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A CRITIQUE OF SOME CLASSICAL THEORIES OF DECISIONS UNDER UNCERTAINTY

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Abstract

The present article describes and examines the orthodox types of uncertainty and shows that they are inadmissibly oversimplified. Real decision situations cannot be classified into the three classes of the orthodox model.

The article describes the most important classical decision criteria based on the orthodox uncertainty types. It reviews briefly some previous criticisms of them, which refer to two aspects:

the different criteria give very different results from the same data,

each of the criteria is incompatible with one or more reasonable requirements of consistent choice.

The paper shows that the problem is not that the mathematical algorithms of these criteria are not good enough, but that their common conceptual starting point is false. Therefore any decision criteria of this kind is unacceptable.

Finally the paper examines the famous St. Petersburg paradox and shows that Bernoulli's equation of the expected payoff is based on a false assumption. Correcting the equation the paradox disappears and the correct equation does not lead to the conclusion that the expected payoff is not appropriate for valuing uncertain prospects.

Keywords: types of uncertainty, decision criteria, St. Petersburg paradox.

1. Introduction

The aim of the present paper is the critical re-examination of some classical theories of decisions under uncertainty:

- the classical types of uncertainty,
- the classical decision criteria,
- Bernoulli's St. Petersburg paradox.

The goal is to judge their relevance to real-life decision making.

2. Notation

The following notation is used:

i = index for actions
m = number of actions ($i = 1...m$)
A_i = action i
j = index for states of nature
n = number of states of nature ($j = 1...n$)
S_j = state j
p_j = probability of S_j
V_{ij} = outcome value of consequence ij
E_i = expected monetary value (EMV) of A_i
opt = index for optimal choice
W_i = Wald's security level of A_i
H_i = Hurwicz's optimism level of A_i
z = Hurwicz's optimism-pessimism index
R_{ij} = Savage's regret (loss) of A_i given that S_j is the true state of nature
Y_i = Savage's index of worst regret of A_i
L_i = Bayes's and Laplace's expected value of A_i
t = index for tosses in Bernoulli's St. Petersburg gamble
T = number of tosses in the modified St. Petersburg gamble ($t = 1...T$)

3. Criticism of the Classical Types of Uncertainty

In structuring a decision problem three factors are identified traditionally:
- states of nature (states of the world),
- courses of action, and
- consequences (often represented by their outcome values).

The well-known classical decision table of these three factors is shown in *Table 1*.

The consequence of any action is determined not just by the action itself but also by a number of external factors, which are beyond the control of the decision maker. By *nature (or the world)* we mean the complex entirety of these external factors, in other words the object about which the person is concerned. A *state* of nature (or the world) means a complete description of the external factors, leaving no relevant aspect undescribed. The true state of nature (or the world) is the state that in fact obtain, i.e., the true description of nature (FRENCH, 1986; SAVAGE, 1972).

Decision problems have conventionally been categorised according to the decision maker's knowledge of the state of nature.

Table 1
A general decision table with the consequences represented by their outcome values

	Values	S_1	S_2	S_3	... S_n
	A_1	V_{11}	V_{12}	V_{13}	... V_{1n}
	A_2	V_{21}	V_{22}	V_{23}	... V_{2n}
	.	.	.	.	... .
Actions	.	.	.	.	... .
	.	.	.	.	... .
	A_m	V_{m1}	V_{m2}	V_{m3}	... V_{mn}

(States of nature span columns S_1 … S_n.)

1st type uncertainty: states of nature are unknown.
2nd type uncertainty: states of nature are known but their probabilities are unknown.
3rd type uncertainty: states of nature and their probabilities are known.

If we know that only one state of nature is possible then we have a decision under certainty.

Unfortunately these types of uncertainty were given different names by different authors, as shown in *Table 2*. For this reason in this paper they are marked with the numbering above.

Are these types of uncertainty appropriate for modelling real-life decision situations? Is reality so black and white that we either know every possible state of nature or don't know any of them, and we either know all probabilities or don't know anything about them?

Let's have an example. We and our competitors want to submit a tender for a contract. States of nature are the possible sets of competitors, i.e. all combinations of the potential applicants. If the number of potential applicants is a, then the number of states of nature is $2a$ because each of them either submits a tender or doesn't.

If we know every competitor then we know every state of nature at the same time. It can happen only if the call for tenders is exclusive or the nature of the business is so special that the existence of new, previously unknown entrants can be precluded (nuclear technology, satellites, etc.). But this case is not typical, to say the least. The other extreme is also very difficult to imagine, when we don't know any competitor of us in our business. In real life we almost always know some of our present competitors and potential new entrants while we don't know some others.

The same applies to the probabilities. If there has been a lot of calls for tenders of the same type then we know the relative frequencies of the

Table 2
Different names of the types of uncertainty

Types	Names	Authors
1st	non-structured uncertainty	Kaufmann, 1968
2nd	structured uncertainty	
3rd	chance	
1st	uncertainty in narrower sense	Szentpéteri, 1980
2nd	uncertainty in wider sense	
3rd	risk	
1st	ignorance	Collingridge, 1982
2nd	restricted uncertainty	
3rd	risk	
2nd	uncertainty	Baird, 1989; Baumol, 1977;
3rd	risk	Dannenbring – Starr, 1981;
		Forgionne, 1986; Jándy, 1975;
		Kmietowicz – Pearman, 1981;
		Knight, 1921; Oberstone, 1990
2nd	strict uncertainty	French, 1986
3rd	risk	
2nd	vague uncertainty	Budescu – Wallsten, 1987
3rd	precise uncertainty	Dummet, 1975; Wallsten, 1990

different sets of applicants so far. But the complete knowledge of the probabilities would imply unchanged set of competitors as potential applicants for a long time, without any new entrants to the business concerned. The other extreme means that this is the very first call for tenders of that type, that's why we don't know relative frequencies at all. In real life there are older and newer competitors, so we know some of the possibilities more precisely, some of them less precisely, and some of them can be unknown.

It follows from the foregoing that the types of uncertainty cannot be arranged along one axis than in the orthodox model. The knowledge of the states of nature and the knowledge of the probabilities are two separate dimensions. On the same level of our knowledge of the states of nature, i.e. the possible sets of applicants, our level of knowledge of their probabilities can differ from each other very much. For instance, if we get newer and more precise business statistics then our knowledge of the probabilities will get more certain, while our knowledge of the states (because of the unknown new entrants) won't change. It is possible that our knowledge about the probabilities of a very limited number of states improves dramatically, while we still don't know the vast majority of the states of nature at all.

It is very easy to give several more similar examples. The extreme cases of the orthodox model occur very rarely in real life. What is typical in reality that our knowledge of both the states of nature and their probabilities is fragmented more or less.

4. Classical Decision Criteria

On the basis of the orthodox types of uncertainty several decision criteria have been developed.

4.1. Decision Criterion under 3rd Type Uncertainty

There is a widely quoted and accepted criterion for decision making under 3rd type uncertainty, usually called the Expected Monetary Value or EMV criterion. (LAPLACE, 1812).

Suppose that we can assign probabilities $p_1, p_2, \ldots, p_n$ to the n states of nature. With the outcome values V_{ij} when the decision maker takes action i and nature is in state j, we can calculate the expected monetary value E_i for the decision-maker's action A_i by *Eq.* (1):

$$E_i = \sum_{j=1}^{n} p_{ij} V_{ij}. \tag{1}$$

The EMV criterion requires that the decision maker takes the action with maximum E_i:

$$choose\ \ A_{opt}\ \ such\ \ that\ \ E_{opt} = \max_{i=1}^{m}\{E_i\}. \tag{2}$$

4.2. Decision Criteria under 2nd Type Uncertainty

The first decision criterion we examine is Wald's maximin return (WALD, 1950). Under the action A_i the worst possible consequence that can occur has a value to the decision maker of

$$W_i = \min_{j=1}^{n}\{V_{ij}\}. \tag{3}$$

W_i is called the *security level of* A_i and means that A_i guarantees the decision maker a return of at least W_i. Wald suggested that the decision maker should choose the action which has the highest security level:

$$choose\ \ A_{opt}\ \ such\ \ that\ \ W_{opt} = \max_{i=1}^{m}\{W_i\}. \tag{4}$$

Hurwicz developed an optimistic criterion by considering the best possible outcome for each action. Define the *optimism level of A_i* to be:

$$H_i = \max_{j=1}^{n}\{V_{ij}\}. \tag{5}$$

Thus H_i is the value of the best consequence that can result if A_i is taken. Hurwicz's *maximax return criterion* is:

$$choose\ \ A_{opt}\ \ such\ \ that\ \ H_{opt} = \max_{i=1}^{m}\{H_i\}. \tag{6}$$

Hurwicz proposed taking a middle course between extreme optimism and pessimism (HURWICZ, 1951). He introduced the *optimism-pessimism index* z which is specific to the individual decision maker and is applicable to any decision problem that faces

$$0 \leq z \geq 1. \tag{7}$$

Hurwicz recommends the decision rule:

$$choose\ \ A_{opt}\ \ such\ \ that\ \ zW_{opt} + (1-z)H_{opt} = \max_{i=1}^{m}\{zW_i + (1-z)H_i\}. \tag{8}$$

Savage proposed comparing the consequence of every action with the consequences of other actions under the same state of nature (SAVAGE, 1951). The *regret* R_{ij} is defined as the difference between the value resulting from the best action given that S_j is the true state of nature and the value resulting from A_i under S_j:

$$R_{ij} = \max_{j=1}^{n}\{V_{ij}\} - V_{ij}. \tag{9}$$

Using the regret table of R_{ij} values each action should be assigned the *index of worst regret* Y_i that can result from action A_i:

$$Y_i = \max_{j=1}^{n}\{R_{ij}\}. \tag{10}$$

Savage's *minimax* regret criterion is:

$$choose\ \ A_{opt}\ \ such\ \ that\ \ Y_{opt} = \min_{i=1}^{m}\{Y_i\}. \tag{11}$$

Savage himself called R_{ij} 'loss', not 'regret', but it became famous as 'regret', against his will. '... some have proposed to call loss 'regret', but that term seems to me charged with emotion and liable to lead to such misinterpretation as that the loss necessarily becomes known to the person.' (SAVAGE, 1972, p. 163).

The Bayes–Laplace criterion (often called just Laplace criterion) employs the principle of insufficient reason which postulates that if no information is available about the probabilities of the states of nature, it is only reasonable to assume that they are equally likely. (BAYES, 1763; LAPLACE, 1825). Thus, if there are n states of nature, the probability of each is $1/n$. Then the expected payoff L_i for each strategy A_i:

$$L_i = \sum_{j=1}^{n} (1/n) V_i j. \tag{12}$$

The criterion goes on to suggest selecting action A_i with the highest expected payoff L_i:

$$choose\ \ A_{opt}\ \ such\ \ that\ \ L_{opt} = \max_{i=1}^{m} \{L_i\}. \tag{13}$$

5. Brief Summary of Earlier Critiques of Decision Criteria

5.1. Limitations of the EMV Criterion

There are some limitations of the application of the EMV criterion which must be taken into consideration.

'Clearly, if the same decision problem is met on a large number of occasions, the EMV criterion will find the action which, when repeatedly used, gives the greatest total payoff. Most decision problems are of a 'one-off' nature or have at most a very limited number of repetitions. In such cases the applicability of the EMV criterion may be questioned. Two points may be made in defence of this criterion. Firstly, although a decision may be 'one off' it is likely that decisions of similar importance in terms of the size of their payoffs will be taken regularly by people in the same organization. Thus, although on a single occasion an action may have been taken which with hindsight appears to be a poor one, the experience has to be balanced against more favourable occasions. (...) The second point is that even when the decision has to be viewed as strictly 'one off' the EMV criterion is a reasonable way out of the inherent dilemma of taking the risk, as it gives an appropriate weight to each possible payoff outcome. Some action has to be taken, and nothing will remove the chance of 'being wrong'.' (GREGORY, 1988).

'... we should point out that the EMV criterion is very widely used in practice. Many people would argue that it is even appropriate to apply it to one-off decisions since, although an individual decision may be unique, a decision maker may, over time, make a large number of decisions that involve similar monetary sums so that returns will still be maximized by the consistent application of this criterion. Moreover, large organizations may be able to sustain losses on projects that represent only a small part of their operations. In these circumstances it may be reasonable to assume that risk neutrality applies, in which case the EMV criterion will be appropriate.' (GOODWIN – WRIGHT, 1991).

So the limitation described above doesn't mean that the EMV would be mathematically or conceptually incorrect.

There are some further objections against the EMV criterion. 'It should be also noted that the EMV criterion assumes that the decision maker has a linear value function for money. (...) A further limitation of the EMV criterion is that it focuses on only one attribute: money.' (GOODWIN – WRIGHT, 1991).

These latter objections cannot be accepted at all. We can use *Eq* (1) and the maximum expected value criterion with utility values as well, using different kinds of utility functions. In this case we call it EU (expected utility) instead of EMV. (NEUMANN – MORGENSTERN, 1944; TVERSKY, 1967; HAMPTON et al., 1973; KMIETOWICZ – PEARMAN, 1981; etc.) We can also use Multiple Criteria Decision Making (MCDM) ranking algorithms for calculating multi attribute utility values. Instead of simple monetary outcome values we can use any kind of scores, so this limitation simply does not exist.

It was substantially important to clarify that the EMV (or EU) criterion is absolutely correct because the conceptual criticism of 2nd type decision criteria (in chapter 6) is based on this assumption.

5.2. Problems with 2nd type Decision Criteria

All of the decision criteria for 2nd type uncertainty have been criticised for two reasons:

- the different criteria give very different results from the same data,
- each of the criteria is incompatible with one or more reasonable requirements of consistent choice.

The classical example of the first point is shown in *Table 3* (MILNOR, 1954).

Clearly Wald's criterion will pick A_2, Savage's criterion will pick A_4, and the Bayes–Laplace criterion will pick A_1. Hurwicz's criterion will pick

Table 3
An example for comparing the results of different decision criteria (MILNOR, 1954)

	S_1	S_2	S_3	S_4	W_i	H_i	R_i	L_i
A_1	2	2	0	1	0	2	2	5/4
A_2	1	1	1	1	1	1	3	1
A_3	0	4	0	0	0	4	2	1
A_4	1	3	0	0	0	3	1	0

A_3 for any $z < 3/4$ and A_2 for any $z > 3/4$. If $z = 3/4$ then we can't choose between A_2 and A_3.

Milnor stated ten requirements for reasonable decision criteria (MILNOR, 1954). French omitted some of them and added some new ones, and stated eight principles of consistent choice as axioms (FRENCH, 1986). They both found that there are some requirements which are met by all criteria, but none of the criteria meets every requirement. *Table 4* shows an extract of Milnor's and French's results, showing only the axioms which are not met by one or more decision criteria (MILNOR, 1954; FRENCH, 1986).

Table 4
Incompatibility of decision criteria with some principles of consistent choice (extract from MILNOR, 1954; FRENCH, 1986) + indicates compatibility, – indicates incompatibility

	Wald	Hurwicz	Savage	Bayes–Laplace
Independence of irrelevant alternatives	+	+	–	+
Independence of addition of constant to a column	–	–	+	+
Independence of row permutation	+	+	–	+
Independence of column duplication	+	+	+	–
Convexity	+	–	+	+

Independence of irrelevant alternatives demands that the ranking of two actions in a decision table should be independent of any other actions that are available. In other words the ordering between old rows should not be changed by adding a new row. This axiom is also called *row adjunction*.

Independence of addition of constant to a column means that the ordering should not be changed by adding a constant to a column. This axiom is also called column linearity.

Independence of row permutation is illustrated in *Table 5*. (FRENCH, 1986). If the decision maker knows nothing about the probabilities then it is reasonable to be indifferent between the two actions in the table.

Table 5

An example of row permutation (FRENCH, 1986)

	S_1	S_2	S_3
A_1	6	0	3
A_2	0	6	3

Independence of column duplication is illustrated in *Table 6* (FRENCH, 1986). If states S_2', S_3', S_4', and S_5' are gathered together and identified with a single composite state S_2'', then *Table 6(b)* becomes identical to *Table 6(a)*. If we cannot say anything about the probabilities then we have no argument that suggests S_2 in *Table 6(a)* is any different to S_2'' in the collapsed table. So adding an identical column should not change the ordering.

Table 6

An example of column duplication (FRENCH, 1986)

(a)

	S_1	S_2
A_1	9	4
A_2	2	6

(b)

	S_1'	S_2''			
		S_2'	S_3'	S_4'	S_5'
A_1'	9	4	4	4	4
A_2'	2	6	6	6	6

Convexity: if A' and A'' are indifferent in the ordering, then neither A' nor A'' should be preferred to $(1/2A', 1/2A'')$. The most seriously criticized criterion is the one by Bayes and Laplace. 'A rule (...) has been widely adopted, generally under the title of the *principle of non-sufficient reason*, down to the present time. This description is clumsy and unsatisfactory, and, if it is justifiable to break away from tradition, I prefer to call it *the principle of indifference*. (...) This rule, as it stands, may lead to paradoxical and even contradictory conclusions.' (KEYNES, 1921).

There are several illustrative examples of unknown but unequal probabilities in the literature, criticizing this disreputable principle. (TODHUNTER, 1865; KEYNES, 1921; SAVAGE, 1954; LANGE, 1964; FRENCH, 1986; etc.) There is no need to quote any of them because anyone can easily create as many examples of this kind as he or she wants. The reason for assuming equal probabilities is obviously exactly as insufficient as any other probabilities in a 2nd type uncertain decision situation. 'In fairness to James Bernoulli (1654–1705) who first stated the principle of insufficient reason, the claim for this principle was merely that if there is no evidence to believe that one outcome is more likely than any other, then all outcomes should be judged to have the same probability. This is certainly not the same thing as complete ignorance.' (GREGORY, 1988)

'A final word: SAVAGE, (1972) renounced the use of minimax regret ideas, and it is far from certain that Laplace, Hurwicz or Wald would still champion the decision rules that bear their names.' (FRENCH, 1986).

6. Criticism of the Basic Concept of 2nd Type Decision Criteria

Is it the problem that the classical algorithms and criteria are not good enough? Should we try to improve them or to develop completely new, better ones?

In paragraph 5.1 we found that the EMV (EU) criterion is absolutely correct, and accepted *Eqs.* (1) and (2) as the appropriate way of choosing the best action. Considering these equations the difference between 2nd and 3rd type uncertainty is that under 2nd type uncertainty we don't know the p_{ij} values. So the common basic principle of every kind of 2nd type decision criteria is that *we can say which sum of products is highest without knowing one of the multiplicands of every product!!!* 'No kind of 'principle' substitutes missing information, just produces the illusion of exactness.' (PATAKI, 1989).

Hurwicz's optimism-pessimism index, z, perhaps might be considered a kind of estimation of the unknown probabilities. This kind of estimation is usually called 'subjective' or 'personal' probability. But this criterion (like Savage's and Wald's) considers only the two extreme payoffs. If we considered all states of nature with their subjective probabilities then we would be back to the EVM (EU) criterion. But Hurwicz neglects every state with non-extreme payoffs, regardless of their potentially extremely high probabilities.

The problem of 2nd type uncertainty can't be solved with tricky algorithms just with information gathering, which means getting closer to

3rd type in the knowledge level spectrum described in the criticism of the classical uncertainty types in chapter 3.

7. Criticism of the St. Petersburg Paradox

One of the most famous theoretical decision situations under uncertainty is the St. Petersburg gamble. (BERNOULLI, 1738)

'Suppose that you have the opportunity of playing the following game. A fair penny will be repeatedly tossed until it falls 'heads'. If this occurs on the t^{th} toss, you will receive £ 2^t. How much would you be prepared to pay to play this game once? Consider your expected payoff. The probability of a fair penny first landing 'heads' on the t^{th} toss is $(1/2)^t$. So your expected payoff is

$$\sum_{t=1}^{\infty}(1/2)^t 2^t = \sum_{t=1}^{\infty} 1 = \infty. \tag{14}$$

Thus, however much you pay to enter the game, you may expect to win more. It would seem to follow, therefore, that you should be willing to risk everything that you possess for the opportunity of playing this game just once. Yet, of course, no one would consider such a course of action to be rational.

The moral of this example is that expected monetary return is not necessarily the appropriate criterion to use in valuing uncertain prospects' (FRENCH, 1986).

Is the St. Petersburg gamble really a paradox? *Eq.* (14) is based on the assumption that we can toss the coin unlimited times, which is impossible in real life. The game simply must come to an end even if the coin has not fallen 'heads' up to that time. Only immortal creatures could toss a coin infinite times. The premise of the example is obviously false, so it does not justify the statement that expected monetary value is not appropriate for valuing this kind of gamble. The St. Petersburg paradox is actually not a paradox but a faulty reasoning based on a false assumption.

If we want to draw a conclusion to the behaviour of real decision makers then we must not set out from the premise of absolutely unreal decision makers. Human beings are mortal, so they have only limited time for playing this game. It means that the number of tosses, T, must be a finite number. If we correct *Eq.* (14) this way than the expected payoff is:

$$\sum_{t=1}^{T}(1/2)^t 2^t = \sum_{t=1}^{T} 1 = T. \tag{15}$$

It means that it is worth to pay for entering this game any amount of money which is less than £ T, supposing that:

- the maximum number of tosses in a game, T, is fixed or at least can be estimated;
- the number of games is high enough to be considered statistical, otherwise probabilities could not be applied. (Infinite number of games, of course, would also require immortal gamblers.)

One game of unlimited number of tosses is absolutely unrealistic. A statistically high number of games of limited tosses is realistic, but does not implicate the conclusion that expected monetary return is not appropriate for valuing uncertain prospects.

8. Concluding Remark

Simon French called decision theory 'the mathematics of rationality' in the subtitle of his book. (FRENCH, 1986). I would call the theories criticized above 'the mathematics of irrationality', no matter how famous they are.

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