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Ecological Footprint Calculation as a Land Demand: Based on the Dynamic Leontief Model

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Abstract

In recent years, researchers have sought to specify precisely what is meant by the ecological footprint, and there are some methods for calculating it. This paper features a new calculation method for determining the ecological footprint (EFP). The basis of our model is the dynamic Leontief model. If our method is applied, one can determine that a dynamic ecological footprint is a sequence of footprints for periods. We also calculate the ecological footprint for both closed and open economies. Our model contains elements taken from the Leontief model: capital accumulation, and integration of exports and imports into the model through input-output panels. All periods are treated as interdependent, rather than as a series of stand-alone data. Most notably, the model separates capital accumulation from the final use of capital, i.e., investment and final consumption. We illustrate the results with numerical examples.

Keywords

ecological footprint, dynamic Leontief model, balanced growth path

1 Introduction

Calculating the ecological footprint (EFP) has been the focus of academic interest for decades. The notion of an ecological footprint sought to give a land-based metric accounting for the impacts of past consumption – thereby attempting to cover all pressure on the environment into the land area needed, indirectly, to provide for consumption. This method evaluates the quantity of land required to compensate for environmental pressures.

In addition, many scientific terms connect to the environment and economy. Fig. 1 shows the frequency of occurrence of the ecological footprint (red), the circular economy (blue) and the sustainable development (green) expressions, which appear as the percentage of documents in the Google Books database (Lin et al., 2012). The explanation for choosing these expressions is the cited number for these expressions in the academic literature.

The ecological footprint began its career as an indicator in the early 1990s, when sustainable development had already become popular. The idea of a circular economy has become increasingly popular recently, but academic researchers still focus on the ecological footprint. According to the Google Books data, the ecological

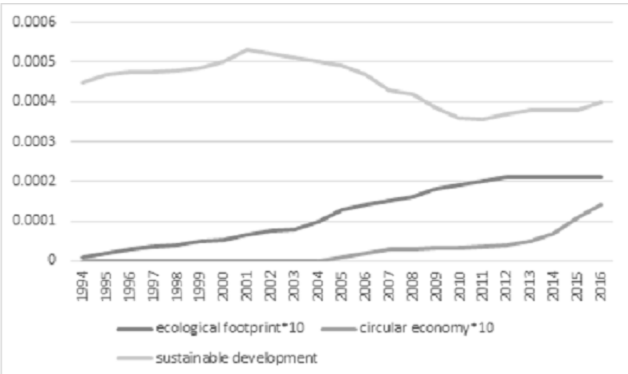

Fig. 1 Frequency of "ecological footprint", "circular economy" and "sustainable development" expressions in the Google Books database between 1994 and 2016

footprint calculation must still be considered. In our opinion, newer methodologies can help researchers to define indicators that are more accurate.

The first representatives of footprint calculation are Rees (1992) and Rees and Wackernagel (1996). The unit of measurement of the ecological footprint is the global hectare, usually estimated, and the method of determining it is constantly being fine-tuned. In this study, we identify the

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ecological footprint of the used land area. This footprint is constructed as a function of the vectors of final consumption, exports and products directly imported for final consumption over the period. We use the dynamic Leontief model to calculate the land demand during the production period for each production period, which is necessary for a national economy. The Leontief model (Leontief, 1986) provides a comprehensive and thorough analysis of the circular economy, and its applicability is gaining ground today (e.g., Lonca et al., 2019; Lucas and Vardon, 2021; Wiebe et al., 2019). Leontief incorporated environmental factors too, he presented a static input-output model with environmental factors through pollution (Leontief, 1970).

Bicknell et al. (1998) and Ferng (2001), who used input-output models, developed the methodology. For a methodological critique of their models, see Dobos (2019a). The paper contains the following sections: Section 2 presents the literature review, and the analysis of Bicknell et al. (1998) and Ferng (2001) models. We highlight the main differences, and we give a critique of the models. Section 3 shows the ecological footprint with a dynamic input-output model for a closed economy. Section 4 shows the dynamic Leontief model-based model for calculating dynamic ecological footprint for an open economy. Then, in Section 5, we calculate the balanced growth path. Section 6 contains numerical examples about the model. We conclude this paper with a summary and outlook in Section 7.

2 Literature review

We can create groups from the papers in the literature which are relevant to our study. There are several criteria for creating groups from the literature (see the Fig. 2; Abbood et al., 2022; Baabou et al., 2017; Bagliani et al., 2003; Begum et al., 2009; Bicknell et al., 1998; Dobos, 2019(b); Dobos and Floriska, 2009; Ferng, 2001; Guan, 2009; Haberl et al., 2001; Hubacek and Giljum, 2003; Jianmin et al., 2004; Kissinger, 2013; Kitzes et al., 2009; Kratena and Wiedmann, 2008; Lenzen et al., 2007; Mattila, 2012; McGregor et al., 2004; Moffatt et al., 2005; Rees, 1992; Rees and Wackernagel, 1996; Tsuchiya et al., 2021; Turner et al., 2007; Wackernagel et al., 2004; Zhou et al., 2006). One of them is the emergence of static and dynamic footprint calculations in the literature. The first group includes the methodological works that distinguish between static and dynamic footprints. The second level of grouping contains works based on empirical/concrete calculations.

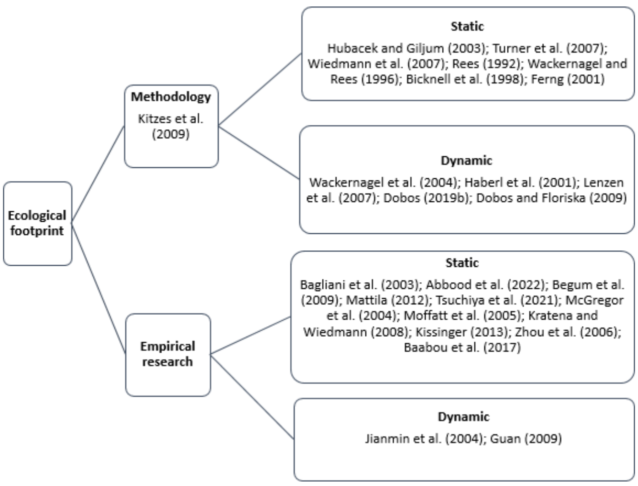

Fig. 2 Literature review for ecological footprint, focusing on input-output models

We focus on studies using the input-output method. The ecological footprint concept approaches in several ways to analyses with input-output models. We focus on the land use models, such as Hubacek and Sun (2001), Ferng (2001), Bicknell et al. (1998), and Eder and Narodoslawsky (1999). Hubacek and Giljum (2003) and Turner et al. (2007) meanwhile used static methods for calculating ecological footprint, taking both pollution and land-use into account.

In Section 3, we seek to answer the question of which factors cause the ecological footprint to be dynamic. In the literature, Wackernagel et al. (2004) and Haberl et al. (2001) have used trend fitting to fit static data dynamically. Meanwhile, by using trend analysis, Lenzen et al. (2007) have given the ecological footprint a more dynamic aspect by using production functions.

Many papers contain specific footprint calculations. Notably, Abbood et al. (2022) used an input-output model to calculate the carbon and energy footprint of the US manufacturing system. Their research quantified the life-cycle carbon and energy footprint impacts of US manufacturing activities, taking international trade relations with the rest of the world into consideration.

We can also find some interesting analysis relating to ecological footprints in different countries, for example:

- Begum et al. (2009) for Malaysia;
- Mattila (2012) for Finland;
- Tsuchiya et al. (2021) for Japan;
- McGregor et al. (2004) and Moffatt et al. (2005) for Scotland;
- Kratena and Wiedmann (2008) for United Kingdom;
- Kissinger (2013) for Canada;
- Baabou et al. (2017) for Mediterranean cities.

A methodology for calculating dynamic ecological footprint using a dynamic input-output model has not yet been applied; examples include Dobos (2019a). Dobos (2019b) derives how it is possible to perform the dynamic footprint calculation without knowing the sectoral linkage matrix of the supplying economy and to produce the specific land demand for each sector. This way, it is possible to split final consumption into capital accumulation and final consumption. The study by Bicknell et al. (1998) is the first known work on this topic. The starting assumption is to consider whether the land-use is incorporated into the model through imports or needs for domestic production. The model represents the share of land that can be allocated to consumption. The total land use consists of three sub-areas:

1. The final amount of direct consumption attributable to imports;
2. Land entering production indirectly to the consumer;
3. Indirectly transferred from imports to exports for the land.

However, we follow Ferng's method about applying a land multiplier (Ferng, 2001). The main difference between Ferng (2001) and Bicknell et al. (1998) is that Ferng modelled the land as a consumption resource rather than a primary resource for modelling purposes. The second difference is that they interpret the share of land in imports differently. However, both models suffer from the same shortcoming: it is impossible to partition sectoral imports between final consumption and exports in a linear algebraic way.

Given that the focus of the ecological footprint is to capture the total (direct plus indirect) resource use embodied in final consumption in an economy, input-output would seem to be the ideal accounting framework.

The basis of input-output analysis is a set of sectorally disaggregated economic accounts, where inputs to each industrial sector and the subsequent uses of those sectors' output are separately identified. The primary function of input-output analysis is to quantify the interdependence of different activities within the economy. It uses straightforward mathematical routines to track all direct, indirect, and, where appropriate, induced, resource use embodied within consumption (Leontief, 1970; Miller and Blair, 2009). The methods in Bicknell et al. (1998) and Ferng (2001) differ sharply in the land requirements of imports used indirectly for consumption and exports through production. Whereas Bicknell et al. (1998) allocated imports used for production by sector in proportion to final consumption and exports; Ferng (2001) reduced total output by exports and did not distinguish between them. Thus it is relevant to know whether imports used for production remain in the economy or are instead allocated to indirect consumption.

Table 1 Main elements of Bicknell et al. (1998) and Ferng (2001) about land requirements

	Bicknell et al. (1998)	Ferng (2001)
Domestic land requirements for final consumption	$\boldsymbol{l}(\boldsymbol{I}-\boldsymbol{A})^{-1}\langle\boldsymbol{c}\rangle$	$\langle\boldsymbol{l}\rangle(\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{c}$
Foreign land requirements for final consumption	$\boldsymbol{l}(\boldsymbol{I}-\boldsymbol{A})^{-1}\langle\boldsymbol{c}_{imp}\rangle$	$\langle\boldsymbol{l}\rangle(\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{c}_{imp}$
Foreign land requirements for production	$\boldsymbol{l}(\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{A}_{imp}\langle\boldsymbol{x}\rangle$	–
Land requirements of final consumption	$\boldsymbol{l}(\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{A}_{imp}\langle\boldsymbol{x}\rangle\langle\boldsymbol{c}\rangle$ $\langle\boldsymbol{c}+\mathbf{exp}\rangle^{-1}$	$\langle\boldsymbol{l}\rangle(I-\boldsymbol{A})^{-1}\boldsymbol{A}_{imp}$ $(\boldsymbol{x}-\mathbf{exp})$
Domestic land requirements for exports	–	$\langle\boldsymbol{l}\rangle(\boldsymbol{I}-\boldsymbol{A})^{-1}\mathbf{exp}$

Table 1 contains the main elements of the cited models about land requirements.

3 Interpretation and definition of the dynamic ecological footprint in a closed economy

The dynamic ecological footprint is based on the well-known dynamic Leontief model. Suppose that there are n economic industries. Each industry produces one type of good and m types of lands released by industries, for example, agricultural or forest. The model is:

$$\begin{aligned} &\boldsymbol{x}_t = \boldsymbol{A}\boldsymbol{x}_t + \boldsymbol{B}\left(\boldsymbol{x}_{t+1} - \boldsymbol{x}_t\right) + \boldsymbol{c}_t, \ t = 1,2,\ldots,T \\ &\boldsymbol{L}\boldsymbol{x}_t \le \boldsymbol{l}, \ t = 1,2,\ldots,T, \end{aligned} \tag{1}$$

where:

- $\boldsymbol{x}_t$ is an n-dimensional vector of gross industrial outputs in period t;
- $\boldsymbol{c}_t$ is the n-dimensional vector of final consumption demands for goods in period t;
- $\boldsymbol{A}$ is the $n \times n$ matrix of conventional input coefficients, showing the input of goods that are required to produce a unit of product; we suppose that this matrix is productive, i.e., it has a non-negative Leontief inverse;
- $\boldsymbol{B}$ is the $n \times n$ matrix of capital coefficients, showing the invested products to increase the output of the producing sectors by a unit;

- T is the length of the planning horizon;
- $\boldsymbol{L}$ is the $m \times n$ matrix of land input coefficients of producing sectors, showing the quantity of land area required during producing a unit of industrial product;
- $\boldsymbol{l}$ is the m-dimensional vector of carrying capacity of the lands.

The first equation contains the basic context from the Leontief model, the second inequality ($\boldsymbol{L}\boldsymbol{x}_t \leq \boldsymbol{l}$, $t = 1,2, \ldots, T$) relates to the land requirements and shows that the land requirement of the economy cannot exceed the carrying capacity of the land. In the following, we calculate the land requirements, which feature two elements: the land requirement of the final demand and the land requirements of the capital accumulation. Hence:

$$\boldsymbol{l}_1\boldsymbol{x}_t = \boldsymbol{l}_1\left(\boldsymbol{I} - \boldsymbol{A}\right)^{-1}\left[\boldsymbol{c}_t + \boldsymbol{B}\left(\boldsymbol{x}_{t+1} - \boldsymbol{x}_t\right)\right]. \tag{2}$$

If the final consumption, $\boldsymbol{c}_t$ is known for $t = 1,2 \ldots, T$, we can determine the dynamic land requirements. We assume that matrix $\boldsymbol{B}$ is invertible and $\boldsymbol{x}_1$ is known.

The output of the economy in the t^{th} period is:

$$\begin{aligned} \boldsymbol{x}_t &= \left[\boldsymbol{B}^{-1}\left(\boldsymbol{I} - \boldsymbol{A}\right) + \boldsymbol{I}\right]^{t-1} \boldsymbol{x}_1 \\ &- \sum_{\tau=1}^{t-1}\left[\boldsymbol{B}^{-1}\left(\boldsymbol{I} - \boldsymbol{A}\right) + \boldsymbol{I}\right]^{t-\tau-1} \boldsymbol{B}c_\tau . \end{aligned} \tag{3}$$

We obtain the dynamic ecological footprint in a closed economy from Eqs. (2) and (3):

$$\text{EFP}_t = \boldsymbol{l}_1\boldsymbol{x}_t = \boldsymbol{l}_1 \begin{Bmatrix} \left[\boldsymbol{B}^{-1}\left(\boldsymbol{I} - \boldsymbol{A}\right) + \boldsymbol{I}\right]^{t-1} \boldsymbol{x}_1 \\ -\sum_{\tau=1}^{t-1}\left[\boldsymbol{B}^{-1}\left(\boldsymbol{I} - \boldsymbol{A}\right) + \boldsymbol{I}\right]^{t-\tau-1} \boldsymbol{B}\boldsymbol{c}_t \end{Bmatrix}. \tag{4}$$

Consequently, using Eq. (4), we can specify the ecological footprint. This formula is easy to apply because the calculation of input-output tables is generally used in economies. We recommend reducing the level of new products in the closed economy (see vector in Eq. (4)).

4 Interpretation and definition of the dynamic ecological footprint for an open economy

Section 4 shows that the dynamic Leontief model causes an endogenous dynamic ecological footprint. Dynamic EFP is more than a succession of static values: the current size of the footprint also depends on the previous period's processes.

In the following, we provide some conditions for the dynamic model based on the dynamic Leontief model (see, for example, Zalai (1989)). The dynamic multi-sector input-output model is based on Leontief (1986:p.31). The basic model has n economic sectors, each producing a single good:

$$\boldsymbol{x}_t = \boldsymbol{A}\boldsymbol{x}_t + \boldsymbol{B}\left(\boldsymbol{x}_{t+1} - \boldsymbol{x}_t\right) + \boldsymbol{c}_t, \tag{5}$$

where $\boldsymbol{B}$ is the matrix of capital coefficients (investment demand matrix), size $n \times n$, is non-negative, non-singular. t, $t + 1$ refer to specific periods. According to Eq. (5), the produced goods are deducted for the necessary intermediate input and final consumption, and the remaining surplus is used to expand production capacity.

Output in the $(t + 1)^{\text{th}}$ period is:

$$\boldsymbol{x}_{t+1} = \left[\boldsymbol{I}_n + \boldsymbol{B}^{-1}\left(\boldsymbol{I}_n - \boldsymbol{A}\right)\right]\boldsymbol{x}_t - \boldsymbol{B}^{-1}\boldsymbol{c}_t, \tag{6}$$

where $\boldsymbol{I}_n$ is an $(n \times n)$-size identity matrix.

The dynamic Leontief model divides the output into intermediate input, consumption, and accumulation. In the latter case, we distinguish between investments that increase stock and investments that create fixed capacities.

In our case, the dynamic model is:

$$\begin{aligned} \boldsymbol{x}_t &= \boldsymbol{A}\boldsymbol{x}_t + \boldsymbol{B}\left(\boldsymbol{x}_{t+1} - \boldsymbol{x}_t\right) + \boldsymbol{c}_t + \mathbf{exp}_t \\ \boldsymbol{imp}_t &= \boldsymbol{A}_{imp}\boldsymbol{x}_t + \boldsymbol{B}_{imp}\left(\boldsymbol{x}_{t+1} - \boldsymbol{x}_t\right) + \boldsymbol{c}_{imp,t} \end{aligned}, \tag{7}$$

where ($t = 0,1, \ldots, (T - 1)$). Suppose that we know $\boldsymbol{x}_0$ vector, which represents the initial output of the planning period. Fig. 3 contains the logic scheme of the model.

The final consumption from the static model (the final consumption vector) splits capital accumulation (vector $\boldsymbol{B}(\boldsymbol{x}_{t+1} - \boldsymbol{x}_t)$ and final consumption ($\boldsymbol{c}_t$). Suppose that $\boldsymbol{T}$ is the end of the planning procession's period, and we know the capital matrix of domestically produced ($\boldsymbol{B}$) and imported goods ($\boldsymbol{B}_{imp}$). The non-singularity of $\boldsymbol{B}$ matrix generates interesting questions in the literature. Suppose that the $\boldsymbol{B}$ matrix is non-singular in this model for linear algebraic manageability. We can find some examples of non-singular capital matrices in the literature (for example Dobos (1987), Ábel and Dobos (2017)). If we were to work with a singular matrix, it would not significantly change what the model says, so we work with a non-singular capital matrix.

Table 2 shows the transaction table for the dynamic case.

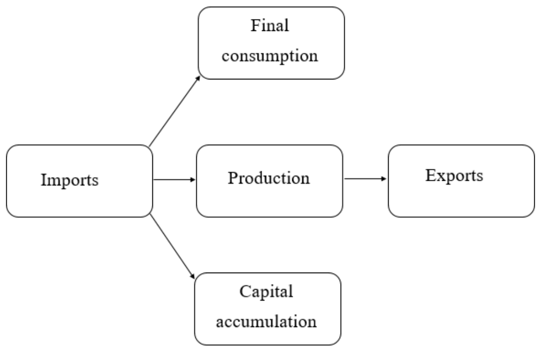

Fig. 3 Material flow diagram of the dynamic input-output model

Table 2 Dynamic input-output (transaction) table for an economy with aggregated foreign trade

	Sectors	Final consumption	Capital accumulation	Export	Total output, import
Sectors	$A\langle x_t \rangle$	c_t	$B\langle x_{t+1} - x_t \rangle$	$\exp_t$	x_t
Import	$A_{imp}\langle x_t \rangle$	$c_{imp,t}$	$B_{imp}\langle x_{t+1} - x_t \rangle$	–	imp_t
Value added	v_t	$v_{c,t}$	–	–	–
Total output	x_t	–	–	–	–
Land area	$l\langle x_t \rangle$	–	–	–	–

We can solve Eq. (7) explicitly, using the inverse of the capital matrix:

$$\boldsymbol{x}_{t+1} = \boldsymbol{B}^{-1}\left(\boldsymbol{I} - \boldsymbol{A} + \boldsymbol{B}\right)\boldsymbol{x}_t - \boldsymbol{B}^{-1}\boldsymbol{c}_t - \boldsymbol{B}^{-1}\mathbf{exp}_t,$$
$$\left(t = 0, 1, \ldots, (T-1)\right).$$

The solution of the dynamic system is:

$$\begin{aligned}\boldsymbol{x}_t &= \left[\boldsymbol{B}^{-1}\left(\boldsymbol{I} - \boldsymbol{A} + \boldsymbol{B}\right)\right]^t \boldsymbol{x}_0 \\ &-\sum_{\tau=0}^{t-1}\left[\boldsymbol{B}^{-1}\left(\boldsymbol{I} - \boldsymbol{A} + \boldsymbol{B}\right)\right]^{t-\tau}\boldsymbol{B}^{-1}\boldsymbol{c}_\tau \\ &-\sum_{\tau=0}^{t-1}\left[\boldsymbol{B}^{-1}\left(\boldsymbol{I} - \boldsymbol{A} + \boldsymbol{B}\right)\right]^{t-\tau}\boldsymbol{B}^{-1}\mathbf{exp}_\tau.\end{aligned} \tag{8}$$

Equation (8) defines the total output, which depends on the initial total output, the final consumption, and the time series of the exports. We must replace Eq. (8) with the equations about imports. In consequence, we get the following expression for imports in the t^{th} period:

$$\begin{aligned}&\boldsymbol{imp}_t = \boldsymbol{A}_{imp}\boldsymbol{x}_t + \boldsymbol{B}_{imp}\boldsymbol{B}^{-1}\left[\left(\boldsymbol{I} - \boldsymbol{A}\right)\boldsymbol{x}_t - \boldsymbol{c}_t - \mathbf{exp}_t\right] \\ &+\boldsymbol{c}_{imp,t} = \left[\boldsymbol{A}_{imp} + \boldsymbol{B}_{imp}\boldsymbol{B}^{-1}\left(\boldsymbol{I} - \boldsymbol{A}\right)\right] \\ &\times\left[\boldsymbol{B}^{-1}\left(I - \boldsymbol{A} + \boldsymbol{B}\right)\right]^t \boldsymbol{x}_0 \\ &-\left[\boldsymbol{A}_{imp} + \boldsymbol{B}_{imp}\boldsymbol{B}^{-1}\left(\boldsymbol{I} - \boldsymbol{A}\right)\right] \\ &\times\sum_{\tau=0}^{t-1}\left[\boldsymbol{B}^{-1}\left(\boldsymbol{I} - \boldsymbol{A} + \boldsymbol{B}\right)\right]^{t-\tau}\boldsymbol{B}^{-1}\boldsymbol{c}_\tau \\ &-\left[\boldsymbol{A}_{imp} + \boldsymbol{B}_{imp}\boldsymbol{B}^{-1}\left(\boldsymbol{I} - \boldsymbol{A}\right)\right] \\ &\times\sum_{\tau=0}^{t-1}\left[\boldsymbol{B}^{-1}\left(\boldsymbol{I} - \boldsymbol{A} + \boldsymbol{B}\right)\right]^{t-\tau-1}\boldsymbol{B}^{-1}\mathbf{exp}_\tau \\ &-\boldsymbol{B}_{imp}\boldsymbol{B}^{-1}\boldsymbol{c}_t - \boldsymbol{B}_{imp}\boldsymbol{B}^{-1}\mathbf{exp}_t + \boldsymbol{c}_{imp,t}.\end{aligned} \tag{9}$$

Now we can determine the land requirements. The basis of our calculation is the total outputs for each period because we can determine the total output using the time series of final consumption and export. The land requirement for domestic production is:

$$\begin{aligned}&\boldsymbol{l}\boldsymbol{x}_t = \boldsymbol{l}\left(\boldsymbol{I} - \boldsymbol{A}\right)^{-1}\boldsymbol{B}\left(\boldsymbol{x}_{t+1} - \boldsymbol{x}_t\right) \\ &+\boldsymbol{l}\left(\boldsymbol{I} - \boldsymbol{A}\right)^{-1}\boldsymbol{c}_t + \boldsymbol{l}\left(\boldsymbol{I} - \boldsymbol{A}\right)^{-1}\mathbf{exp}_t.\end{aligned} \tag{10}$$

The dynamic internal consumption is:

$$\begin{aligned}&\boldsymbol{l}\boldsymbol{x}_t - \boldsymbol{l}\left(\boldsymbol{I} - \boldsymbol{A}\right)^{-1}\mathbf{exp}_t + \boldsymbol{l}\left(\boldsymbol{I} - \boldsymbol{A}\right)^{-1}\boldsymbol{imp}_t \\ &= \boldsymbol{l}\left(\boldsymbol{I} - \boldsymbol{A}\right)^{-1}\boldsymbol{B}\left(\boldsymbol{x}_{t+1} - \boldsymbol{x}_t\right) + \boldsymbol{l}\left(\boldsymbol{I} - \boldsymbol{A}\right)^{-1}\boldsymbol{c}_t \\ &+\boldsymbol{l}\left(\boldsymbol{I} - \boldsymbol{A}\right)^{-1}\boldsymbol{A}_{imp}\boldsymbol{x}_t + \boldsymbol{l}\left(\boldsymbol{I} - \boldsymbol{A}\right)^{-1}\boldsymbol{B}_{imp} \\ &\times\left[\boldsymbol{B}^{-1}\left(\boldsymbol{I} - \boldsymbol{A}\right)\boldsymbol{x}_t - \boldsymbol{B}^{-1}\boldsymbol{c}_t - \boldsymbol{B}^{-1}\mathbf{exp}_t\right] \\ &+\boldsymbol{l}\left(\boldsymbol{I} - \boldsymbol{A}\right)^{-1}\boldsymbol{c}_{imp,t}.\end{aligned} \tag{11}$$

The difference between Eqs. (15) and (16) is:

$$\begin{aligned}&\boldsymbol{l}\left(\boldsymbol{I} - \boldsymbol{A}\right)^{-1}\mathbf{exp}_t - \boldsymbol{l}\left(\boldsymbol{I} - \boldsymbol{A}\right)^{-1}\boldsymbol{imp}_t \\ &= \boldsymbol{l}\left(\boldsymbol{I} - \boldsymbol{A}\right)^{-1}\left(\mathbf{exp}_t - \boldsymbol{imp}_t\right).\end{aligned} \tag{12}$$

Equation (12) shows that does the economy imports or exports land area in the t^{th} period. The optimal case for an economy is the used land export because it means that the economy does not use more land area than is available. Economic decision-makers are advised to keep their land demand under the critical size of the ecological footprint. An open economy must reduce the consumption of imported and newly produced goods to reduce the dynamic ecological footprint.

5 Ecological footprint and the balanced growth path

5.1 Balanced growth path in the closed economy

We assume that $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{L}$ matrices are non-negative, $\boldsymbol{B}$ is nonsingular, $\boldsymbol{c}_t$ is a non-negative vector. We can find analysis in the literature like the follows in Dobos and Floriska (2007) and Schoonbeek (1990). Suppose that there is a given growth rate α, $\alpha \geq 0$ and the total output and the final consumption increase with this rate. Under these assumptions the balanced growth solution Eq. (1) is:

$$\boldsymbol{x}_t = \left(1 + \alpha\right)^t \boldsymbol{x}_0 \text{ and } \boldsymbol{c}_t = \left(1 + \alpha\right)^t \boldsymbol{c}_0. \tag{13}$$

After substitution of Eq. (13) into Eq. (1), we obtain Eq. (14):

$$\left(\boldsymbol{I} - \boldsymbol{A} - \alpha\boldsymbol{B}\right)\boldsymbol{x}_0 = \boldsymbol{c}_0. \tag{14}$$

After that, we have established conditions for the existence of non-negative output configuration $\boldsymbol{x}_0$. The output configuration $\boldsymbol{x}_0$ corresponding to Eq. (6) exists and it is non-negative if $\alpha \in [0, \alpha_0]$, where α_0 is the marginal growth rate such that λ_i $(\boldsymbol{A} + \alpha_0\boldsymbol{B}) = 1$, i.e., it is the balanced growth rate of the closed dynamic Leontief model. Where λ_1 $(\boldsymbol{M})$ denotes the Frobenius root of an arbitrary non-negative square matrix $\boldsymbol{M}$, it is the non-negative real dominant eigenvalue of $\boldsymbol{M}$. If the former condition for the existence of non-negative $\boldsymbol{x}_0$ is fulfilled, then the output configuration $\boldsymbol{x}_0$ has the following form:

$$\boldsymbol{x}_0\left(\alpha, \boldsymbol{c}_0\right) = \left(\boldsymbol{I} - \boldsymbol{A} - \alpha \boldsymbol{B}\right)^{-1} \boldsymbol{c}_0. \quad (15)$$

We assume that the carrying capacity of the land is constant in the planning horizon. Then the vector of land capacity is a known vector, $\boldsymbol{l}$. Let us substitute this expression and Eqs. (13) and (15) in the inequality $\boldsymbol{L}\boldsymbol{x}_t \leq \boldsymbol{l}$, $t = 1,2, \ldots, T$, we obtain the following inequality:

$$\left(1+\alpha\right)^t \boldsymbol{L}\left(\boldsymbol{I} - \boldsymbol{A} - \alpha \boldsymbol{B}\right)^{-1} \boldsymbol{c}_0 \leq \boldsymbol{l}. \quad (16)$$

Lemma 1. Suppose that the growth rate of production and consumption α is limited by an upper bound α^* due to the land capacity. That is the following limitation must hold $0 \leq \alpha \leq \alpha^*$ where:

$$\max_i \left(\frac{\left(\boldsymbol{L}\left(\boldsymbol{I} - \boldsymbol{A} - \alpha^* \boldsymbol{B}\right)^{-1} \boldsymbol{c}_0\right)_i}{\left(\boldsymbol{l}\right)_i} \right) = 1, \quad (17)$$

where $(l)_i$ denotes the i^{th} component of the respective vector.

Proof. We assume that we are at the beginning of the examined time period, i.e., $t = 0$. Using the relation (Eq (16)), we determine the maximal growth rate α^* for which the quantity of the land-use generated is not more than the allowed limit. Then for this α^* must hold the equality (Eq. (17)).

Remark 1. We should impose more strict restriction on the chosen growth rate than we have made previously (according to Dobos and Floriska (2009) the upper bound for α is the marginal growth rate α_0, i.e., the balanced growth rate for the closed dynamic Leontief model). Considering α_0 for the value of α^* in Eq. (4), the left-hand side of it will be an unbounded function for α_0. This implies that α^* should be less than α_0. That is the following inequalities must hold: $0 \leq \alpha \leq \alpha^* < \alpha_0$. Under these assumptions, there will become a time t^* such that the amount of one type of land-use generated by industries will be equal to the allowed carrying capacity.

Lemma 2. The time t^* can be calculated by the following formula:

$$t^* = \frac{1}{\ln\left(1+\alpha\right)} \min_i \left(\ln \frac{\left(\boldsymbol{l}\right)_i}{\left(\boldsymbol{L}\right)_i \left(\boldsymbol{I} - \boldsymbol{A} - \alpha \boldsymbol{B}\right)^{-1} \boldsymbol{c}_0} \right), \quad (18)$$

where $(\boldsymbol{L})_i$ denotes the i^{th} component of the respective vector and the row of the respective matrix.

Proof. By a simple mathematical calculation, we express t^* from the inequality (Eq. (16)).

This lemma gives estimation for the time interval without an adjustment process on land-use. After this point of time the economy must change either production level or consumption rate, or both. In our model, we assume that first the production rate is adjusted to the carrying capacity and then the consumption level. It can be proven that this kind of adjustment process leads to a higher consumption level than another choice, i.e., first adjusted consumption, then afterwards production.

Lemma 3. After the time t^* (i.e., for $t \geq t^*$), the maximum growth rate of production is zero.

Proof. Denote γ the growth rate of production after the time t^*. Then the balanced growth path of production has the form $\boldsymbol{x}_t = \left(1+\gamma\right)^{t-t^*} \boldsymbol{x}_{t^*}$ for $t \geq t^*$. Substituting these expressions for x_t and $\boldsymbol{l}$ the inequality $\boldsymbol{L}\boldsymbol{x}_t \leq \boldsymbol{l}$ we obtain:

$$\left(\frac{1}{1+\gamma}\right)^{t-t^*} \boldsymbol{L}\boldsymbol{x}_{t^*} \leq \boldsymbol{l} \text{ for } t \geq t^*.$$

If $\gamma > 0$, for $t \to \infty$ the former inequality is $\boldsymbol{L}\boldsymbol{x}_{t^*} \leq 0$. This obviously is not fulfilled for every $\boldsymbol{x}_{t^*} \neq 0$. If $\gamma = 0$, the former inequality is $\boldsymbol{L}\boldsymbol{x}_{t^*} \leq \boldsymbol{l}$. This is obviously fulfilled for the time t^*. This concludes that the maximal value for γ, i.e., the maximum growth rate of production is zero. This lemma allows us to construct the path of the production level. The production level is grown with a growth rate α until point of time t^* and after this point the growth rate is 0. The growth rate can be determined as follows: $\boldsymbol{x}_t = (1 + \alpha)^t \boldsymbol{x}_0$, $t < t^*$ and $\boldsymbol{x}_t = \left(1+\alpha\right)^{t^*} \boldsymbol{x}_0$ $t^* \leq t \leq T$. The next proposition makes it possible to calculate the consumption levels along the planning horizon. Let us now define the growth rate of the production level in the planning horizon as function of the time:

$$\gamma_t = \alpha, t < t^* \text{ and } \gamma_t = 0,\ t^* \leq t \leq T.$$

We use the *Lemma 1*, *2*, *3* for the following proposition.

Proposition 1. In case of a balanced growth solution of the model (Eq. (1)) for a given rates of growth, the following Eq. (19) must hold:

$$\left(\boldsymbol{I} - \boldsymbol{A} - \gamma_t \boldsymbol{B}\right) \boldsymbol{x}_t = \boldsymbol{c}_t, \text{ for } t = 1,2,\ldots,T, \quad (19)$$

Proof. This relation can be proved in similar way as we have got Eq. (15). The consumption rate can be constructed as:

$$\boldsymbol{c}_t = \left(1+\alpha\right)^t \boldsymbol{c}_0,\ t < t^*,$$

and:

$$\boldsymbol{c}_t = (1+\alpha)^{t^*} (\boldsymbol{I}-\boldsymbol{A})(\boldsymbol{I}-\boldsymbol{A}-\alpha\boldsymbol{B})^{-1} \boldsymbol{c}_0,\ t^* \le t \le T.$$

Remark 2. An overview of the open economy model. The growth rate α of the balanced growth path of the system (Eq. (1)) could be at most α^* according to the *Lemma 1*. In so far as this rate of growth is greater than the biocapacity, then this balanced path with rate of growth α, can be continued at most to the time t^* according to *Lemma 2*. After the time t^* the maximal growth rate of the balanced path is 0, according to *Lemma 3*. The production corresponding to such a path is growing with a rate of growth α until the time t^*, and with a rate of growth 0 after this time. In the case of different growth rates, a given level of production will correspond to different levels of consumption at a given time. In *Lemma 4*, we analyse this change in the consumption level.

Lemma 4. The consumption level at the time t^* is not less than it was at the time $t^* - 1$. That is $\boldsymbol{c}_{t^*} \ge \boldsymbol{c}_{t^*-1}$.

Proof. For $\alpha \ge 0$, the next inequality is obviously fulfilled:

$$\boldsymbol{c}_{t^*} - \boldsymbol{c}_{t^*-1} \ge \boldsymbol{c}_{t^*} - (1+\alpha)\boldsymbol{c}_{t^*-1}. \tag{20}$$

Using Eq. (19), *Lemma 3* and the *Remark 2* we obtain:

$$\begin{aligned} &\boldsymbol{c}_{t^*} - (1+\alpha)\boldsymbol{c}_{t^*-1} \ge (\boldsymbol{I}-\boldsymbol{A})\boldsymbol{x}_{t^*} \\ &-(1+\alpha)(\boldsymbol{I}-\boldsymbol{A}-\alpha\boldsymbol{B})\boldsymbol{x}_{t^*-1}. \end{aligned} \tag{21}$$

For the rate of growth α we have:

$$(1+\alpha)\boldsymbol{x}_{t^*-1} = \boldsymbol{x}_{t^*}. \tag{22}$$

By substituting Eq. (22) into the inequality (Eq. (21)) finally we obtain the inequality (Eq. (18)) in the following form:

$$\boldsymbol{c}_{t^*} - \boldsymbol{c}_{t^*-1} \ge \alpha\boldsymbol{B}\boldsymbol{x}_{t^*}.$$

The right-hand side of the previous inequality is non-negative for $\alpha > 0$, $\boldsymbol{B}$ is a non-negative matrix and $\boldsymbol{x}_{t^*}$ is a non-negative vector. This concludes that $\boldsymbol{c}_{t^*} - \boldsymbol{c}_{t^*-1} \ge 0$.

Remark 3. The decrease of the growth rate of production from the value α to the value 0 results in an excess supply of economic products in the year t^*. Because the less growth rate of production induces less investments in capital goods. This surplus of goods results a sudden growth of the consumption level in the year t^*. In the closed dynamic Leontief model, the carrying capacity cannot be exceeded, as we have shown for this model type. In Section 5.2, we will introduce imports and exports to the model. In this case, the biocapacity of the land does not place an upper limit on the economic growth of a country. The necessary "land" could be imported from oversea to satisfy the final demand and capital accumulation of the economy.

5.2 Balanced growth path in the open economy

Suppose that final consumption (domestic and import) and export grow at the given same rate, α. For an arbitrary period, $\boldsymbol{c}_t = (1+\alpha)^t \boldsymbol{c}_0$:

$$\mathbf{exp}_t = (1+\alpha)^t \mathbf{exp_0} \text{ and } \boldsymbol{c}_{\boldsymbol{imp},t} = (1+\alpha)^t \boldsymbol{c}_{\boldsymbol{imp},0}.$$

We examine a special case, where the production and the import increase this α rate too, i.e., $\boldsymbol{imp}_t = (1+\alpha)^t \boldsymbol{imp}_0$ and $\boldsymbol{x}_t = (1+\alpha)^t \boldsymbol{x}_0$.

The dynamic system is:

$$\begin{aligned} \begin{bmatrix} \boldsymbol{x}_0 \\ \boldsymbol{imp}_0 \end{bmatrix} &= \begin{bmatrix} \boldsymbol{A} & 0 \\ \boldsymbol{A}_{imp} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_0 \\ \boldsymbol{imp}_0 \end{bmatrix} \\ &+\alpha \begin{bmatrix} \boldsymbol{B} & 0 \\ \boldsymbol{B}_{imp} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_0 \\ \boldsymbol{imp}_0 \end{bmatrix} + \begin{bmatrix} \boldsymbol{c}_0 + \mathbf{exp_0} \\ \boldsymbol{c}_{\boldsymbol{imp},0} \end{bmatrix}. \end{aligned} \tag{23}$$

If we would like to have initial total output and import, which consumption and export are needed for this case? For the solution, we must solve the following linear equation system:

$$\begin{bmatrix} \boldsymbol{I}-\boldsymbol{A}-\alpha\boldsymbol{B} & 0 \\ -\boldsymbol{A}_{imp}-\alpha\boldsymbol{B}_{imp} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_0 \\ \boldsymbol{imp}_0 \end{bmatrix} = \begin{bmatrix} \boldsymbol{c}_0 + \mathbf{exp_0} \\ \boldsymbol{c}_{\boldsymbol{imp},0} \end{bmatrix}. \tag{24}$$

The inverse of the left-side matrix for the solution is:

$$\begin{aligned} &\begin{bmatrix} \boldsymbol{I}-\boldsymbol{A}-\alpha\boldsymbol{B} & 0 \\ -\boldsymbol{A}_{imp}-\alpha\boldsymbol{B}_{imp} & \boldsymbol{I} \end{bmatrix}^{-1} \\ &= \begin{bmatrix} (\boldsymbol{I}-\boldsymbol{A}-\alpha\boldsymbol{B})^{-1} & 0 \\ (\boldsymbol{A}_{imp}+\alpha\boldsymbol{B}_{imp})(\boldsymbol{I}-\boldsymbol{A}-\alpha\boldsymbol{B})^{-1} & \boldsymbol{I} \end{bmatrix}. \end{aligned}$$

Using the inverse matrix, we obtain the following expression:

$$\begin{aligned} \begin{bmatrix} \boldsymbol{x}_0 \\ \boldsymbol{imp}_0 \end{bmatrix} &= \begin{bmatrix} (\boldsymbol{I}-\boldsymbol{A}-\alpha\boldsymbol{B})^{-1} & 0 \\ (\boldsymbol{A}_{imp}+\alpha\boldsymbol{B}_{imp})(\boldsymbol{I}-\boldsymbol{A}-\alpha\boldsymbol{B})^{-1} & \boldsymbol{I} \end{bmatrix} \\ &\times \begin{bmatrix} \boldsymbol{c}_0 + \mathbf{exp_0} \\ \boldsymbol{c}_{\boldsymbol{imp},0} \end{bmatrix}. \end{aligned} \tag{25}$$

We can determine the maximal non-negative growth path of the economy by solving the following eigenvalue problem:

$$\frac{1}{\alpha^0}\begin{bmatrix}\boldsymbol{x_0}\\ \boldsymbol{imp_0}\end{bmatrix}=\begin{bmatrix}(\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{B} & 0\\ \boldsymbol{A}_{imp}(\boldsymbol{I}-\boldsymbol{A})\boldsymbol{B}+\boldsymbol{B}_{imp} & 0\end{bmatrix}\begin{bmatrix}\boldsymbol{x_0}\\ \boldsymbol{imp_0}\end{bmatrix}.$$

The non-negative solution is the smallest positive eigenvalue of the economy and the eigenvector, which is associated with the eigenvalue. According to the Perron-Frobenius theorem, this eigenvalue is non-negative (more detail, for example, Zalai, 1989:p.61). Suppose that this eigenvalue is α^0. If we choose a smaller growth path than α^0 in the economy, a non-negative solution exists to the equation system. We examine the land requirement along this path. The land requirement for domestic production is:

$$\langle \boldsymbol{l}\rangle \boldsymbol{x_0}=\langle \boldsymbol{l}\rangle(\boldsymbol{I}-\boldsymbol{A}-\alpha\boldsymbol{B})^{-1}\boldsymbol{c}_0+\langle \boldsymbol{l}\rangle(\boldsymbol{I}-\boldsymbol{A}-\alpha\boldsymbol{B})^{-1}\mathbf{exp_0}. \quad (26)$$

The internal consumption is:

$$\begin{aligned}&\langle \boldsymbol{l}\rangle(\boldsymbol{I}-\boldsymbol{A}-\alpha\boldsymbol{B})^{-1}\boldsymbol{c}_0+\langle \boldsymbol{l}\rangle(\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{imp_0}\\ &=\langle \boldsymbol{l}\rangle(\boldsymbol{I}-\boldsymbol{A}-\alpha\boldsymbol{B})^{-1}\boldsymbol{c}_0+\langle \boldsymbol{l}\rangle(\boldsymbol{I}-\boldsymbol{A})^{-1}\left(\boldsymbol{A}_{imp}+\alpha\boldsymbol{B}_{imp}\right)\\ &\times(\boldsymbol{I}-\boldsymbol{A}-\alpha\boldsymbol{B})^{-1}(\boldsymbol{c}_0+\mathbf{exp_0})+\langle \boldsymbol{l}\rangle(\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{c}_{\boldsymbol{imp},0}.\end{aligned} \quad (27)$$

The ecological footprint is Eqs. (27)–(26):

$$\begin{aligned}&\langle \boldsymbol{l}\rangle(\boldsymbol{I}-\boldsymbol{A}-\alpha\boldsymbol{B})^{-1}\mathbf{exp_0}-\langle \boldsymbol{l}\rangle(\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{imp_0}\\ &=\langle \boldsymbol{l}\rangle(\boldsymbol{I}-\boldsymbol{A}-\alpha\boldsymbol{B})^{-1}\mathbf{exp_0}-\langle \boldsymbol{l}\rangle(\boldsymbol{I}-\boldsymbol{A})^{-1}\\ &\times\left[\left(\boldsymbol{A}_{imp}+\alpha\boldsymbol{B}_{imp}\right)(\boldsymbol{I}-\boldsymbol{A}-\alpha\boldsymbol{B})^{-1}(\boldsymbol{c}_0+\mathbf{exp_0})+\boldsymbol{c}_{\boldsymbol{imp},0}\right].\end{aligned} \quad (28)$$

This expression can be further modified to take into account the effects of each factor, i.e., export, internal consumption and products imported directly for consumption:

$$\begin{aligned}&\langle \boldsymbol{l}\rangle\left[\boldsymbol{I}-(\boldsymbol{I}-\boldsymbol{A})^{-1}\left(\boldsymbol{A}_{imp}+\alpha\boldsymbol{B}_{imp}\right)\right]\\ &\times(\boldsymbol{I}-\boldsymbol{A}-\alpha\boldsymbol{B})^{-1}\mathbf{exp_0}-\langle \boldsymbol{l}\rangle(\boldsymbol{I}-\boldsymbol{A})^{-1}\left(\boldsymbol{A}_{imp}+\alpha\boldsymbol{B}_{imp}\right)\\ &\times(\boldsymbol{I}-\boldsymbol{A}-\alpha\boldsymbol{B})^{-1}\boldsymbol{c}_0-\langle \boldsymbol{l}\rangle(\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{c}_{\boldsymbol{imp},0}.\end{aligned} \quad (29)$$

Investigating at what growth rate such an economy will become self-sustaining requires further research. The difficulty in carrying out such an investigation is that the derivation of the growth rate derivative of the former term is not a simple task.

6 Numerical example

6.1 Closed economy case

In Section 6.1, we demonstrate numerical examples of the model. Let us assume that the investigated economy produces three goods and two types of land. The matrices of input coefficients, capital coefficients and land-use coefficients are as follows:

$$\boldsymbol{A}=\begin{bmatrix}0.2 & 0.3 & 0.2\\ 0.5 & 0.3 & 0.1\\ 0.2 & 0.3 & 0.3\end{bmatrix},$$

$$\boldsymbol{B}=\begin{bmatrix}0.07 & 0.03 & 0.02\\ 0.06 & 0.07 & 0.04\\ 0.07 & 0.06 & 0.03\end{bmatrix},$$

and:

$$\boldsymbol{L}=\begin{bmatrix}0.6 & 0.2 & 0.1\\ 0.4 & 0.5 & 0.3\end{bmatrix}.$$

Suppose that the marginal growth rate of the model is 0.376 ($\alpha_0 = 0.376$). It means that a rational growth rate must be lower than this growth rate.

Let us assume next that the balanced growth rate α is equal to 0.10, i.e., 10%, and the vectors of initial consumption level $\boldsymbol{c_0}$ and biocapacity of land $\boldsymbol{l}$ are:

$$\boldsymbol{c}_0=\begin{bmatrix}5\\ 7\\ 6\end{bmatrix} \text{ and } \boldsymbol{l}=\begin{bmatrix}200\\ 300\end{bmatrix}.$$

The planning horizon of the economy is $T = 35$ years. Applying Eq. (15) the initial output of the economy is:

$$\boldsymbol{x}_0(0.10)=\begin{bmatrix}28.215\\ 35.597\\ 32.616\end{bmatrix}.$$

The balanced growth path for the economy is as follows:

$$\boldsymbol{c_t}=(1+\alpha)^t\boldsymbol{c}_0,\ t<21,$$

$$\boldsymbol{c_t}=\boldsymbol{c_e},\ 21\le t\le 35,$$

where the new initial consumption rate $\boldsymbol{c_e} = (1 + \alpha)^{20}\boldsymbol{c}_0$ is:

$$\boldsymbol{c_e}=\begin{bmatrix}36.123\\ 50.785\\ 43.789\end{bmatrix}.$$

The land-use path for the first type of land is depicted in the Fig. 4 with the land capacity as an upper bound. (The capacity is depicted with a dotted line). In our example, the land-use doesn't reach the upper bound over the time interval under study. This is an example for that it is achievable to maintain the land used at an appropriate level in an economy.

Fig. 5 presents the production level for the first activity. The dotted line shows a balanced growth path without

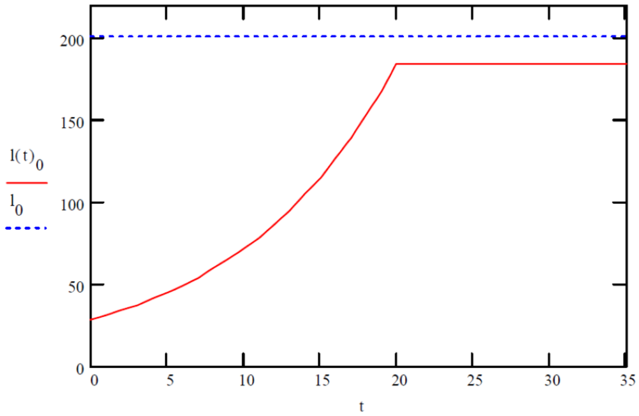

Fig. 4 The land-use and the capacity of land 1

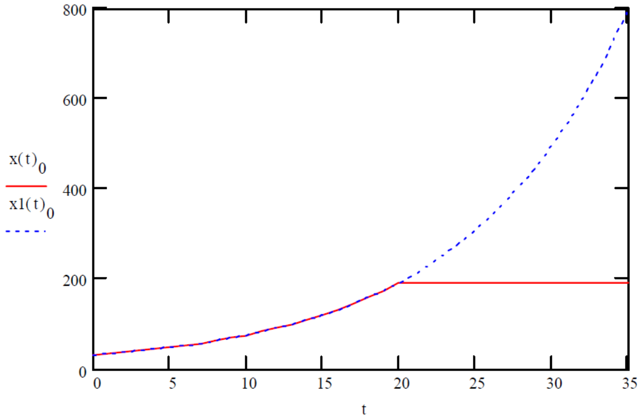

Fig. 5 The balanced growth path for the first sector both without and with capacity, as an upper limit

carrying capacity constraints. As can be seen, the growth path will be significantly lower after capacity is attained.

Fig. 6 shows the development of the consumption level over time. The dotted line represents the consumption level in case of no upper bound on the land-use. After the carrying capacity is achieved, consumption is higher than without it. However, after three periods, the consumption level is lower than with environmental standard. The consumption increases because less goods will be invested to increase production. It is a positive effect of capacity on consumption.

6.2 Open economy case

In the following, suppose that there are three goods and two types of land in the economy. The matrices of input coefficients of import and capital coefficients of import are the following:

$$\boldsymbol{C}_A = \begin{bmatrix} 0.02 & 0.03 & 0.02 \\ 0.05 & 0.03 & 0.01 \\ 0.02 & 0.03 & 0.03 \end{bmatrix},$$

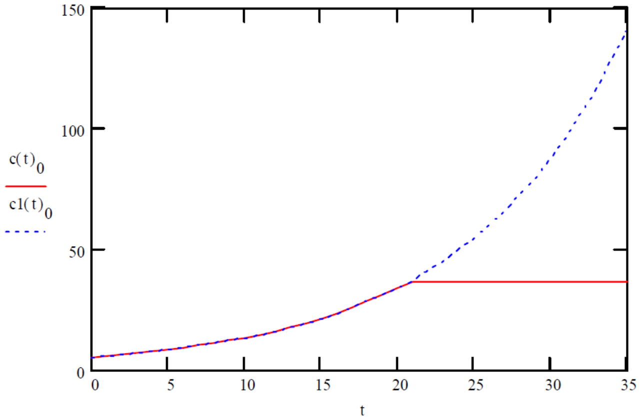

Fig. 6 The path of the consumption level for the first good without and with carrying capacity constraints

$$\boldsymbol{C}_B = \begin{bmatrix} 0.007 & 0.003 & 0.002 \\ 0.006 & 0.007 & 0.004 \\ 0.007 & 0.006 & 0.003 \end{bmatrix},$$

i.e., we have assumed that the import matrices are equal to $\boldsymbol{C}_A = 0.1\ \boldsymbol{A}$, $\boldsymbol{C}_B = 0.1\ \boldsymbol{B}$. The initial import level to final demand and export level are known:

$$\boldsymbol{c}_{i,0} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \text{ and } \boldsymbol{e}_0 = \begin{bmatrix} 7 \\ 7 \\ 8 \end{bmatrix}.$$

The land-use path for the first type of land is depicted in Fig. 7, with the biocapacity as an upper bound. (Capacity is depicted with a dotted line.) This economy exceeds the carrying capacity after the 33 period. It means that the economy after this period uses the land of other countries. The consequence of this example is that the economy can reduce long-term negative impacts through appropriate short-term measures.

We have depicted the balance of the export and import for the analysed economy. As can be seen, the export of the first product of the economy exceeds the imports along the planning horizon. This economy imports more products from the second product than it exports (Fig. 8).

7 Conclusion

In this paper, we have investigated the effect of the ecological footprint on production and consumption in a dynamic Leontief model in the case of a balanced growth path. Our study aims to extend the methodology of ecological footprint calculation. We have shown that the dynamic input-output model can be used to determine the size of the ecological footprint in relation to the land area used.

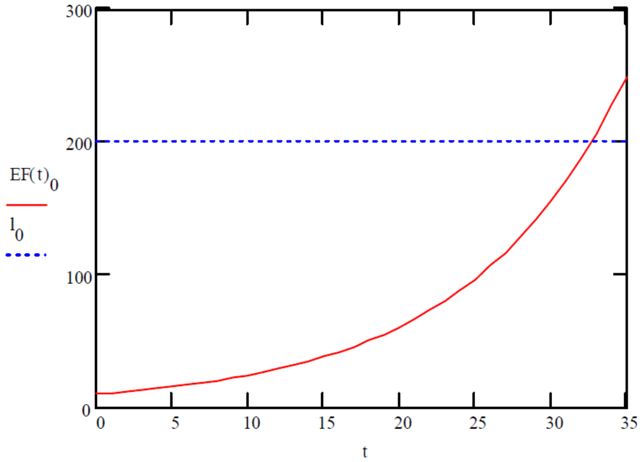

Fig. 7 The dynamic ecological footprint with carrying capacity constraints for the first type of land

If the land's carrying capacity is adequate, i.e., it is a constraint on the production, then the production and consumption growth rate will be lower after the allowed levels are attained. Of course, we cannot offer the restriction of consumption in this process. Our policy proposal is to encourage planned action in the short term to ensure that long term impacts are minimised and not harmful. Using the methodology, new indicators can be used to provide information to policy makers.

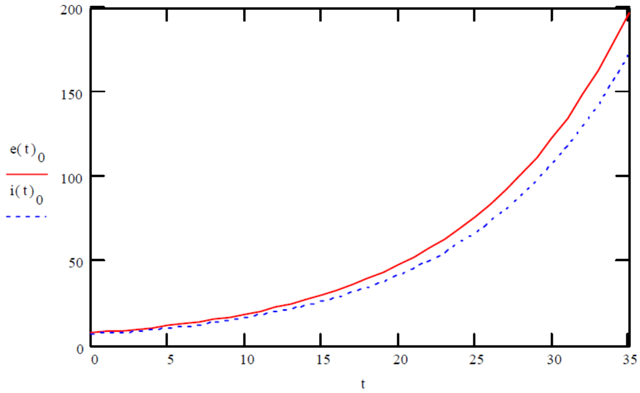

Fig. 8 Export and import path for the first product of the economy

The investigated model assumes no technological development in the economy. In further research, we could introduce technological development into the economic model, i.e., the matrices of the model could be changed in time. In a modern economy, research and development will develop new technologies to save the environment.

A second extension would be to investigate the path of the ecological footprint in a dynamic empirical model. To achieve this, we can apply the Leontief generalised inverse to this dynamic input-output model. Such a dynamic analysis would make it possible to examine the changes in land-use within a specified time interval.

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