Abstract
The paper focuses on the development of buyer – supplier relationships over time. Although there is an ongoing debate about the nature and characteristics of relationship life cycle, the existence of some kind of life cycle is usually assumed when timely development of these relationships is investigated. The objective of our analysis is to investigate this hidden hypothesis using quantitative research methodology. The research focusing on the development of business relationships over time has mainly used qualitative research methods and to the best of our knowledge, no one have yet attempted to measure any relevant variables and the pattern of their development over time using a quantitative methodology. In order to be able to test this hypothesis the level of relation-specific investments generated in the relationship, called the explicit investment measure, was chosen. We have conceptualized and measured it and tested empirically to what extent its development over time fits the pattern of the traditional mathematical model (representation) of the life cycle, which is usually described with a logistics curve. The development of this explicit investment measure over time is a dynamic phenomenon the analysis of which is not without methodological problem. We suggest and apply a procedure developed in the field of population dynamics that makes it possible to use cross sectional data for such dynamic analysis.

Keywords
supply chain relationship · life cycle model · relation-specific investments · empirical testing · quantitative analysis

1 Introduction
Resource dependence theory states that companies on their own are not capable to produce complex product and service offerings suitable to create customer value. Therefore companies continually attempt to build interorganizational ties, business relationships in order to acquire specific resources and capabilities. Linking these resources and capabilities to the internal ones and using them are basis for competitiveness (Pfeffer and Salancik, 1978; Dyer and Singh, 1998). Interfirm relationships are a company’s most important assets. Developing and managing these relationships influences competitiveness directly on both company and supply chain level.

Development of the concept interfirm relationship has been triggered by the critics towards the understanding of traditional economic exchange expounded by the theory of transaction cost economics (Coase, 1937; Williamson, 1975). In this interpretation economic exchanges between a customer and a supplier firm are although essential part of operation but they do not have influence on any of the cooperating parties. Economic theory and management efforts therefore should be aimed at the companies themselves and not at the transactions taking place and the ties developing between them. Resource dependence and also network theory (Granovetter, 1973; Gemünden et al., 1997) contradict this approach emphasizing that interfirm transactions in a networked economy can not be understood as discrete exchange episodes. From transactions and separate exchange episodes interest is shifted to the interaction taking place between cooperating parties. Interaction is conceptualized as a set of interrelated processes that occurs between business actors and through which all of the aspects and elements of business take their concrete form, they are changed and transformed (Turnbull et al., 1996; Ford et al., 2003). The concept of interaction is essentially different from economic exchange, since it requires change and adaptation from both parties. The consequences of these changes, the ongoing adaptation is fundamental, it leads to the birth of something new, the business relationship which is not equal to any of the cooperating parties, not even the sum of them (Blois, 1972, Ford, 1980). Business relationship is a substantive phenomenon that calls for attention and deeper understanding.
This paper focuses on business relationship, especially on their development over time. Consequently we address the problem of dynamics these relationships have, a crucial theoretical issue in relationship literature (Halinen, 1998; Naude and Turnbull, 1998; Medlin, 2004). Interactions occur in time; thus, the development of business relationships also has a time dimension. This time dimension of relationship development is a long-standing research topic and is usually linked to the concept of relationship life cycle (Batonda and Perry, 2003; Ford et al. 2003; Zerbini and Castaldo, 2007). Although there is an ongoing debate about the nature and characteristics of relationship life cycle, several researches accept its existence and take the hidden assumption that a typical development path of business relationships exists (Eggert et al., 2006; Clements et al., 2007; Wagner, 2011; Law et al., 2011). Our objective is to investigate this hypothesis using quantitative research methodology.

The research focusing on the development of business relationships over time has mainly used qualitative research methods (Sutton-Brady, 2008). Our paper also deals with the question of how business relationships develop over time, but we use a quantitative research methodology. Wilson (1995) argues that any business relationship has the following core relational attributes: level of commitment, satisfaction and trust, joint goal setting, power relations, adaptation, relation specific investments, technology sharing, structural and social bonds. Ford et al. (2003) point out that learning and communication are also among those theoretical constructs along which the life cycle of a supply chain relationship can be conceptualized and described. To the best of our knowledge, researchers have not yet attempted to measure these variables and investigated the pattern of their development over time using quantitative methodology. In order to be able to test the applicability of the mathematical representation of the life cycle model in a supply chain relationship setting, we had to choose one relevant method. Otto and Obermaier (2009) argue that the AAR model developed by Håkansson and Johanson (1992) is appropriate for capturing the investments generated and accumulated in business relationships. The AAR model describes the internal build up and the content of business relationships in general. It specifies three different building blocks of any business relationships: Actor bonds, Activity links and Resource ties. Actor bonds evolve among employees of the cooperating firms. The strength of these bonds depends on the extent to which cooperating employees trust each other and are satisfied with each other’s work as well as on the level of mutual commitment. Activity links include different types of processes

2 Conceptualizing and Measuring Relation-Specific Investments in Buyer – Supplier Relationships

The life cycle model plays a crucial role in several disciplines, including management, where the life cycle model has been proven to be applicable and useful. Think, for example, of the diffuse character of the spread of innovation (Utterback and Abernathy, 1975) or the product life cycle model in marketing management, which captures the relevant characteristics of the product’s market penetration (time and sales volume or increase in revenues). The cumulative character of the variables analyzed is a central point of the life cycle model. (Cumulativity in time is typically present in the first three stages of a life cycle and missing in the stage of decline.) Analyzing any type of life cycle necessitates selecting an important variable that can be measured and examined as it develops over time. As already introduced, the specific relational attribute analyzed in this paper is the level of relation-specific investments. As mentioned in the Introduction, we analyze empirically whether the development of these relation-specific investments in supply chain relationships over time can be described by the traditional mathematical representation of the life cycle model.

Relation-specific investments are the costs of ongoing and long-standing relationships and are very diverse and difficult to measure. Otto and Obermaier (2009) argue that the AAR model developed by Håkansson and Johanson (1992) is appropriate for capturing the investments generated and accumulated in business relationships. The AAR model describes the internal build up and the content of business relationships in general. It specifies three different building blocks of any business relationships: Actor bonds, Activity links and Resource ties. Actor bonds evolve among employees of the cooperating firms. The strength of these bonds depends on the extent to which cooperating employees trust each other and are satisfied with each other’s work as well as on the level of mutual commitment. Activity links include different types of processes
performed within the relationship. Negotiations, information exchange and joint problem solving and adaptation are specific forms of such activities. These activities inevitably generate relation-specific investments. Resource ties also must be developed in all kinds of business relationships. Matching these supplementary resources requires adaptation from both parties and generates investments in the relationship. The development of actor bonds, activity links and resource ties goes hand in hand. The stronger the actor bonds, resource ties and activity links are in a relationship, the more relation-specific investment is generated in that relationship. The overall level of relation-specific investment in a given relationship is consequently determined by the sum of relation-specific investments generated by the three AAR constructs over time.

We agree with Otto and Obermaier (2009) and think that the AAR model is conceptually appropriate for measuring the level of relation-specific investments accumulated in business relationships. Applying this model we have developed a scale using which the level of accumulated relation-specific investments in a given relationship can be measured. We have validated and tested this scale and published the results of this validation process in a separate working paper (Gelei – Dobos, 2013). Our validation and testing has backed our hypothesis that the AAR model and the chosen scale can be used for further empirical research, including our research related to the life cycle model of business relationships.

The aim of our paper is to analyze whether the mathematical representation of the life cycle model, also known as the diffusion model, is suitable to describe the development of relation-specific investments in business relationships over time. To test this hypothesis, we developed and validated a scale, and then used it to actually measure these investments. Only after measuring them can we empirically test how well our data fit the traditional life cycle model developed by Bass (1969).

We have developed a questionnaire and conducted a survey. Presentation of the applied methodology in this survey is also published in the above mentioned working paper (Gelei – Dobos, 2013). We could gather 46 complete questionnaires. Our sample size can be considered small, although small sample size is not defined exactly in the literature. Bock and Sergeant (2002) for example considers a sample as small that has fewer than 30 observations; this minimum level is exceeded in our analysis.

Our data base describes 46 relationships from the perspective of the strength of different relational ties of the AAR model and so the level of relation-specific investments accumulated in these relationships. Our sample is cross-sectional, but we want to analyze the dynamic development of relation-specific investments generated by these relational ties; thus, we had to overcome the problem of how to use a static sample to understand a dynamic phenomenon. We applied a methodology that made it possible to use this cross-sectional, static sample for testing our life cycle hypothesis. We assumed that the development of relation-specific investment in business relationships over time can be described by an unknown but existing development pattern. If this is a case, we can interpret the 46 concrete observations in our sample as 46 different and specific representative values of this unknown development pattern. Using this assumption, we were able to analyze how well our data fit different possible development patterns, among them the development pattern of the traditional life cycle model developed by the Bass model.

This assumption, the suggested method is not without antecedents, although we could not find any economic and management related article referring to it. This method is widely used by researchers in the field of population dynamics and is called life table construction (Coale, 1984; Bellows, Jr. et al., 1992). In these analyses the same problem is present: a static, cross functional sample is given, that is used then for developing a dynamic curve. The application of this assumption and method is suggested and used in our paper without specifying the concrete parameters of the dynamic curve that describe the relation-specific investments over time.

3 Research Methodology and Results

In this part of the paper, we describe the empirical research and the quantitative analysis conducted. In the first subsection of the chapter, we describe how the overall level of relation-specific investments in relationships, called the explicit investment measure, was actually calculated. We demonstrate in detail how we tested our hypothesis: the pattern of development of this explicit investment measure over time can be described using the traditional mathematical representation of the life cycle model developed by Bass (For the mathematical solution of the Bass model see Appendix 1).

3.1 Testing the Life Cycle Hypothesis of Relation-Specific Investments in Business Relationships

The objective of our analysis is to test whether the well-known life cycle model is applicable to the development of relation-specific investments between buyer–supplier relationships. This development is a dynamic phenomenon. As mentioned above, we had a static, cross-sectional database from which to perform our analyses. However, we interpreted the 46 concrete observations in our sample as 46 specific representatives of a hypothetical pattern the development of these relation-specific investments over time may have. So we could test the fit of our actual data to different potential development patterns, among them the pattern known as the traditional mathematical representation of the life cycle model.

In order to be able to measure this fit, the level of overall relation-specific investments, called explicit investment measure had to be calculated for all the 46 relationships. In order to be able to calculate this explicit measure we developed three sub-measures for relation-specific investments generated by
the three different relationship ties defined by the AAR model (actor bonds, activity links and resource ties).

We conducted a questionnaire and used three separate questions for capturing relation-specific investments generated by the development of the three relational ties of the AAR model. The question used for measuring Actor bond had four dimensions (or sub-questions), while question used for measuring relation-specific investments generated by Activity links had seven dimensions. Finally question used for measuring relation-specific investments due to strengthening Resource ties had also four dimensions. For details see Gelei – Dobos (2013). In this way we had three sub-measures. These sub-measures–just like the explicit investment measure developed using these sub-measures–are interpreted as preference relations (namely, utilities) as defined in the standard microeconomic theory. The overall level of relation-specific investments in a given relationship was expressed as a function of these three utility values; the complex explicit investment measure was computed as the sum of these sub-measures. In this way, we assigned one specific overall utility value to each of the relationships in our sample, indicating the overall level of relation-specific investment.

While developing the sub-measures of the overall explicit investment measure we used both the linear and logarithmic utility functions in our calculations. We chose these functions because utilities are described using concave curves both in microeconomic theory and in decision science. These concave curves can fulfill the necessary and sufficient condition of maximality. From an analytical perspective, the logarithmic utility functions for all three sub-measures were defined as follows:

$$U_{soc, j}^{soc} = \frac{1}{4} \sum_{i=1}^{4} U_{ij}^{soc},$$

$$U_{act, j}^{act} = \frac{1}{7} \sum_{i=1}^{7} U_{ij}^{act},$$

$$U_{res, j}^{res} = \frac{1}{4} \sum_{i=1}^{4} U_{ij}^{res},$$

where $U_{soc, j}^{soc}$, $U_{act, j}^{act}$ and $U_{res, j}^{res}$ are the created utility values measuring the level of relation-specific investments generated by the development of actor bonds, activity links and resource ties in the relationship, respectively. These values determine the overall level of relation-specific investment level of a given relationship. In the formulas above, $j$ always indicates the identification number of the relationship in our sample, and index $i$ indicates the number of sub-questions in the questions used in our questionnaire. So $U_{ij}^{soc}$ $(i=1,\ldots,4)$, $U_{ij}^{act}$ $(i=1,\ldots,7)$ and $U_{ij}^{res}$ $(i=1,\ldots,4)$ are derived based on the respondents’ concrete answers to the specific questions.

For the linear utility function, the utilities were defined as follows:

$$U_{soc, j}^{lin} = \frac{1}{4} \sum_{i=1}^{4} U_{ij}^{soc},$$

$$U_{act, j}^{lin} = \frac{1}{7} \sum_{i=1}^{7} U_{ij}^{act},$$

$$U_{res, j}^{lin} = \frac{1}{4} \sum_{i=1}^{4} U_{ij}^{res},$$

where $U_{soc, j}^{lin}$, $U_{act, j}^{lin}$ and $U_{res, j}^{lin}$ are linear utility values developed from the answers given to the three questions. These linear utility values are derived values that measure the levels of relation-specific investments in relationship $j$. $U_{ij}^{soc}$ $(i=1,\ldots,4)$, $U_{ij}^{act}$ $(i=1,\ldots,7)$ and $U_{ij}^{res}$ $(i=1,\ldots,4)$ determine the overall relation-specific investment level of a relationship and are also derived from and based on the concrete answers of our respondents.

The values indicating the levels of relation-specific investments generated by the three relational ties in any supply chain relationship are characteristic utilities. The sum of all three characteristic utilities defines the overall level of relation-specific investment for relationships in our sample and is called explicit investment measures. In case of a logarithmic utility function, the explicit investment measure was calculated as follows:

$$U_{log, j} = U_{soc, j}^{log} + U_{act, j}^{log} + U_{res, j}^{log},$$

In case of a linear utility function, the following formula was used:

$$U_{lin, j} = U_{soc, j}^{lin} + U_{act, j}^{lin} + U_{res, j}^{lin},$$

where index $j$ indicates the identification number of the relationship in our sample $(j=1,\ldots,46)$.

We calculated twice 46 overall utility values (using first linear, then logarithmic utility functions), overall explicit investment measure for all the relationships in our sample. Based on these utility values, the explicit measure of investment level, all 46 relationships were dedicated to a specific discretized life cycle phase (introduction, growth or maturity; see Appendix 2). (We ignored the last phase of the traditional life cycle, the decline phase, because the relationships evaluated by our respondents were ongoing and had strategic importance, meaning that they were necessarily not in the decline phase.) The relationships were classified in two ways by applying two different utility functions: logarithmic and linear.
Using these two utility functions and developing two sets of explicit investment measures of relationships, we could also investigate to what extent the two different analyses led to similar or different results regarding our core research question: To what extent does the development of relation-specific investments over time follow different hypothetical patterns, among them the pattern of the life cycle model, namely, the logistic curve suggested in the model developed by Bass?

Concrete relationships of our sample were grouped and dedicated to specific discretized life cycle phases based on the difference between the actual utility value and its expected utility value. This means that a given calculated proportion of the deviation is added to and deducted from the expected utility value. The two values calculated in this way break down the real line into three phases. In this way, we can identify the three phases of the life cycles. (Appendix 2 describes the classification process in detail and presents its results.)

After calculating and classifying our overall utility values, using the explicit investment measures of relationships in our sample and assigning these relationships to specific discretized life cycle phases, we tested the fit of the data to different potential development patterns among them the logistic distribution curve suggested by Bass. In SPSS, the fit to different distribution functions can be tested visually by using the P-P plots in the Graphs menu. These analyses unfortunately do not indicate whether the fit is statistically significant or not. Therefore in those cases when results of the P-P plot analysis indicated a visual fit we tested the statistical significance of this fit using the \( \chi^2 \) tests in SPlus program.

### 3.2 Results of Empirical Investigation

As mentioned, we tested the distribution of the utility values, explicit investment measures, developed for measuring the level of the overall relation-specific investments generated in business relationships. In the case of both the logarithmic and the linear explicit measure, a P-P plot was created in SPSS, and the graphical results were analyzed. In case of both explicit heaviness measures—developed using the logarithmic and the linear utility—the visual fit to the following development patterns were investigated: logistic, linear, exponential, Pareto, Student t and normal distribution curve. Figure 1 and 2 show the results of this P-P plot analysis.

Results of this analysis indicate a high level of visual fit only in case of the logistic and normal distribution but for both explicit investment measures; thus, we can assume that the distribution of these explicit investment measures describe such curves. Let us mention here that the logistic curve is suggested in the model developed by Bass but it is very close to the normal distribution as well.

As a next step, we tested the statistical fit of our explicit investment measures to these two distribution curves using the \( \chi^2 \) test in the SPlus software. In the case of the logarithmic utility function, our explicit investment measure has a mean of 3.3778 and a standard deviation of 0.72372. The expected value of our explicit measure is the same in both distribution functions. In the case of the normal distribution, the standard deviation is again used to describe the distribution function, whereas the parameter \( s \) describing the logistic distribution has a value of 0.399.

First, we tested the statistical fit to the logistic distribution function. The fit was calculated by the SPlus program with 12 independent intervals, which means that, in general, 4 elements were dedicated to each interval. According to this analysis, the \( \chi^2 \) test had 11 degrees of freedom. The empirical value of \( \chi^2 \) was 6.6957, and the probability value (or p-value) was 0.8232.

The p-value ranges between 0 and 1 and represents the level of fitness of the analyzed value, i.e., the overall level of relation-specific investment. The higher this p-value is, the better the analyzed fitness is (Anderson et al., 2010). Based on our results, we can accept the assumption that the explicit investment measure fits the pattern of the logistic distribution curve.

In the case of the normal distribution, the SPlus program yielded 12 independent intervals. The empirical value of \( \chi^2 \) was 7.2174, leading to a p-value of 0.7812. The assumption concerning the distribution can again be accepted. We can state with a high level of confidence that we cannot rule out the possibility that the distribution of the explicit investment measures analyzed follows a logistic or a normal distribution curve.

We tested the distribution of the linear utility function in a similar way. The mean of the linear utility values was 10.1578, and its standard deviation was 1.098. Here, we also dealt with both logistic and normal distribution functions. The number of intervals was 12 again, with 11 degrees of freedom. In the case of the normal distribution, the empirical value of \( \chi^2 \) was 3.8085, corresponding to a p-value of 0.9752. In the case of the logistic distribution, the empirical value of \( \chi^2 \) was 6.8723 (with a p-value of 0.8093).

In the case of the linear utility values, our previous statement also holds; namely, we cannot exclude the possibility that the distribution of the explicit investment measures analyzed is either logistic or normal. We should mention again that the shapes of the two distribution functions are very similar.

Literature on business relationships uses the life cycle concept frequently, but its empirical applicability has not been examined yet. The aim of our paper was to investigate the development of a key relationship attribute, the level and development of relation-specific investments over time. Using an online questionnaire, we measured perceived levels of relation-specific investments in 46 concrete buyer-supplier relationships. These data were static, but the phenomena analyzed were dynamic in nature. We assumed that the development of explicit investment measure in business relationships over time can be described with a hypothetical typical development pattern, the shape of which is not known in advance. This assumption allowed us to interpret the 46 concrete relationships in our sample as 46 different observations that represent this typical development pattern. As a next step, we tested whether...
Fig. 1. Result of a P-P plot analysis using logarithmic utilities of explicit investment measure.
Fig. 2. Result of a P-P plot analysis using linear utilities of explicit investment measure
the mathematical representation of the life cycle model developed by Bass (1969) can be applied for describing the observed concrete development pattern of these explicit investment measures. The tests and the results were presented above in detail. Based on our results, we cannot exclude the possibility that the distribution of the explicit investment measures developed follow the pattern of the logistic or the normal distribution curve. Consequently, we can state that the development pattern of the overall level of relation-specific investment in business relationships over time seems to follow the shape of the traditional life cycle model developed by Bass (1969). This result provides empirical support for previous research arguing that relationship development may have a life cycle (Ford, 1980; Dwyer et al., 1987; Brennan and Turnbull, 1999; Batonda and Perry, 2003; Eggert et al., 2006; Clements et al., 2007; Wagner, 2011).

4 Summary – Limitations and Implications

We focused our attention on a phenomenon—relationship development—that is a long-standing research issue. Relationships are building blocks of supply chains and networks and are regarded as key sources of competitiveness in today’s turbulent economy. Managing relationships efficiently is a real challenge because their development has a time element; they are dynamic in nature, and their characteristics and the ways in which they operate change over time. Consequently, it is of vital importance to gain deeper insight into the dynamics of relationship development.

We chose the explicit investment measure interpreted as the sum of all relation-specific investments generated by the three constructs of the AAR model (actor bonds, activity links and resource ties) as the focal relationship characteristic for our analysis. Its interpretation and measurement is based on the interpretation by Håkansson and Ford (2002) and the suggestion of Otto and Obermaier (2009). Based on this we could successfully conceptualize and validates our measure, the reliability of which is also acceptable. Main limitation of the research presented lies in its relatively small sample size. Although we could argue for acceptance of our sample size, we still think generalizability of research results necessitate increase in this sample size. Even if research results are not generalizable we think our research represents value. First of all because it demonstrate a useful methodology that in some cases can solve the problem of analyzing dynamic problems quantitatively using cross sectional data.

Our paper therefore is academic in nature and its primary contribution is theoretical and methodological. The life cycle concept is widely used in the B2B literature and has been the subject of extensive empirical investigation (Batonda and Perry, 2003). Up to the present, this empirical research has been limited in the sense that it involved exclusively qualitative research methods. To our knowledge, nobody has tried to apply a quantitative research methodology to analyze the development of business relationship or some of its characteristics over time. In this paper, we attempted to fill this gap. The main contribution of the paper lies in this attempt and in the fact that we have quantified a specific relationship attribute that changes over time and used our data for a quantitative analysis.

With this we do not question the necessity and usefulness of qualitative research in the field of supply chain relationship management; in fact, we do quite the opposite. However, we also think that quantitative research has great value. It necessitates and triggers an unambiguous use of key terms and is in some problem settings inevitable for increasing the reliability and generalizability of research results (Malhotra, 1999).

In our empirical research we collected static data about the development of relation-specific investments between cooperating parties in supply chain relationships. We assumed that the development of this explicit investment measure over time could be described by an unknown but existing development pattern. We did not need to have any concrete information concerning the actual shape of this pattern. By accepting this assumption, we could interpret the concrete relationships in our sample as different observations of such a typical development pattern. Using this assumption, we were able to analyze to what extent our data fit different possible patterns, among them the traditional life cycle pattern that is usually described with a logistic curve. The method suggested and applied in this article has not been used in operations and supply chain management context but is widely accepted in population dynamics, a field where collecting dynamic data is not feasible. Using it may help researchers because it makes possible to use cross sectional data in empirical analyses of dynamic phenomena that are key in an economy where the development has been accelerated enormously. The methodology suggested can not only be applied for investigating the development of explicit investment measure, the overall level of relation-specific investments in a relationship. It can be applied for all the elements of it from trust to commitment and the level of different activities.

The main contribution of the paper, as mentioned, is theoretical and methodological. However, this does not mean that it has no managerial implications at all. One of these managerial implications is the fact that our analysis could empirically back the hidden hypothesis of several management efforts, namely that these relationships have a life cycle. Supplier development programs, customer value analysis are among those practical issues that have presumed the existence of different life cycle phases or stages of relationship development and suggest different approach and treatment to them; without any empirical verification of this assumption.

Here we have tested the life cycle hypothesis of business relationships for the variable of explicit investment measure, the overall level of relation-specific investment accumulated in the relationship. We argue that all managerial considerations that are related to the actual level of this explicit investment measure should really be aware of the fact that here a life cycle
Relationship heaviness and consequently relationship stability are among these considerations. Relationship heaviness is interpreted as the sum of relation-specific investments generated by the cooperating parties and accumulated in the relationship and is one of the two influencing factors of relationship stability Håkansson and Ford (2002). According to our results, the more mature a given buyer – supplier relationship is, the higher the level of generated relation-specific investments will be. Increasing heaviness of relationships along the life cycle really tie together cooperating firms that may have positive and negative consequences too. Increasing level of relation-specific investment is a sign for strong mutual commitment that is a prerequisite for future development. On the other hand due to the intensive increase of heaviness partners may get locked in a given relationship that may hinder development of other potential relationships.

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Appendix 1: Mathematical Representation of the Traditional Life Cycle Model Developed by Bass

The concept of the life cycle is widely used in business research for modeling diffusion processes. In our analysis, we use the life cycle model that was developed to understand such diffusion processes in marketing management and applied in other management fields, among them innovation management. In marketing management, the product life cycle traditionally describes the development pattern (or diffusion process) of revenue or of the sales volume generated over time by the specific product analyzed. This life cycle was first specified and expressed using mathematical tools by Bass (1969). He suggested using the differential equation resulting in a logistic curve. Applicability and generalization of Bass’ model are described by Radas (2005). Our description of the life cycle model is based on the latter article.

The life cycle hypothesis in Bass’ model focuses on one product and analyzes how its sales volume develops over time. According to Bass, this sales volume describes an S-curve. This hypothesis is also backed by empirical research. The model is based on the latter article.

The life cycle hypothesis in Bass’ model focuses on the market or the maximum number of products that can be sold on the market. The solution of this differential equation with the following formula:

$$\frac{dF(t)}{dt} = (m - F(t)) \cdot \left( p + \frac{q}{m} \cdot F(t) \right),$$

where parameter $p$ reflects the fraction of the adopter type of customers, and parameter $q$ represents the follower type of customers, while $F(t)$ is the cumulative number of previous buyers (sold products) at a point $t$ in time, and $m$ denotes the size of the market or the maximum number of products that can be sold on the market. The solution of this differential equation with an initial value of $F(0)$ is the following:

$$F(t) = m \cdot \frac{1 - \frac{p}{q} \cdot e^{-(p+q)(t+c)}}{1 + e^{-(p+q)(t+c)}},$$

where the value of $c$ can be calculated according to the following equation:

$$c = -\frac{1}{p+q} \cdot \ln \left( \frac{q}{p} \right).$$

The $F(t)$ solution function gives the logistic curve demonstrated in Figure 1. The curve consists of three well-defined phases. The first phase is characterized by a relatively low growth rate; in the second phase, the growth rate is much higher; and growth slows down again in the last phase. Although marketing management theory typically breaks the product life cycle into four phases—market introduction, growth, maturity and decline (Kotler, 1988)—due to the mathematical structure of the logistic curve, the Bass model can capture only the first three stages. In the case of the logistic curve, the last phase—decline—is missing because the logistic curve is monotonously increasing. In the different management disciplines, life cycle phases are differentiated based on the characteristics of the logistic curve: introduction and maturity are relatively flat, whereas the phase of growth shows a sharper increase (Polli and Cook, 1969).

In management studies, such diffusive processes are not always connected to a variable that is continuous and easily measurable, such as sales volume or revenues in case of the product life cycle model. In some instances, these variables are interpreted and measured on an ordinal scale. This was the case in our research, where we measured relationship heaviness on a 5-point Likert scale. In such cases, we cannot apply a proportion scale. However, we can assign numbers to the phases from 1 to 3 (from introduction to maturity) by performing a transformation into an ordinal scale. With this transformation, we transform the logistic curve into a time-dependent life cycle measured on an ordinal scale. This transformation is illustrated in Figure 1. Function $F(t)$ represents the logistic curve, while $G(t)$ is the transformed discretized version of $F(t)$.
Figure 1 shows that in the \((0, t_1)\) time interval, the product is in the phase of market introduction. This phase is identified by the value 1. The time interval \((t_1, t_2)\) indicates the growth phase of the product’s life cycle. This phase is identified by value 2. Value 3 is dedicated to the maturity phase of the life cycle on time interval \((t_2, +\infty)\). The starting point of our transformation was the logistic curve; the transformed version of this curve is interpreted as the discretized version of that logistic curve.

Appendix 2: Assigning Relationships in our Sample to Specific Life Cycle Stages

An important part of our empirical analysis involved assigning business relationships to specific stages or phases of their life cycle. This process was based on the utility values representing the explicit heaviness measures of the relationships in our sample. Here, we describe in detail the method used in this assignment process.

As mentioned in the main text, each relationship’s assignment was based on its deviation from the mean. The boundaries of the three groups were defined as follows:

- **introduction**: \([0, \bar{x} - a \cdot s)\),
- **growth**: \([\bar{x} - a \cdot s, \bar{x} + a \cdot s)\),
- **maturity**: \([\bar{x} + a \cdot s, +\infty)\),

where \(\bar{x}\) denotes the mean, and \(s\) the standard deviation of the sample. The mean and deviation were calculated using both linear and logarithmic utility values. The \(\bar{x}\) value is the inflection point of the distribution function of the analyzed variable, relationship heaviness. Value \(a\) indicates how many times the deviation was added to or deducted from this mean. Choosing a higher \(a\) value leads to a broader growth phase within the life cycle, and on the contrary, using a lower \(a\) value shortens the growth phase. We chose and analyzed two cases, one in which \(a_1 = 1\) and a second in which \(a_2 = 1/3\). Linear and logarithmic groupings were compared using the same values of \(a\).

In the special situation, when the standard deviation is added to or deducted from the mean—i.e., \(a_1 = 1\)—the result of grouping the business relationships of the sample gives the same result irrespective of whether a linear or logarithmic utility function is used. The results are summarized in Table A4.1: 7 business relationships from our sample fall into the introduction phase, 31 fall into the growth phase and 8 fall into the maturity phase.

The following lists the results of our grouping assuming \(a_2 = 1/3\). In this case, applying logarithmic and linear utility functions leads to different grouping patterns. The grouping pattern obtained using a linear utility function is shown in Table A3.2.

In this case, 18 relationships fall into the introduction phase, 7 fall into the growth phase and 21 fall into the maturity phase. The results obtained using the logarithmic utility function are summarized in Table A3.3.

When \(a_2 = 1/3\) and a logarithmic utility function is used, 19 of the relationships in our sample fall into the introduction phase, 9 fall into the growth phase and 18 fall into the phase of maturity.

Table A3.4 is the cross-table of the above two grouping procedures. We analyzed to what extent these groupings overlap. To this end, we calculated association indices in SPSS. Our data were transformed to an ordinal scale to enable us to measure their association, indicating the strength of the relation between the two groupings, with Kendall’s \(\tau_b\) and the gamma association index. Kendall’s \(\tau_b\) was 0.681, and the gamma association index was 0.883. Both of these indices had an empirical level of significance of 0.000. These results support a strong association between the results of the two groupings obtained applying logarithmic and linear utility functions when \(a_2 = 1/3\).

Both values of \(a\) \((a_1 = 1\) and \(a_2 = 1/3\)) resulted in substantively identical phasing patterns of the investigated business relationships, indicating a low sensibility to the exact value of \(a\).
Tab. A.1. The populations of specific life cycle phases based on the analyses using **linear** and **logarithmic** utility functions, $a_2 = 1$

<table>
<thead>
<tr>
<th>Introduction</th>
<th>Growth</th>
<th>Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear: (0, 2.65)*</td>
<td>Linear: (2.65, 4.10)</td>
<td>Linear: (4.10, +∞)</td>
</tr>
<tr>
<td>Logarithmic: (0, 8.12)</td>
<td>Logarithmic: (8.12, 12.09)</td>
<td>Logarithmic: (12.09, +∞)</td>
</tr>
</tbody>
</table>

Identification numbers of the specific business relationships in the sample

| 7, 18, 39, 48, 49, 71, 73 | 2, 3, 4, 5, 6, 8, 9, 11, 13, 20, 21, 23, 24, 26, 29, 36, 38, 40, 42, 44, 45, 51, 52, 55, 57, 61, 63, 64, 65, 66, 67 | 1, 12, 17, 22, 37, 50, 54, 62 |

* (the lower bound of the interval, the upper bound of the interval)

Tab. A4.2. The population of specific life cycle phases based on the analysis using a **linear** utility function, $a_2 = 1/3$

<table>
<thead>
<tr>
<th>Introduction</th>
<th>Growth</th>
<th>Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear: (0, 9.49)</td>
<td>Linear: (9.49, 10.82)</td>
<td>Linear: (10.82, +∞)</td>
</tr>
</tbody>
</table>

Identification numbers of the specific business relationships in the sample

| 7, 18, 39, 48, 49, 71, 73 | 2, 3, 4, 5, 6, 8, 9, 11, 13, 20, 21, 23, 24, 26, 29, 36, 38, 40, 42, 44, 45, 51, 52, 55, 57, 61, 63, 64, 65, 66, 67 | 1, 5, 12, 13, 17, 21, 22, 24, 36, 37, 40, 42, 44, 45, 50, 51, 52, 54, 61, 62, 64 |

Tab. A4.3. The population of specific life cycle phases based on the analysis using a **logarithmic** utility function, $a_2 = 1/3$

<table>
<thead>
<tr>
<th>Introduction</th>
<th>Growth</th>
<th>Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear: (0, 3.14)</td>
<td>Logarithmic: (3.14, 3.62)</td>
<td>Logarithmic: (3.62, +∞)</td>
</tr>
</tbody>
</table>

Identification numbers of the specific business relationships in the sample

| 2, 3, 6, 7, 8, 9, 18, 23, 26, 29, 38, 39, 43, 48, 49, 55, 63, 66, 71 | 4, 11, 13, 20, 38, 44, 45, 57, 67 | 1, 5, 12, 17, 21, 22, 24, 36, 37, 40, 42, 50, 51, 52, 54, 61, 62, 64 |

Tab. A4.4. Comparison of the groups resulting from the explicit analysis with **linear** and **logarithmic utilities**, assuming $a_2 = 1/3$

<table>
<thead>
<tr>
<th>Linear</th>
<th>Logarithmic</th>
<th>Phase 1</th>
<th>Phase 2</th>
<th>Phase 3</th>
<th>Total:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase 1</td>
<td>18</td>
<td>3</td>
<td>0</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>Phase 2</td>
<td>0</td>
<td>4</td>
<td>3</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Phase 3</td>
<td>0</td>
<td>2</td>
<td>16</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>Total:</td>
<td>18</td>
<td>9</td>
<td>19</td>
<td>46</td>
<td></td>
</tr>
</tbody>
</table>