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RESEARCH ARTICLE

Forecasting Failure Rates of Electronic Goods by Using Decomposition and Fuzzy Clustering of Empirical Failure Rate Curves

Tamás Jónás[1*], Gábor Árva[1], Zsuzsanna Eszter Tóth[1]

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Abstract

In this paper a novel methodology founded on the joint application of analytic decomposition of empirical failure rate time series and soft computational techniques is introduced in order to predict bathtub-shaped failure rate curves of consumer electronic goods. Empirical failure rate time series are modeled by a flexible function the parameters of which have geometric interpretations, and so the model parameters grab the characteristics of bathtub-shaped failure rate curves. The so-called typical standardized failure rate curve models, which are derived from the model functions through standardization and fuzzy clustering processes, are applied to predict failure rate curves of consumer electronics in a method that combines analytic curve fitting and soft computing techniques. The forecasting capability of the introduced method was tested on real-life data. Based on the empirical results from practical applications, the introduced method can be considered as a new, alternative reliability prediction technique the application of which can support the electronic repair service providers to plan their resources in the long run.

Keywords

reliability, failure rate curve model, fuzzy clustering, forecasting empirical failure rates, consumer electronics

[1] Department of Management and Corporate Economics,
Faculty of Economics and Social Sciences,
Budapest University of Technology and Economics,
1117 Budapest, Magyar Tudósok krt. 2., Hungary

[*] Corresponding author, e-mail: jonas@mvt.bme.hu

1 Introduction

Nowadays, the level of after sales service is an important quality factor to which significant attention is paid in the market of consumer electronic goods. New products are launched with an ever increasing pace within decreasing periods of time. The constantly increasing demand for electronics and the current dynamism of product development result in shortening product life-cycles. In such a highly competitive environment not just the manufacturing, but the aftermarket services are also outsourced to service provider companies that provide electronic repair services – as after sales services – on behalf of the product brand owners (original equipment manufacturers, OEM). Companies dealing with electronic repair services process field returns which enable them to produce empirical failure rate curves for consumer electronic goods. These goods are typically tested functionally, the application of specific reliability tests in case of these products is not a common practice. In addition to that, considering the shortening product life-cycles, the curves of failure rate functions of consumer electronics are bathtub-shaped with all the three characteristic phases of the traditional bathtub curve: the decreasing first phase, called infancy period, the quasi constant second phase, which is also referenced as the period of normal operation or useful life, and the third, increasing one representing the wear-out period. The failure rate functions, that are also called hazard functions, represent the characteristics of product life (Goel and Graves, 2006; Economou, 2004; Campbell et al., 1992). Similarly to product- or business related life-cycles (Gelei and Dobos, 2014), the failure rate functions can be considered as characteristics that reflect the product reliability over the product life-cycle.

Product failure rates are time-dependent, and so they can be considered as time series. The complete empirical failure rate time series of end-of-life consumer electronic products of the same commodity can be taken as an empirical knowledge base of product reliability. This knowledge can be built up from field data and can be used to predict the unknown failure rate curves of newly marketed products of the same commodity.

Our approach is founded on an analytic decomposition of empirical failure rate time series by using a flexible model function that can describe all phases of the traditional bathtub-shaped hazard function curve well. The most important property of the introduced model function is that each of its parameters has a geometric interpretation that is related to the shape of the failure rate curve. After applying appropriate transformations to the model functions, those can be standardized and the standardized models can be clustered based on their parameters. This process results in typical standardized failure rate curve models that can be used to predict failure rates of active products. The developed forecasting method is founded on measuring fuzzy similarity between the known fraction of failure rate time series of the studied active product and each fitted typical standardized failure rate curve model. In this sense, our method is a hybrid one that combines times series, analytic curve fitting and soft computing techniques. The introduced method was tested on real life data and its goodness was compared to widely used forecasting techniques such as the moving average, the exponential smoothing, the linear regression and the auto-regressive integrated moving average (ARIMA) methods. Results of practical application are discussed in a case study. Our main conclusion is that the forecast based on typical standardized failure rate curve models is able to indicate the turning points of the bathtub-shaped failure rate curve in advance in contrary to the traditional statistical forecasting techniques that do not have such a capability. Another advantage of our method is that it does not require the knowledge of the failure rate probability distribution, or the knowledge of the so-called stress factors that the product will operate along with. Based on the empirical results from practical applications, our method can be considered as a viable alternative reliability prediction technique. Application of our method can support the electronic repair service providers to plan their resources in the long run.

The remaining part of our paper is structured as follows. Section 2 discusses the state of the field. In Section 3, the failure rate curve model is introduced that we apply for decomposing the empirical failure rate time series and characterize them by parameter vectors. In this section, we also discuss the standardization of the failure rate curve models, their fuzzy clustering, and our prediction method that is based on typical standardized failure rate curve models. Our method is demonstrated through a real-life example in Section 4. Finally, key conclusions and the managerial implications are discussed in Section 5.

2 Literature review

Economou (2004) distinguishes four categories of reliability methods: reliability predictions, qualitative methods, quantitative methods and analytic models. Reliability methods are based on database tools, such as the US Navy's Military Handbook (MIL-HDBK) 217 or Telcordia SR 322, etc. Qualitative methods involve aggressive testing such as Highly Accelerated Life Test (HALT), or Highly Accelerated Stress Screening (HASS), or techniques based on engineers' experience like Failure Mode and Effect Analysis (FMEA). Quantitative methods deal with techniques like Finite Element Analysis (FEA) or Physics of Failure. Analytic models are a blend of reliability prediction tools and quantitative methods, such as Weibull analysis and life stress distribution (Economou, 2004).

Many papers deal with the extensions of analytic models that are based on Weibull-, exponential, Marshall-Olkin extended uniform, and log-normal distributions (e.g. Marshall and Olkin 1996; Abid and Hassan, 2015). In some models, new parameters are introduced that refer to the circumstances of usage, production, etc. It could be said that properly modeled environmental stress is fundamental of varied environment oriented reliability prediction (Yi-kun et al., 2015).

According to Lee and Lee (2008) there are three different techniques for prediction: statistical methods, similarity analyses using failure rate databases, and physics-of-failure methods. Goel and Graves (2006) classify the techniques into two groups: empirical-based models and physics-of-failure models.

The Military Handbook 217 by the US Navy in 1965 is regarded as a milestone in the history of failure rate forecasting. Since then it has become a widely accepted standard for reliability prediction in industrial electronics. Many methods which can be used in the electronic industry were developed based on this standard. Two notable works are the 217Plus(TM) Handbook of Reliability Prediction Models (Denson, 2006) and the FIDES Guide 2009 (Fides Gropus, 2009). Held and Fritz (2009) studied and evaluated the FIDES Guide 2004 (previous release of FIDES Guide) and RIAC-Handbook-217Plus (2006) models by comparing their results to field data. Both models require system decomposition and a wide range of known circumstances which represent environmental and operational influences.

Another approach was followed by Perera (2006) who created a new predictor called reliability index to predict failure rates of mobile phones. However, a significant correlation was found between the reliability index and the failure rate, the method requires a series of different types of tests.

During the past few years, besides adding new parameters or factors to already existing models, new approaches that are founded on soft computing methods such as fuzzy logic or artificial neural networks have been discovered (e.g. Chen, 2007; Xue et al., 2003, Al-Garni and Jamal, 2011). Nowadays, the above mentioned techniques are widely used as mathematical tools that offer an alternative way to deal with complex systems. Recently, artificial neural networks have been widely used not only in the electronic industry, but also in several other industries to predict failure rates. Kutylowskla (2015) used artificial neural networks to predict failure rates of water-pipe networks. Son et al. (2009) presented a soft computing technique for acquiring a proper maintenance plan for individual parts in a complex system. They used a combination of neural network and evolutionary algorithm to discover the relationship between individual parts of a complex system to optimize its reliability.

Our method can be taken as a hybrid one that involves historical failure rates represented by time series as inputs, and combines analytic curve fitting and soft computing techniques to generate the typical standardized failure rate curve models that are the base of our forecasting algorithm. The standardized failure rate curve model, which we introduce here, can be considered as an alternative of the standard line segments failure rate curve model presented by Dombi et al. (2015). The novelty of our method lies in the application of our failure rate curve model, which due to its flexibility, can fit well to a very wide range of bathtub-shaped failure rate time series. The developed method is mainly advantageous in long-term forecasting of failure rates. It is able to indicate the turning points of the bathtub-shaped failure rate curves in advance, while traditional statistical forecasting techniques lack this capability. Application of our method can support the electronic repair service provider companies to plan their resources based on long-term predictions of failure rates of products that they need to repair.

3 The methodology

3.1 Inputs

Let us assume, that we have the

$$\lambda_{i,t_0}, \lambda_{i,t_1}, \ldots, \lambda_{i,t_{n_i}}$$

time series $(i = 1, 2, ..., m)$, each of them represents the complete empirical failure rate curve of a product, and the studied products are all from the same well-defined product category. The $\lambda_{i,t_0}, \lambda_{i,t_1}, \ldots, \lambda_{i,t_n}$ values denote the failure rates of the ith product week by week, and from this point, the simplified $\lambda_{i,0}, \lambda_{i,1}, \ldots, \lambda_{i,n_i}$ notation is used for time series $\lambda_{i,t_0}, \lambda_{i,t_1}, \ldots, \lambda_{i,t_{n_i}}$. The approach introduced here is based on the phenomenon that failure rate curves of the studied consumer electronic products are bathtub-shaped with three characteristic parts: the first decreasing, the second quasi constant, and the third increasing part as depicted in Fig. 1.

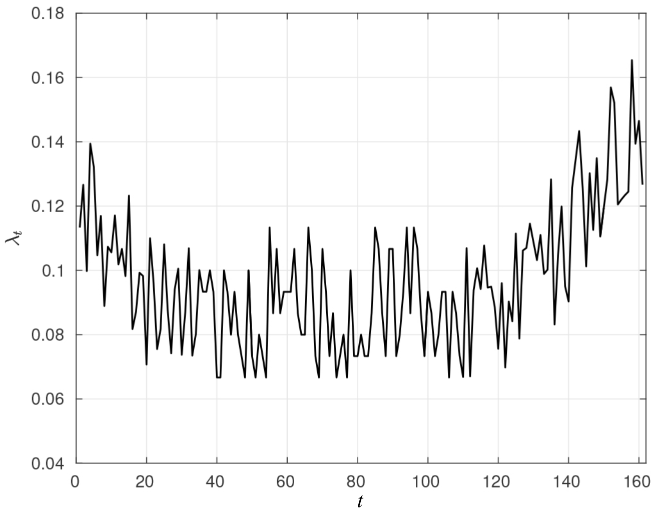

Fig. 1 Example for a bathtub-shaped failure rate curve

3.2 Fitting a model function to empirical failure rate time series

In our approach, a parametric function is applied as a model of each $\lambda_{i,0}, \lambda_{i,1}, \ldots, \lambda_{i,n_i}$ historical failure rate time series of end-of-life products $(i = 1, ..., m)$ which is based on the following $g_{\mu,\omega} : [0,1] \to [0,1],\ x \mapsto g_{\mu,\omega}(x)$ function:

$$g_{\mu,\omega}(x) = \begin{cases} 0, & \text{if } (x = 0 \text{ and } \omega > 0) \\ & \text{or } (x = 1 \text{ and } \omega < 0) \\ \dfrac{1}{1 + \left(\frac{\mu}{1-\mu}\frac{1-x}{x}\right)^{\omega}}, & \text{if } 0 < x < 1,\ \omega \neq 0 \\ 1, & \text{if } (x = 0 \text{ and } \omega < 0) \\ & \text{or } (x = 1,\ \omega > 0), \end{cases} \tag{1}$$

where $0 < \mu < 1$. Function $g_{\mu,\omega}(x)$ is derived from Dombi's kappa function that can be used as a unary operator in fuzzy theory (Dombi, 2012a; 2012b). It can be seen that function $g_{\mu,\omega}(x)$ is monotonously increasing from 0 to 1 if the parameter ω is positive, and it is monotonously decreasing from 1 to 0 if ω is negative. Parameter ω determines the slope of the function curve in the $(\mu, 0.5)$ point. The function has a value of 0.5 in the locus μ. If $|\omega| \neq 1$, then the curve has an inflection point in the $(0,1)$ interval. If $|\omega| = 1$, then $g_{\mu,\omega}(x)$ is either convex or concave or a line in the $(0,1)$ interval, depending on the value of μ. If $\omega = 0$, then $g_{\mu,\omega}(x)$ is constant with a value of 0.5. Main properties of function $g_{\mu,\omega}(x)$ are summarized in Table 1.

Table 1 Main properties of function $g_{\mu,\omega}(x)$

ω	μ	monotony	shape in 0,1
$0 < \omega < 1$	$0 < \mu < 1$	increasing	turns from concave to convex
$\omega = 1$	$0 < \mu < 0.5$	increasing	concave
$\omega = 1$	$\mu = 0.5$	increasing	line
$\omega = 1$	$0.5 < \mu < 1$	increasing	convex
$\omega > 1$	$0 < \mu < 1$	increasing	turns from convex to concave
$-1 < \omega < 0$	$0 < \mu < 1$	decreasing	turns from convex to concave
$\omega = -1$	$0 < \mu < 0.5$	decreasing	convex
$\omega = -1$	$\mu = 0.5$	decreasing	line
$\omega = -1$	$0.5 < \mu < 1$	decreasing	concave
$\omega < -1$	$0 < \mu < 1$	decreasing	turns from concave to convex

Figure 2 depicts different examples for curves of function $g_{\mu,\omega}(x)$.

It is to be highlighted that the curve of function $g_{\mu,\omega}(x)$ can have various shapes and so its appropriate linearly transformed variants are suitable to model the decreasing first and the increasing third phases of the bathtub-shaped failure rate curves of electronic products. It is also worth mentioning that parameters ω and μ are responsible for the shape of the function curve, that is, these have geometric interpretations, and so modeling based on function $g_{\mu,\omega}(x)$ has certain semantics.

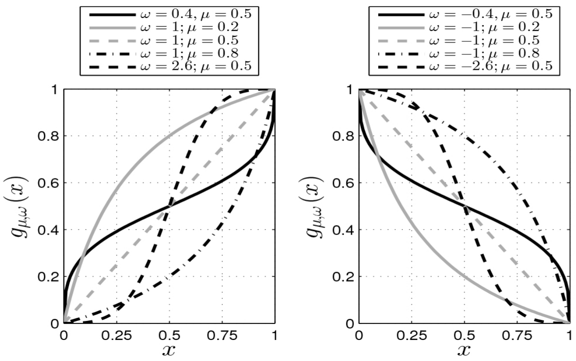

Fig. 2 Examples for curves of function $g_{\mu,\omega}(x)$

Let $\lambda_0, \lambda_1, \ldots, \lambda_n$ be the complete historical failure rate time series of a product. As a model for time series $\lambda_1, \lambda_2, \ldots, \lambda_n$, we use the following $f(t)$ function that is built upon appropriate linear transformations of function $g_{\mu,\omega}(x)$. Function $f(t)$ consists of three main parts representing the three sections of a traditional bathtub-shaped failure rate curve: the declining left phase $l(t)$, the constant mid phase λ_c, and the increasing right phase $r(t)$, and λ_l is the leftest value of the model function, that is, $f(0) = \lambda_l$.

$$f(t) = \begin{cases} \lambda_l, & \text{if } t = 0 \\ l(t), & \text{if } 0 < t < t_{e,l} \\ \lambda_c, & \text{if } t_{e,l} \le t \le t_{s,r} \\ r(t), & \text{if } t_{s,r} < t \le t_{e,r} \end{cases} \tag{2}$$

where

$$l(t) = \lambda_c + (\lambda_l - \lambda_c) \frac{1}{1 + \left(\frac{t_{a,l}}{t_{e,l} - t_{a,l}} \frac{t_{e,l} - t}{t} \right)^{-\omega_l}} \tag{3}$$

$$r(t) = \lambda_c + (\lambda_r - \lambda_c) \frac{1}{1 + \left(\frac{t_{a,r} - t_{s,r}}{t_{e,r} - t_{a,r}} \frac{t_{e,r} - t}{t - t_{s,r}} \right)^{\omega_r}}. \tag{4}$$

The model parameters need to meet the following criteria:

$$\begin{gathered} 0 < t_{a,l} < t_{e,l} < t_{s,r} < t_{a,r} < t_{e,r} \\ \lambda_c < \lambda_l, \lambda_r \\ \omega_l, \omega_r > 0. \end{gathered} \tag{5}$$

$l(t)$ is defined in the $(0, t_{e,l})$ domain and has the λ_l, λ_c, $t_{a,l}$, $t_{e,l}$ and ω_l parameters with the following roles:

- λ_c: lowest value of $l(t)$ as well as the value of constant part of $f(t)$
- $t_{a,l}$: locus where $l(t) = (\lambda_l + \lambda_c)/2$
- $t_{e,l}$: locus of the end of the left side curve
- ω_l: slope of $l(t)$ in locus a_l is proportional to ω_l

$r(t)$ is defined in the $(t_{s,r}, t_{e,r}]$ domain and has the λ_r, λ_c, $t_{s,r}$, $t_{a,r}$, $t_{e,r}$ and ω_r parameters with the following roles:

- λ_r: last value of $r(t)$, that is, it is the end value of the third segment of the life-cycle curve
- λ_c: lowest value of $r(t)$ as well as the value of constant part of $f(t)$
- $t_{s,r}$: locus of the start of the right side curve, same as the end locus of the constant middle segment of $f(t)$
- $t_{a,r}$: locus where $r(t) = (\lambda_r + \lambda_c)/2$
- $t_{e,r}$: locus of the end of the right side curve, $t_{e,r} = n$
- ω_r: slope of $r(t)$ in locus a_r is proportional to ω_r, ω_r is defined as $0 < \omega_r$.

The unknown model parameters can be determined by minimizing the

$$\sum_{i=0}^{n} \left(f(i) - \lambda_i \right)^2 \tag{6}$$

quantity. It can be done by using the Interior Point Algorithm (Bazaraa et al., 2006; Byrd et al., 1999). Function $f(t)$ is the *failure rate curve model (FCM)* of the empirical failure rate time series $\lambda_0, \lambda_1, \ldots, \lambda_n$. Figure 3 shows how well function $f(t)$ can be used to model an empirical failure rate time series.

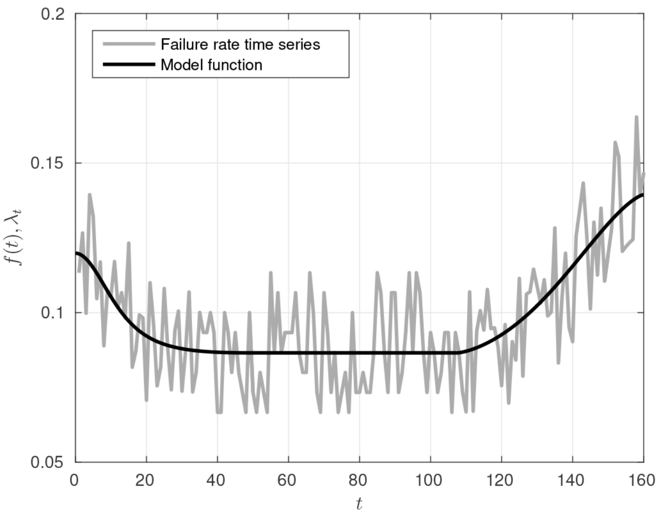

Fig. 3 An empirical failure rate time series and its $f(t)$ model

3.3 Standardizing the failure rate curve models

Once the parameters of $f(t)$ for a particular failure rate times series $\lambda_0, \lambda_1, \ldots, \lambda_n$ have been identified, the $f(t)$ model can be standardized to the $s : [0,1] \rightarrow [0,1]$, $x \mapsto s(x)$ function by applying the following transformation:

$$x = \frac{t}{n} \tag{7}$$

$$s(x) = \frac{f(nx) - \lambda_c}{\max\{\lambda_l, \lambda_r\} - \lambda_c}. \tag{8}$$

Applying the transformation given by (7) and the min-max standardization given by (8) to the model function $f(t)$ result in the following parameters of the $s(x)$ *standardized failure rate curve model (SFCM)* function.

$$y_l = \frac{\lambda_l - \lambda_c}{\max\{\lambda_l, \lambda_r\} - \lambda_c}, y_c = 0, x_{s,l} = \frac{t_{s,l}}{n} = 0, x_{e,l} = \frac{t_{e,l}}{n}, x_{a,l} = \frac{t_{a,l}}{n}$$
$$y_r = \frac{\lambda_r - \lambda_c}{\max\{\lambda_l, \lambda_r\} - \lambda_c}, x_{s,r} = \frac{t_{s,r}}{n}, x_{e,r} = \frac{t_{e,r}}{n} = 1, x_{a,r} = \frac{t_{a,r}}{n} \quad (9)$$

The transformation given by (7) and (8) does not modify ω_l and ω_r, so function $s(x)$ is as follows:

$$s(x) = \begin{cases} y_l, & \text{if } x = 0 \\ s_l(x), & \text{if } 0 < x < x_{e,l} \\ 0, & \text{if } x_{e,l} \le x \le x_{s,r} \\ s_r(x), & \text{if } x_{s,r} < x \le 1, \end{cases} \quad (10)$$

where

$$s_l(x) = y_l \frac{1}{1 + \left(\frac{x_{a,l}}{x_{e,l} - x_{a,l}} \frac{x_{e,l} - x}{x} \right)^{-\omega_l}} \quad (11)$$

$$s_r(x) = y_r \frac{1}{1 + \left(\frac{x_{a,r} - x_{s,r}}{1 - x_{a,r}} \frac{1 - x}{x - x_{s,r}} \right)^{\omega_r}} \quad (12)$$

It is worth noting that due to the min-max standardization applied to $f(nx)$, one of the y_l, y_r values is 1. Each standardized failure rate curve model has eight parameters:

$$y_l, x_{a,l}, x_{e,l}, \omega_l, y_r, x_{a,r}, x_{s,r}, \omega_r,$$

and each parameter has a geometric interpretation related to the shape of the model curve. These semantics of the model parameters are important properties of the standardized failure rate curve functions, namely, they render clustering them based on their parameters possible.

3.4 Clustering the standardized failure rate curve models

Let $s_{\mathrm{p}_i}(x)$ denote the standardized failure rate curve model for the empirical failure rate time series $\lambda_{i,0}, \lambda_{i,1}, \ldots, \lambda_{i,n_i}$, $(i = 1, 2, ..., m)$, where the parameter vector $\mathbf{p}_i$ is

$$\mathbf{p}_i = \left(y_{l,i}, x_{a,l,i}, x_{e,l,i}, \omega_{l,i}, y_{r,i}, x_{a,r,i}, x_{s,r,i}, \omega_{r,i} \right). \quad (13)$$

In order to identify typical standardized failure rate curve models, we cluster the $s_{\mathrm{p}_i}(x)$ models based on their parameter vectors $\mathbf{p}_i$ by applying the fuzzy C-means clustering method (Bezdek, 1981; Chiu, 1994).

Let us assume that the $\mathbf{C}_1, \mathbf{C}_2, \ldots, \mathbf{C}_N$ clusters $(N \le m)$ of the standardized failure rate curve models are formed. Let $\Im_r$ be the index set of standardized failure rate curve models $s_{\mathrm{p}_i}(x)$ that belong to cluster $\mathbf{C}_N$ (r ∈ 1, 2, …, N), that is,

$$\Im_r = \left\{ i : \mathbf{p}_i \in \mathbf{C}_r, i \in \{1, 2, \ldots, m\} \right\} \quad (14)$$

and let $\mathbf{c}_r$ be the centroid of $\mathbf{p}_i$ vectors in cluster $\mathbf{C}_r$. Vector $\mathbf{c}_r$ contains the parameters of the *cluster characteristic standardized failure rate curve model* $s_{\mathrm{c}_r}(x)$.

The $s_{\mathrm{c}_1}(x), s_{\mathrm{c}_2}(x), \ldots, s_{\mathrm{c}_N}(x)$ functions represent the *typical standardized failure rate curve models*, and as such can be taken as representative models of the empirical failure rate time series $\lambda_{i,0}, \lambda_{i,1}, \ldots, \lambda_{i,n_i}$ $(i = 1, 2, ..., m)$. The typical SFCMs are generated from complete historical failure rate time series of a consumer electronic commodity, that is, they represent historical knowledge on failure rate curves of the studied product category. The knowledge represented by the typical standardized failure rate curve models can be used to predict the unknown failure rates of active products of which empirical failure rate time series are not complete yet.

3.5 Predicting failure rate curves of active products

In our approach, active products of the studied product category are defined as ones with empirical failure rate time series that are not complete, that is, only a fraction of their failure rate time series is known. We may assume that products in the same product category have similar reliability properties. This assumption, which is empirically justified, lays the foundation of using the identified typical standardized failure rate models to predict the unknown continuations of failure rate curves of active products.

Let $\lambda_{F,0}, \ldots, \lambda_{F,M}$ be a fractional failure rate time series of an active product. For each typical SFCM $s_{\mathrm{c}_r}(x)$ the $\alpha_r \ge M$, $\beta_r \ge 0$ and $\gamma_r \ge 0$ parameters are identified so that

$$g_r : [0, \alpha_r] \to \mathbb{R}^+ \cup \{0\} \quad (15)$$

$$g_r(t) = \gamma_r s_{\mathbf{c}_r}\left(\frac{t}{\alpha_r} \right) + \beta_r \quad (16)$$

$$d_r = \sum_{i=0}^{M} \left(g_r(i) - \lambda_{F,i} \right)^2 \to \min. \quad (17)$$

Solution for each fitting problem described by (15), (16) and (17) can be found by applying the same Interior Point Algorithm referenced in Section 3.2. The d_r distance measures the level of dissimilarity between $g_r(t)$ and the fractional failure rate time series $\lambda_{F,0}, \ldots, \lambda_{F,M}$ $(r = 1, 2, ..., N)$. The normalized dissimilarity d_r^* is derived from d_r by applying the following transformation:

$$d_r^* = \begin{cases} \frac{d_r - \min(d_r)}{\max(d_r) - \min(d_r)}, & \text{if } \max(d_r) > \min(d_r) \\ 0, & \text{if } \max(d_r) = \min(d_r) \end{cases} \quad (18)$$

Each normalized dissimilarity d_r^* is turned to similarity w_r as follows (Tan et al., 2006):

$$w_r = \frac{e^{-d_r^*}}{\sum_{u=1}^{N} e^{-d_u^*}}. \tag{19}$$

Similarity w_r can be taken as a weight that expresses how well $s_{c_r}(x)$ can be used as a model of fractional failure rate time series $\lambda_{F,0}, \ldots, \lambda_{F,M}$. Based on this, we define the *prediction model* $p(x)$ for $\lambda_{F,0}, \ldots, \lambda_{F,M}$ as

$$p(x) = \sum_{r=1}^{N} w_r s_{c_r}(x), \tag{20}$$

and identify the $\alpha \geq M$, $\beta \geq 0$ and $\gamma \geq 0$ parameters so that

$$F : [0, \alpha] \to \mathbb{R}^+ \cup \{0\} \tag{21}$$

$$F(t) = \gamma\, p\left(\frac{t}{\alpha}\right) + \beta \tag{22}$$

$$\sum_{i=0}^{M} \left(F(i) - \lambda_{F,i} \right)^2 \to \min. \tag{22}$$

Solution for the fitting problem given by (21), (22) and (23) can be found by applying the same Interior Point Algorithm referenced in Section 3.2. As $\alpha \geq M$, the $F(i)$ values for $\lfloor \alpha \rfloor \geq i > M$ can be taken as predictions of the unknown $\lambda_{F,M+1}, \ldots, \lambda_{F,\lfloor \alpha \rfloor}$ future values, that is, the $\lambda_{F,M+1}, \ldots, \lambda_{F,\lfloor \alpha \rfloor}$ series can be taken as a possible continuation of the $\lambda_{F,0}, \ldots, \lambda_{F,M}$ fractional failure rate time series.

4 A case study

The method discussed so far was applied to real-life empirical failure rate curves. 43 complete empirical failure rate curves of consumer electronic goods of the same commodity were used to generate typical standardized failure rate curve models. Each complete empirical failure rate time series represents weekly failure rates of a product. The standardized failure rate curve models of the studied 43 failure rate times series were clustered into 12 clusters, that is, 12 typical SFCMs were generated. Parameters of the typical SFCMs are collected in Table 2.

Figure 4 depicts graphs of the individual standardized failure rate curve models in each cluster (grey colored curves) and the cluster characteristic (typical) standardized failure rate curve models (black colored curves).

An empirical failure rate time series containing 176 weekly failure rates of a product, which had not been involved into establishing the typical standardized failure rate curve models, was selected to demonstrate how the typical SFCMs can be used to forecast future failure rates. The cluster characteristic SFCM-based prediction was compared to four widely applied forecasting methods: the moving average, the exponential smoothing, the linear regression and the autoregressive integrated moving average (ARIMA) method. The moving average was applied with span of 5, default weight for the exponential smoothing was computed by fitting an ARIMA (0,1,1) model to the data, and back-casting was used to calculate the initial smoothed value. The number of autoregressive terms (p), non-seasonal differences needed for stationarity (d), and lagged forecast errors (q) were indicated for the best fitting ARIMA model for each time series.

Table 2 Parameters of the cluster characteristic failure rate curve models

cluster	y_l	$x_{a,l}$	$x_{e,l}$	ω_l	y_r	$x_{a,r}$	$x_{s,r}$	ω_r
1	0.7904	0.2232	0.3297	0.7426	0.9652	0.8311	0.3535	4.6672
2	0.9047	0.1898	0.3185	3.5168	1.0000	0.8631	0.3185	3.1601
3	1.0000	0.1965	0.4129	1.5842	0.7932	0.8959	0.6406	0.9354
4	1.0000	0.1167	0.3153	2.7837	0.7023	0.7501	0.7279	1.8343
5	1.0000	0.0973	0.1933	0.9909	0.5487	0.8503	0.6901	1.0315
6	0.4686	0.0616	0.4130	3.5978	1.0000	0.8363	0.6821	1.1576
7	0.4369	0.0840	0.1077	8.0754	1.0000	0.8993	0.7112	1.7103
8	0.7751	0.1210	0.2330	1.1132	1.0000	0.8193	0.6377	0.9918
9	0.4705	0.1053	0.2184	0.9736	1.0000	0.8595	0.7039	1.0257
10	0.9290	0.2258	0.9540	4.6764	1.0000	0.9809	0.9497	0.2577
11	0.6771	0.1342	0.2971	1.7623	1.0000	0.8448	0.6432	1.3206
12	0.9902	0.1223	0.2334	0.9709	0.9255	0.8417	0.6553	0.9751

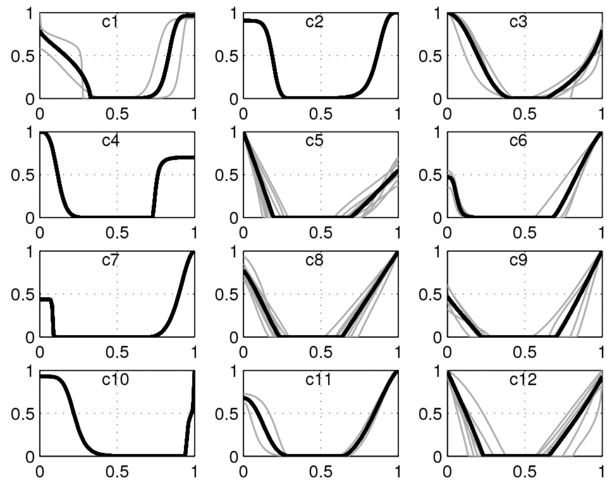

Fig. 4 Clustered standardized failure rate curve models

In order to evaluate goodness of the methods, we created forecasts based on data of the first 30, 80 and 130 weeks for the next 50 weeks, that is, we carried out almost year ahead forecasts. When we used the first 130 data as a known fraction of the studied failure rate time series, we were able to generate forecast only for the next 31 periods, as parameter α of function $F(t)$ in this case was 160.4382. Note that the starting index of periods (weeks) is zero, and so if $\alpha = 160.4382$, then the last forecast period has the index of 160, that is, the last forecast is for the 161st week. The mean squared error (MSE) of the fitted and predicted values were calculated for each forecast to characterize goodness of the applied forecasting methods. The values of α, β and γ parameters of function $F(t)$ for the three forecasts are summarized in Table 3.

Table 3 Parameters of the cluster characteristic failure rate curve models

Parameters of F(t)	A	B	C
α	161.8701	176.2402	160.4382
β	0.0278	0.0221	0.0227
γ	0.0374	0.0449	0.0442

A: forecast based on data of the first 30 weeks for the next 50 weeks
B: forecast based on data of the first 80 weeks for the next 50 weeks
C: forecast based on data of the first 130 weeks for the next 31 weeks

The MSE values for fittings to the known fraction of the studied failure time series and the MSE values for the forecast failure rates are in Table 4. The abbreviations used in this table are as follows. MA(5) stands for the moving average with span of 5, Exp. S. is the exponential smoothing, and Lin. Reg. is the linear regression.

Table 4 MSE values for fits and forecast

	$F(t)$	MA(5)	Exp. S.	Lin. Reg.	ARIMA
Fits*	1.175E-05	1.936E-05	1.823E-05	1.174E-05	1.217E-05
Forecasts*	4.165E-05	2.434E-04	2.739E-04	1.074E-04	1.052E-04
Fits**	1.657E-05	2.334E-05	2.290E-05	3.339E-05	2.062E-05
Forecasts**	3.398E-05	4.486E-05	3.767E-05	7.382E-04	3.135E-04
Fits***	1.626E-05	2.314E-05	2.226E-05	1.579E-05	2.716E-05
Forecasts***	1.260E-05	2.608E-04	2.213E-04	2.803E-05	8.087E-05

*first 30 weeks → next 50 weeks, best fitting ARIMA is ARIMA(1,1,1)
**first 80 weeks → next 50 weeks, best fitting ARIMA is ARIMA(1,1,1)
***first 130 weeks → next 31 weeks, best fitting ARIMA is ARIMA(0,2,3)

Figure 5 depicts the forecast results of the five studied methods. There are a couple of notable properties of the typical standardized failure rate curve model based forecasts, that is, the function $F(t)$ based predictions.

In the first forecast case, when predictions for period from week 30 to week 79 are given based on data of period from week 0 to week 29, the known part of the failure rate time series is in the first declining phase of the bathtub curve. In this case the linear regression gives the best fitting for the known fraction of the failure rate time series, and function $F(t)$ brings the forecast with the least MSE. The moving average and exponential smoothing methods show similar fitting results for the known fraction of the failure rate time series as the linear regression. As the failure rate time series is decreasing in the first 30 weeks and the moving average and exponential smoothing methods give constant forecasts, the latter two methods result in relatively weak forecast accuracy. The linear regression gives a decreasing forecast while the actual failure rates are getting quasi constant from week 50, that is why the accuracy of linear regression based forecast is far behind the accuracy of function $F(t)$ based one. It is important to highlight that even though the typical standardized failure rate curve model based forecast overestimates the actual values from week 40, it indicates that the failure rate curve turns from its decreasing phase to its quasi constant phase at around week 50. None of the other four methods is able to predict this turning point of the bathtub curve.

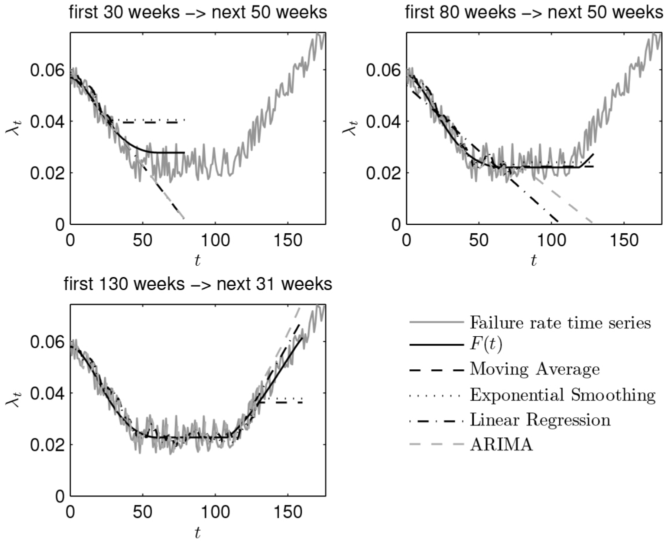

Fig. 5 Results from different forecast methods

In the second forecast case, when predictions for period from week 80 to week 129 are given based on data of period from week 0 to week 79, approximately first half of the known part of the failure rate time series is in the first declining, while the other half is in the second, almost constant phase of the bathtub curve. The actual failure rate values start to increase at around week 115, and so the moving average, the exponential smoothing and function $F(t)$ give similarly good forecasting results for the period from week 80 to week 129. For this period, the linear regression and ARIMA result one order of magnitude worse forecast accuracy in terms of MSE than those of the moving average, exponential smoothing or function $F(t)$. It is important to mention that the function $F(t)$ based forecast is the only one among the studied five methods which is able to indicate that the failure rate curve will turn from its quasi constant second phase to its increasing third phase. Function $F(t)$ suggests that the failure rate will take an increasing trend from approximately week 120, the actual figures show that in reality it happened a bit earlier, from approximately week 115.

In the third forecast case, predictions for period from week 130 to week 160 are given based on the first 130 weekly failure rates of the studied product. We know that from approximately week 115 to week 130 the failure rate curve is in its increasing third phase, thus, the linear regression was applied for the period from week 115 to week 130 instead of using the first 130 data. As the moving average and exponential smoothing methods give constant forecasts, while the actual failure rate is increasing, these two methods are not suitable to predict well the failure rates in this phase of the failure rate curve. The ARIMA, the linear regression and the function $F(t)$ based forecasts follow well the increasing trend of failure rate time series. A shortcoming of the typical standardized failure rate curve model based forecast is that the end of the forecast period is determined by parameter α of function $F(t)$. In our case, $\lfloor \alpha \rfloor = 160$, and so our method cannot give any prediction for weeks that have index greater than 160.

The case study demonstrates the main advantages of our forecast method well. Namely, the typical standardized failure rate curve model based forecast is able to indicate the turning points of the bathtub-shaped failure rate curve in advance, while the traditional statistical forecasting techniques do not have such a capability. In general, machine learning, fuzzy, neural, and fuzzy-neural hybrid techniques have similar capabilities. Our method can be taken as a hybrid one founded on an analytic model of the failure rate time series and complemented with fuzzy clustering to discover typical standard models.

5 Conclusions and managerial implications

In this paper we presented a hybrid technique for modeling and forecasting bathtub-shaped failure rate curves of consumer electronics. Empirical failure rate curves describing the whole life-cycles of on-the-market electronic products considered as time series were considered as inputs for typifying SFCMs. These typical SFCMs are applied for predicting unknown continuations of failure rates of active products of which complete empirical failure rate time series are unknown, that is, only a fraction of their failure rate time series is known.

Similarities among historical SFCMs are characterized by eight model parameters having geometric interpretations and cognitive aspects relating to the shape of the model curve and so clustering the SFCMs results in typical SFCMs that can be applied for predicting the future values of failure rate curves. In this sense, the cluster characteristic SFCMs represent the knowledge of failure rate curves gained from historical data. From a managerial perspective, discovering similarities among empirical failure rate curves generates added information both for the repair service providers and for the original equipment manufacturers. Electronic repair service provider companies can utilize it to predict resource needs for particular repair services, the latter ones can conclude on typical reliability characteristics of their products.

The case study for illustrating the practical application of the introduced methodology underlines that the SFCMs can be used to discover similarities of the studied failure rate curves. The introduced method is founded on a model that is mathematically simple, but suites well the needs of industrial applications. The accuracy of our methodology was compared to moving average, exponential smoothing, linear regression and ARIMA methods. In contrary to the traditional statistical forecasting methods, the presented forecasting methodology can indicate the turning points of the traditional bathtub-shaped failure rate curve in advance. Based on the results we can conclude that our modeling results are encouraging and it has the potential to be a suitable alternative predicting technique.

Conclusions concerning model flexibility should be added. Taking an active electronic product with an incomplete failure rate curve into consideration, management can get further information about the continuation of actual failure rates in the course of time. From time to time we can reconsider which typical SFCM fits the actual curve of the failure rate the best. This kind of fitting by moving forward in the active product life-cycle can change and can be revised according to new information. On the other hand, with the production of new products due to shortening life-cycles in the electronic industry the database of curves can be complemented and historical failure rate curves can be clustered again resulting in the re-identification of typical curves as time goes on. One of the restrictions of the introduced methodology is that it goes along with the condition of presuming bathtub-shaped failure rate curves. As a future research plan the methodology should be tested on other types of products as well. Another possible future research direction is the consideration of functional and technological features of the studied products; however, one can say that products with similar functional and technological parameters presumably have similar failure rate curves. It is worth studying whether products that are functionally and technologically similar also show similarities in their SFCM parameters. For this purpose, self-organizing feature maps could be applied to study how effectively the product SFCMs could be typified according to clustering criteria learnt from their functional and technological attributes.

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