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MARKOV TYPE DECISION SUPPORTING SYSTEM

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Abstract

A decision model for pavement management has been developed herein based on linear programming formulation. Markov transition probability matrices are introduced to model the deterioration process of the road sections. To every type of road surfaces and class of traffic amount belongs a certain Markov matrix. The presented model and methodology is used to determine the optimal rehabilitation and maintenance policy in network level. Depending on the objective function two types of problems could be solved by the model : the necessary funds calculation and the optimal budget allocation for the entire network. We have developed the computer program on microcomputer and it has been used by the Ministry of Transport who is responsible for the 30000 km road network of Hungary.

Keywords: Markov process, decision support system, pavement management system, network - level model, linear programming.

Introduction

The current budget condition in the Eastern European Countries needs an effective economical politics. It is true about Hungary too. We try to use the most powerful optimizaation models in every possible field. In this paper we present an optimal decision supporting model that is used to maintain our highway network. Its length is 30000 kilometres. The works began some years ago. First a large scale Road Data Bank was developed (BAKÓ et all, 1989). The second step was to develop a network level Pavement Management System (sc. PMS).

Concerning the PMS problem several types of models are known. One of them is the network level model, the other is a project level one. The network level model deals with the whole network. Its aim is to determine the most advantageous maintenance technique for every subset of the road having the same type of surface, the same condition parameters and the same traffic category. This type of model is a budget planning tool capable of estimating the total lengths and costs of works required on the network for pavement rehabilitation, resurfacing and routine maintenance.

One type of financial planning is generally connected with the determination of the level funding needed to maintain the health of the pavement network at a desirable level. In the other type of model the available budget is given and we have to determine the maintenance politics that fulfil the required constraint of conditions and optimize the total benefit of the society. In the project level model a maintenance and rehabilitation program are determined for each pavement section. We usually use this model in a district.

Several types of solution algorithms can be used depending on the given task, the available data, the budget constraints, etc. (FEIGHAN et all, 1988a, BAKÓ, 1989, COOK at all 1988). Two main types are the heuristic and the optimisation algorithms. The heuristic technique is usually used in project level, but it could be used in network level, too. The optimisation models are solved by the traditional optimisation algorithms. Depending on the problem to be solved we use integer (CHESTER–HARRISON, 1987), a linear (KELVIN et all, 1993) or a dynamic programming algorithm (FEIGHAN, 1988b, MARKOW et all, 1988).

Our model is a linear programming one which has some stochastic elements. Namely the road deterioration process is described by the Markov transition probability matrices. In the second chapter we describe this probability supposition. The third chapter deals with the model formulation. In the last chapter we summarise the applied model itself. The engineering part of the model was developed by GÁSPÁR (1991), the program system was written by SZÁNTAI, T. (1990). Similar model was proposed in Arizona (KELVIN et all, 1993) and in Finland (TALVITIE et all, 1988).

Markov Transition Probability Matrix

In the model we will use the theory of Markov chains (PREKOPA, 1972). To demonstrate this let us suppose that the pavement conditions are described by a certain discrete state. This state contains a discrete set. Let us denote these states by numbers 1,2,.... The change of the system condition in time is probabilistic, and we fix these states in the time $t = 1, 2, \dots$.

The probabilistic variables $x_0, x_1, \dots$ are defined in the following way: $x_n = i$ when the system is in state i in time period $t = n$. The system conditions are described by the $x_0, x_1, \dots$ variables.

We can suppose, that the initial state e.g. x_0 is fixed.

The set of $x = (x_0, x_1, \dots)$ is called a Markov chain, when any integer time set $t_0 < t_1 < \cdots < t_{n+1}$ and states $k_1, k_2, \dots, k_{n+1}$ the following condition is satisfied:

$$P(x_{t_n+1} = k_{n+1}|x_{t_1} = k_1, x_{t_2} = k_2, \dots x_{t_n} = k_n) = \\ = P(x_{t_{n+1}} = k_{n+1}|x_{t_n} = k_n). \tag{1}$$

This condition means that the probability that the system in time t_{n+1} is in state k_{n+1} depends only on the previous state, and independent from the earlier states. Now we define the Markov transition matrix. The r-step homogeneous transition probability is defined by

$$q_{ik}^{(r)} = P(x_{n+r} = k|x_n = i).$$

The $q_{ik}^{(r)}$ values are ordered into a matrix $\mathbf{Q}_r$ which is called the transition probability matrix

$$\mathbf{Q}_r = \begin{pmatrix} q_{11}^{(r)} & q_{12}^{(r)} & \dots \\ q_{21}^{(r)} & q_{22}^{(r)} & \dots \\ \dots & \dots & \dots \end{pmatrix}. \tag{2}$$

This matrix is a stochastic matrix because it is quadratic, its element are non negative and the sum of the columns is equal to 1. It could be shown that the product of two stochastic matrices is also stochastic. We will use this result later.

It can be proved the r-step transition probability matrix is equal to the rth power of the one-step transition probability matrix:

$$\mathbf{Q}_r = \mathbf{Q}^r. \tag{3}$$

The system is ergodic, we can reach every state by positive probability.

On the basis of this theorem we build up the matrix which is used in our model. In this case a state corresponds to a certain condition of a set of sections which has the same type of surface, amount of traffic and quality. The number of rows (and columns) is equal to the number of discrete road states. The $q_{ij} \in \mathbf{Q}_{ij}$ is the probability that the road being in state j will be in state i at the end of the planning period.

Let us suppose that the initial distribution $\mathbf{X} = (\mathbf{X}_{01}, \mathbf{X}_{02}, \dots, \mathbf{X}_{0m})$ is known. We compute the distribution $\mathbf{X}_1$ at the end of the planning period using the Markov matrix $\mathbf{Q}$:

$$\mathbf{X}_1 = \mathbf{X}_0\mathbf{Q}. \tag{4}$$

If there is m planning period, $t = 1, 2, \dots, m$, then the corresponding distributions $\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_m$ are determined by a recursive procedure:

$$\begin{aligned} \mathbf{X}_1 &= \mathbf{Q}\mathbf{X}_0, \\ \mathbf{X}_2 &= \mathbf{Q}\mathbf{X}_1 = \mathbf{Q}\mathbf{Q}\mathbf{X}_0 = \mathbf{Q}^2\mathbf{X}_0, \\ \mathbf{X}_3 &= \mathbf{Q}\mathbf{X}_2 = \mathbf{Q}\mathbf{Q}^2\mathbf{X}_0 = \mathbf{Q}^3\mathbf{X}_0, \\ &\dots \quad \dots \quad \dots \quad \dots \\ \mathbf{X}_m &= \mathbf{Q}\mathbf{X}_{m-1} = \mathbf{Q}^m\mathbf{X}_0. \end{aligned} \tag{5}$$

Model Formulation

The Markov matrix depends on the pavement type, the volume of traffic and the maintenance actions.

In the model we suppose that there are $\underline{s}$ different type of pavement, $\underline{f}$ class of traffic volume and $\underline{t}$ type of maintenance politics. In this case we have $s * f * t$ different $\mathbf{Q}$ matrix. Let us denote the Markov matrix by $\mathbf{Q}_{sft}$, which belongs to the pavement type s, traffic class f and maintenance politics t.

There are several constraints to be fulfilled. We will denote the unknown variable by x_{ijk} which belongs to the pavement type i, to the traffic volume j and to the maintenance politics. The solution have to be Markov stabile. The Markovian stability constraint is

$$\sum_{i=1}^{s}\sum_{j=1}^{f}\sum_{k=1}^{t}(\mathbf{Q}_{ijk} - E)x_{ijk} = 0. \tag{6}$$

where $\mathbf{E}$ is a unit matrix.

Because the equality is usually not fulfilled or not desirable, we use $\leq$ or $\geq$ relation instead of equality in (6). There are several further constraints which are connected with other suppositions. We suppose that the traffic volume will not change during the planning period:

$$\sum_{k=1}^{t} x_{ijk} = b_{ij}, \qquad i = 1, 2, \dots, s, \quad j = 1, 2, \dots, f, \tag{7}$$

where b_{ij} belongs to the pavement type i and the traffic volume j.

The total area of the road surface type i will remain the same at the end of the planning period

$$\sum_{j=1}^{f}\sum_{k=1}^{t} x_{ijk} = d_i, \qquad i = 1, 2, \dots, s, \tag{8}$$

where d_i belongs to the pavement type i and $\sum_{i=1}^{s} d_i = \underline{1}$.

We have to apply one of the maintenance politics on every road section

$$\sum_{i=1}^{s}\sum_{j=1}^{f}\sum_{k=1}^{t} x_{ijk} = \underline{1}. \tag{9}$$

We divide the segments into 3 groups: acceptable (good), unacceptable (bad) and the rest. Let us denote the tree set by $\mathbb{J}$ (good) by $\mathbb{R}$ (bad) and by $\mathbb{E}$ (rest of the segments) and by $\mathbb{H}$ the whole set of segments. The relations for these sets are given by

$$\mathbb{J} \cap \mathbb{R} = \emptyset, \qquad \mathbb{J} \cap \mathbb{E} = 0,$$
$$\mathbb{R} \cap \mathbb{E} = \emptyset, \qquad \mathbb{J} \cup \mathbb{R} \cup \mathbb{E} = \mathbb{H}.$$

The following conditions are related to these sets

$$\sum_{i,j,k \in J} x_{ijk} \geq \nu_J,$$
$$\sum_{i,j,k \in R} x_{ijk} \leq \nu_R,$$
$$\underline{\nu}_E \leq \sum x_{ijk} \leq \overline{\nu}_E,$$

where $\mathbb{J}$, $\mathbb{R}$, $\mathbb{E}$ are given above, and

ν_J the total length of the good road after the planning period
ν_R the total length of the bad road after the planning period
$\underline{\nu}_E$ the lower bound of the other road
$\overline{\nu}_E$ the upper bound of the other road

The meaning of the first condition is that the amount of good segment has to be greater than or equal to a given value. The second relation does not allow more bad roads than it is fixed in advance. The third relation gives an upper and lower limit to the amount of the rest road.

Let us denote by c_{ijk} the unit cost of the maintenance politics k on the pavement type i and traffic volume j. Our objective is to choose such an $\mathbb{X}$ which:

- fulfils the conditions given above
- with minimal rehabilitation cost.

The objective is

$$\sum_{i=1}^{s}\sum_{j=1}^{f}\sum_{k=1}^{t} x_{ijk}c_{ijk} \to \min!$$

Let us denote this value by $\mathbb{C}$. The budget $\mathbb{C}^*$ which is available for the maintenance purpose is usually less than $\mathbb{C}$, so $\mathbb{C}^* < \mathbb{C}$. In this case we modify our model: the above mentioned rehabilitation cost function becomes constrained:

$$\sum x_{ijk} c_{ijk} \leq C^* \tag{11}$$

and we use another objective. Let us denote the benefit by h_{ijk}, where this is the benefit of the societies when we apply on the pavement type i and with the traffic volume j the maintenance politics k.

Our aim is to determine such a solution $\mathbb{X}$ which fulfils the constraints (6)–(10) and (11) and maximises the total benefit of the society.

The objective in this case is

$$\sum x_{ijk} h_{ijk} \rightarrow \max! \tag{12}$$

Two Types of Optimisation Models

We could build up two different types of models using the element given above. One of them is the Necessary Funds Model (NFM), the other is the Budget Bound Model (BBM). In the NFM model we determine the necessary funds needed for ensuring a given condition level of roads with minimal cost. The BBM model is used to distribute a certain amount of money with the given constraints and maximises the benefit of the road users.

The NFM Model

Let us determine the unknown variable matrix $\mathbb{X} = (x_{ijk})$ that fulfils the following conditions

$$\sum_{i,j,k} (\mathbb{Q}_{ijk} - E) x_{ijk} = 0,$$

$$\sum_{k} x_{ijk} = b_{ij}, \qquad i = 1, 2, \ldots, s, \qquad j = 1, 2, \ldots, f,$$

$$\sum_{jk} x_{ijk} = d_i, \qquad i = 1, 2, \ldots, s,$$

$$\sum_{i,j,k} x_{ijk} = \underline{1}, \tag{13}$$

$$\sum_{i,j,k \in J} x_{ijk} \geq \nu_J,$$

$$\sum x_{ijk} \leq \nu_R,$$

$$\underline{\nu}_E \leq \sum_{i,j,k} x_{ijk} \leq \overline{\nu}_E,$$

and

$$\sum_{i,j,k} x_{ijk} c_{ijk} \rightarrow \min!$$

The Budget Bound Model

Determining the unknown matrix $\mathbb{X} = (x_{ijk})$ which fulfils the condition (13) and the following budget limit condition:

$$\sum_{i,j,k} x_{ijk} c_{ijk} \leq C$$

and

$$\sum_{i,j,k} x_{ijk} h_{ijk} \rightarrow \max!$$

Application

The two models have been applied for solving the Hungarian network level Pavement Management System. The road network is divided into smaller groups which depend on the pavement type, the traffic volume and the maintenance action. Two pavement types were taken into consideration, the asphalt concrete and the asphalt macadam. Three traffic classes were chosen. These are low, medium and high traffic category. In our model we use three type maintenance actions. Theoretically 2x3x3=18 different categories were formed but two of them are unrealistic. So the aim was to elaborate the 16 categories. One Markov matrix belongs to each category.

The condition of a road section is described by 3 parameters: bearing capacity (5 classes), longitudinal unevenness note (3 classes), pavement surface quality note (5 classes). The number of the condition states are 3x5x5=75. For practical reason and simplification we reduce this number to 41.

The NFM model was used to determine the necessary funds needed to held the road network at a desired condition level. The available budget for that purpose was lower, that is why we use the BBM model with a fixed budget limit. Instead of the benefit h_{ijk}, we apply the vehicle operating cost in the objective.

Firstly we distribute the available budget country-wide according to the maintenance actions, pavement types, and traffic categories. Thereafter we distribute the result among regional traffic agencies. This distribution was based on the area shares of sections with given characteristics (traffic volume, pavement type, pavement condition).

Both problems can be solved by a linear program package. This package consists of two parts: data generation and optimisation. The data generation uses the Hungarian Road Data Bank. Depending on the constraints a selection and a data aggregation is used to generate the proper data to the model.

The size of the matrix is quite large

- the number of columns in both models is 734 (18×41),
- the number of rows in
  NFM model is 91
  BBM model is 92.

The computer solves the problem in 10–30 minutes depending on the output and the structure of the matrix. The LP code was written in FORTRAN by T. Szantai. (1990)

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