnitude of error depends on the cash flow pattern and the probability distribution of the life of the asset. The relative error was used during the comparison of the methods, which is defined as the deviation of the ratio of the estimated and the theoretically accurate present value from one. The relative error depends on three main parameters in this case: the growth rate, the discount rate and the expected life of the asset. The studied interval of the growth rate is between 0 % and 20 % and the examined interval of the discount rate is between 0 % and 30 %. The examined expected lifespan was 5, 10, 20 and 40 years. After determining the relative error of each method, I analyzed the case when the "textbook" method is more accurate than the calculation until the expected life with the correct cash flow pattern. Then, I examine if the accuracy of the "textbook" method can be improved by weighting the end-of-period cash flows by the period ends' probability of occurrence associated with the life uncertainty.

The correct cash flow pattern gives the minimum relative error in absolute sense when the growth rate is greater than the discount rate and it gives zero error when the growth rate is equal to the discount rate as it was stated by Andor and Dülk (2013a) as well, while the "textbook" method gives the minimum relative error in absolute sense when the growth rate is smaller than the discount

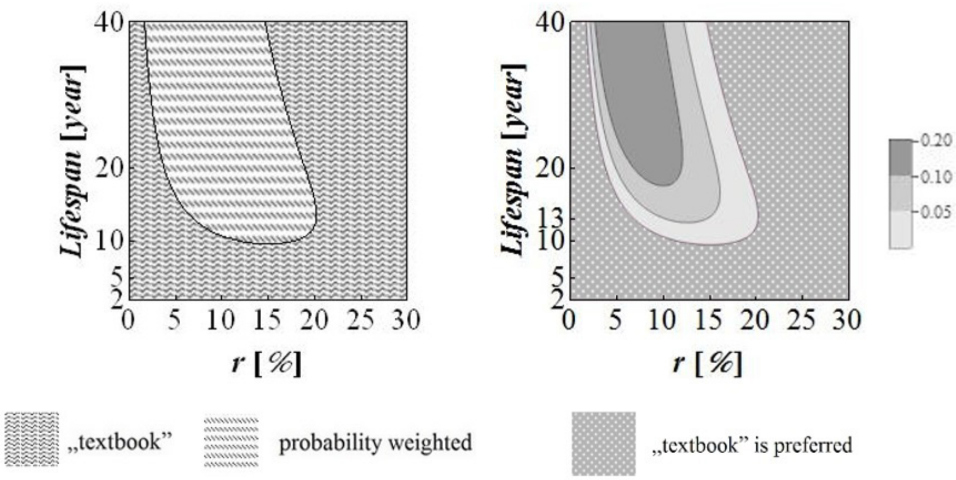

Fig. 8 The difference between the absolute value of relative errors of the "textbook" and probability weighted cash flow method when the growth rate is equal to zero

rate and the expected life of the asset is less than 10 years, however when the expected life is more than 10 years, the probability weighted cash flow pattern method starts to become more and more preferable. The priority was not only finding the best method, but also to analyze the difference between the methods, because whenever the "textbook" method's relative error gives less the 5 % deviation compared to one of the other two methods, then it will be preferable on that interval of the parameters due to its simplicity. I define the deviation between two methods significant if the difference of the absolute values of the relative errors is more than 5 %. The difference between the "textbook" and the correct cash flow pattern method was examined when the growth rate is greater than the discount rate and when the growth rate is equal to the discount rate, while the difference between the "textbook" and the probability weighted cash flow method was examined when the growth rate is smaller than the discount rate and when the growth rate is equal to zero.

When $j < r$ (including the subcase when $j = 0$) and the expected life of the asset is less than 13 years, then the "textbook" method is the most accurate approach, or at least it is really close to the most accurate approach, i.e., the absolute difference between the two relative errors is less than 5 %. When the expected life exceeds 13 years the probability weighted method is becoming more and more preferable. When $j \geq r$ for each examined expected lifespan the case according to the correct cash flow pattern is the most accurate and the "textbook" method is only the second best, but if the growth rate is less than 10 %, which is typical in practice, then the difference between the two methods is less than 5 %, so we can say that the "textbook" method is preferable considering its simplicity.

Based on the results of the comparison, I conclude that for typical values of the parameters in practice, it is not worth to take either the correct cash flow pattern or the life uncertainty into account beyond the "textbook" method.

Acknowledgement
I gratefully acknowledge the helpful inputs in developing the ideas presented here from Marcell Dülk without which my study could not have been completed. I am also grateful to Imre Tarafás and Péter Juhász for their constructive criticism.

References

Andor, G., Dülk, M. (2013a) "Present value under uncertain asset life: an evaluation of relative error", Periodica Polytechnica Social and Management Sciences, 21(2), pp. 71–82. https://doi.org/10.3311/PPso.7086

Andor, G., Dülk, M. (2013b) "Harmonic Mean as an Approximation for Discounting Intraperiod Cash Flows", The Engineering Economist, 58(1), pp. 3–18. https://doi.org/10.1080/0013791X.2012.742607

Andor, G., Dülk, M. (2015) "Cost of capital estimation for energy efficiency projects through a cash flow beta approach", Energy Efficiency, 8(2), pp. 365–384. https://doi.org/10.1007/s12053-014-9298-y

Chen, K. C. W., Manes, R. P. (1986) "A Note on Bias in Capital Budgeting Introduced by Stochastic Life", The Engineering Economist, 31(2), pp. 165–174. https://doi.org/10.1080/00137918608902934

Eschenbach, T. G. (2011) "Engineering Economy: Applying Theory to Practice", Oxford University Press, New York, NY, USA.

Fleischer, G. A., Mason, A. K., Zhou, X. Y. (1998) "Technical Note - The Mid-period and Other Approximations of Relative Error", The Engineering Economist, 43(4), pp. 369–377. http://doi.org/10.1080/00137919808903205

Horvath, P. A. (1995) "Compounding/discounting in continuous time", The Quarterly Review of Economics and Finance, 35(3), pp. 315–325. https://doi.org/10.1016/1062-9769(95)90072-1

Lawrence, E. C. (2009) "Biases in Mid-Year and End-of-Year Conventions in Discounted Cash Flow Models for Corporate Valuations", Journal of Legal Economics, 16(1), pp. 1–15. [online] Available at: https://pdfs.semanticscholar.org/7678/ea3eedc900098e-767129a66bd51591994b4e.pdf [Accessed: 03 May 2019]

Lohmann, J. R., Oakford, R. V. (1984) "Errors in Present Worth Evaluations Attributable to the End-of-year and Mid-year Cash Flow Conventions", The Engineering Economist, 29(4), pp. 303–309. https://doi.org/10.1080/00137918408967726

Park, C. S., Sharp-Bette, G. P. (1990) "Advanced Engineering Economics", John Wiley & Sons, New York, NY, USA.

Remer, D. S., Tu, J. C., Carson, D. E., Ganiy, S. A. (1984) "The state of the art of present worth analysis of cash flow distributions", Engineering Cost and Production Economics, 7(4), pp. 257–278. https://doi.org/10.1016/0167-188X(84)90044-2

Van Horne, J. C. (1972) "Capital Budgeting under Conditions of Uncertainty as to Project Life", The Engineering Economist, 17(3), pp. 189–199. https://doi.org/10.1080/00137917208902716

Zinn, C. D., Lesso, W. G., Motazed, B. (1977) "A Probabilistic Approach to Risk Analysis in Capital Investment Projects", The Engineering Economist, 22(4), pp. 239–260. https://doi.org/10.1080/00137917708965183