Periodica Polytechnica
Transportation Engineering

47(3), pp. 171-177, 2019
https://doi.org/10.3311/PPtr.10838

RESEARCH ARTICLE

Easy Tracking of UAV Using PID Controller

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Received 02 April 2017; accepted 22 February 2018

Abstract
The main objective of the present paper is to design a mathematical model to estimate the behavior of flying robots with four motors (quadcopters) controlled by three algorithms; P depends on the present errors; I on the accumulation of past errors, and D a prediction of future errors (PID controller design) with simple strategy. In this regard, a governing equation of motion based on Newton Euler's formularies for rigid body dynamics is presented. In order to design the control algorithm some assumptions are made such as the ignorance of the blade flapping, surrounding fluid velocities. This exclusion of parameters makes the model flexible, simple, and allows the control to be more efficiency and easy to designed without the need of expensive computation. The simulation studies are carried out using MATLAB program.

Keywords
Quadcopters, PID controller, Newton Euler's Equations, Matlab

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1 Introduction

The terminology UAV (unmanned aircraft vehicle) refers not just to the aircraft, but for all fly machines controlled from the ground with the use of a controller with WIFI-connection and is small, without the need of any pilot and is mostly called, drone. UAVs are using new technologies of sensors, micro-controllers, Control software, communication hardware, and use interfaces. In researchers, UAV can generally be divided into two types or categories fixed-wings and rotorcraft; this paper focuses on the quad-copter.

Quadrotor is not as fast as others types of UAVs like fixed-wing and standard helicopters, but it is largely considered a very communal and advantageous (VTOL) Vertical Take Off and Landing concept especially for specific type of missions and operations needed to do it's. Last years, autonomous control of this vehicle has been worked by many laboratories and organizations. Although quadrotor have many advantageous properties, it has a highly nonlinear coupled and under actuated dynamics. Therefore, control of the vehicle is not straightforward and many researchers and science test interested in designing and verifying control methods for quadrotors. There are a large of texts that cover quadrotor dynamics and control.

However, various techniques of control were tested for examined behavior of quadrotor platforms (Al-Younes et al., 2010). Nonlinear control technique was developed and based on the equations of motion of rigid body Newton Euler's formalism (Mian and Daobo, 2008). A multi-layer aerial vehicle was studied with new modeling simulation and control of rotorcraft with four motors (Mahony et al., 2012). Process of modeling and designing control laws for four-rotor type of the Parrot UAV with state space model is obtained by using several phenomena like gyroscopic effects for rigid bodies, and controlled by PID algorithm (Koszewnik, 2014). Predictive Functional Control (PFC) was made for the Parrot Drone follow a red ball and controlling the Position and Velocity in Space of the Quad-Rotor has an in-built proportional-integral-derivative (PID) controller with image processing open cv (Jamkhandi et al., 2012). New model design method for the flight control of an autonomous quad rotor; this study was described the controller PID architecture for the quad rotor with Matlab/Simulik (Salih et al., 2010).

Some researches of science test and laboratories have published their experience results in designing quadrotor prototypes. Their work integrated on the rise of the developing and simulation models, utilizing different controllers, equipments and materials. In Pennsylvania State University, different studies had been done on quadrotors, two methods of control are studied; one of them uses a series of mode-based, feedback linearizing controllers, the other using a back-stepping control law tested on DraganFlyer model (Altuğ, 2003). In Middle East Technical University, three orthogonal piezoelectric gyro used in the system designed to control the attitude with an LQR and PD controller of the quadrotor (Çamlıca, 2004). Study was presented from Swiss Federal Institute of Technology, including the mechanical design, dynamic modeling, sensing, and control method for autonomous take-off of an indoor VTOL (OS43) only the 3 DOF are looked (Bouabdallah et al., 2004). American University at Sharja has published their experience work gained in the design and control of several quadrotor generations (Al-Younes, 2009).

In this paper we will present a very simplified study of quadcopter dynamics with design controllers PID for our dynamics modal to follow a designated trajectory. Then we will test our controllers with a numerical simulation of a quadcopter in flight.

The paper is organized as follows: In Section 2, we describe the mathematical modeling with principal physical, kinematics and dynamics flight mechanical of the quadrotor. In Section 3, we present the fundamentals of PID controller. In Section 4 we illustrate the essentials simulation results of present paper. In Section 5 concludes the paper.

2 Mathematical modeling

In order to create a flying controller, we should have a good understanding of the quadrotor movement, and its dynamics to extract the mathematical model, this understanding it's not just necessary for the creating of the controller but also needs to insure that the simulation behavior will be in good agreement with the reality of the applicable command.

The dynamics of the quadrotor subject to external forces applied to the mass center and expressed in the body-fixed reference frame in Newton-Euler can be formulated as:

$$\begin{bmatrix} MI_{3\times3} & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} \dot{V} \\ \dot{\Omega} \end{bmatrix}+\begin{bmatrix} \Omega\times M\dot{\xi} \\ \Omega\times I\dot{\eta} \end{bmatrix}=\begin{bmatrix} F \\ \tau \end{bmatrix}. \tag{1}$$

There are 12 states that describe quadrotors dynamic behavior: Space position $\xi = [X\ Y\ Z]$, Linear velocity $V = [u\ v\ w]$, Rotational angles $\eta = [\phi\ \theta\ \psi\ ]$ (Roll, Pitch, and Yaw), and angular velocities $\Omega = [\Omega_1\ \Omega_2\ \Omega_3]$. These can be considered as the plant's outputs while the inputs are the applied forces and torques (F, τ) generated by the four motor's rotation. M mass of quadcopter and I is the inertia matrix which given by:

$$I=\begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix}. \tag{2}$$

The quadrotor is considered like a rigid body with constant mass and axis aligned with the principal axis of inertia and symmetric geometry (see Fig. 1), then the inertia tensor I becomes a diagonal matrix containing only the principal moments of inertia:

$$I=\begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}. \tag{3}$$

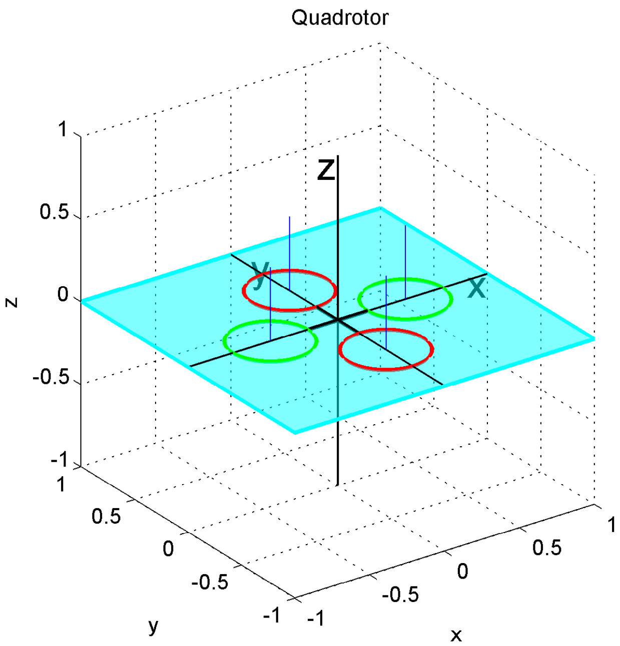

Fig. 1 Quadcopter schematic and principal axis (Mode +).

Taking account the properties of the design, the quadrotor is only controlled by varying separately the speed of the four rotors (see Fig. 2). Let τ_i and F_i be the torque and thrust for i th rotor respectively (i = 1 ... 4) (these values are normalized with the moment of inertia and mass, corresponding). Denoting L the distance between the rotor and center of mass, we can now start a set of four control inputs U_i, as functions of normalized character thrusts and torques as follows (Cooke and Fitzpatrick, 2002).

$$U_1 = F_1 + F_2 + F_3 + F_4. \tag{4}$$

U_1: Is the total thrust. The roll moment is obtained by unstable the left (motor 4) and right (motor 2) rotor speeds:

$$U_2 = L(F_4 - F_2). \tag{5}$$

A pitch moment is achieved by varying the ratio of the front (motor 1) and back (motor 3) rotor speeds.

$$U_3 = L\left(F_1 - F_3\right). \tag{6}$$

Finally, a yaw moment is produced from the torque resulting from the subtracting counterclockwise (front and back) from the clockwise (left and right) speeds.

$$U_4 = \left(\tau_1 + \tau_3 - \tau_2 - \tau_4\right). \quad (7)$$

Moreover, the motor's thrusts are related to the motor's angular velocity as:

$$F_i = b\left(\omega_i\right)^2. \quad (8)$$

The motor's torques acting on the quadrotor is expressed as:

$$\tau_i = d * F_i. \quad (9)$$

Where b is the static thrust constant, and d is a constant that relates moment and thrust of a propeller.

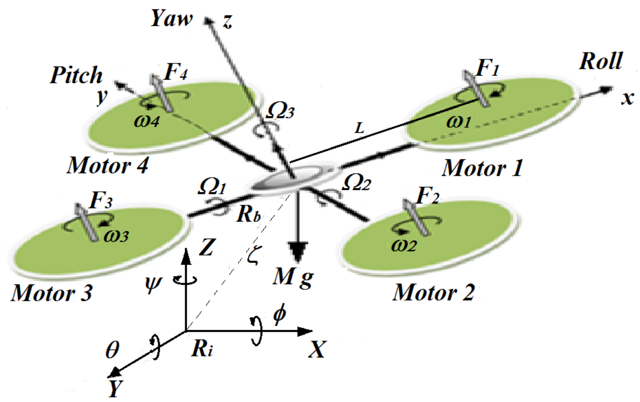

Fig. 2 The structure of quadrotor and relative coordinate systems.

The vector U is the vector of control inputs defining by:

$$U = \left[U_1, U_2, U_3, U_4\right]^T. \quad (10)$$

From the right hand rule; the body of drone fixed coordinates is defined as show in (Fig. 2).

To successfully drive our quadrotor, the fellow steps are:

- As a starting the mobile frame is coincided with that of the inertial frame, then the mobile frame start a rotation along x direction with a roll angle:
  $(-\pi/2 < \phi < \pi/2)$.
- A rotation along y direction with a pitch angle:
  $(-\pi/2 < \theta < \pi/2)$.
- A rotation along z direction with a yaw angle:
  $(-\pi < \psi < \pi)$.

It must be notified that each rotation must be effectuated with corresponding axe from the fixe frame R_i (Fig. 3).

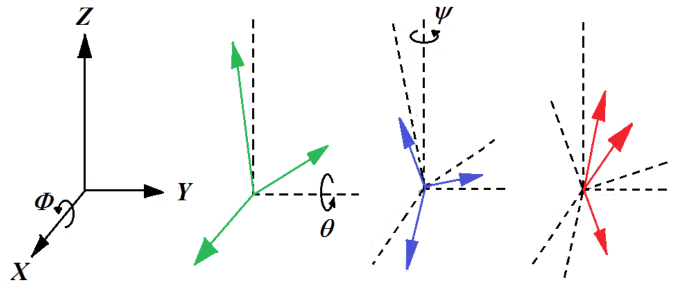

Fig. 3 Angles presentation (roll, pitch, and yaw).

From this chosen modeling of angles the rotation matrix R can be formulated by:

$$R = Rot_z(\psi) * Rot_y(\theta) * Rot_x(\phi)$$
$$= \begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix} \quad (11)$$

$$R = \begin{bmatrix} c\psi c\theta & s\phi s\theta c\psi - s\psi c\phi & c\phi s\theta c\psi + s\psi s\phi \\ s\psi c\theta & s\phi s\theta s\psi + c\psi c\theta & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix}. \quad (12)$$

With $s \equiv sin$ and $c \equiv cos$.

By taking account the influence the gravitational force (Fig. 2) and vector of controller inputs Eq (10), the dynamic model Eq (1) of drone is driven using Newton-Euler approach a simplified mathematical model has been chosen (Khatoon et al., 2014). The differential equation of motion of the modeled can be formulated as following:

$$\begin{cases} \ddot{X} = \left(\sin\psi \sin\phi + \cos\phi \sin\theta \cos\psi\right)\dfrac{U_1}{M} \\ \ddot{Y} = \left(\sin\psi \cos\phi - \sin\phi \sin\theta \cos\psi\right)\dfrac{U_1}{M} \\ \ddot{Z} = -g + \left(\cos\phi \cos\theta\right)\dfrac{U_1}{M} \\ \ddot{\phi} = \dfrac{U_2}{I_{xx}} \\ \ddot{\theta} = \dfrac{U_3}{I_{yy}} \\ \ddot{\psi} = \dfrac{U_4}{I_{zz}} \end{cases}. \quad (13)$$

3 The control low

In the present section, we provide an overview of PID algorithm contr oller with an emphasis on simple model reference adaptive control (MRAC) (Ghaffar and Richardson, 2015). Its main objective is to design of adaptive flight PID controller for the quadrotor drone. The control input u used to control the position and angle of the drone respect to the reference input designed as follows:

$$u(t) = K_p e(t) + K_i \int e(t)\,dt + K_d \dot{e}(t). \quad (14)$$

K_p: is the proportional gain;
K_i: is the integral gain.
And K_d: is the derivation gain.

With error can be formulated as:

$$e(t) = s_p - p_v(t), \quad (15)$$

s_p: is the setpoint or desired position.

And p_v (t): is the process variable at instantaneous time according to s_p.

A high-quality controller must be able to establish a desired position, in which the yaw, pitch and roll angles stay constant and stable.

By using Pythagoras theorem and implementing the following assumptions and cancellations:

1. The quadrotor is considered like a rigid body with constant mass and symmetrical structure.
2. The Inertia matrix (I) of the vehicle is very small and to be neglected.
3. The center of mass and center of gravity coincides.
4. Thrust is proportional to the square of the propellers speed.

Based on the above assumption and considering the drone as a material point, which their rotational angles can be identified using a desired position, the whole methodology is depicted in the (Fig. 4). The desired angles (Roll, Pitch, and Yaw) can be extracted in the following expressions:

$$\phi_d = \tan^{-1}(Z_d / Y_d) \quad (16)$$

$$\theta_d = \sin^{-1}(X_d / \sqrt{Z_d^2 + X_d^2}). \quad (17)$$

And:

$$\psi_d = \cos^{-1}(Y_d / \sqrt{X_d^2 + Y_d^2 + Z_d^2}). \quad (18)$$

Fig. 4 shows Roll, Pitch, and Yaw angles during the motion of quadrotor. It can be seen, from this figure, that a very good tracking of the desired angles can be established to get better position. In this step we considered every instantaneous position create a cube with inertial frame, the dimensions of this cube are changed with every instant chosen for recalculate rotational angles, hence, for eliminate blockage case we taking account critical value of pitch angle $\theta < \pi/2$.

From the geometry modeling of our model, the input needs to control the spatial location (X, Y and Z) are given in the following expressions:

$$u_X = K_p(X_d - X) + K_i \int_0^t (X_d - X) + K_d \frac{d(X_d - X)}{dt} \quad (19)$$

$$u_Y = K_p(Y_d - Y) + K_i \int_0^t (Y_d - Y) + K_d \frac{d(Y_d - Y)}{dt} \quad (20)$$

$$u_Z = K_p(Z_d - Z) + K_i \int_0^t (Z_d - Z) + K_d \frac{d(Z_d - Z)}{dt}. \quad (21)$$

With K_p, K_i and K_d are PID controller gains for coordinate position.

The orientation angles are controlled as described in next equations:

$$u_\phi = K_{pa}(\phi_d - \phi) + K_{ia} \int_0^t (\phi_d - \phi) + K_{da} \frac{d(\phi_d - \phi)}{dt} \quad (22)$$

$$u_\theta = K_{pa}(\theta_d - \theta) + K_{ia} \int_0^t (\theta_d - \theta) + K_{da} \frac{d(\theta_d - \theta)}{dt} \quad (23)$$

$$u_\psi = K_{pa}(\psi_d - \psi) + K_{ia} \int_0^t (\psi_d - \psi) + K_{da} \frac{d(\psi_d - \psi)}{dt}. \quad (24)$$

Where K_{pa}, K_{ia} and K_{da} are parameters of PID controller forth control of Roll, Pitch and Yaw angles.

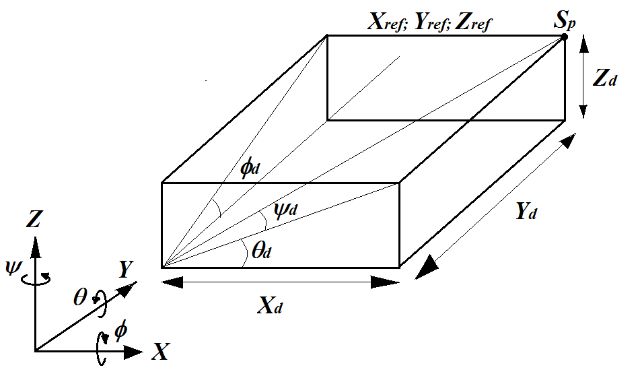

Fig. 4 Description of rotational angles.

4 Simulation result and analysis

In order to analyze the dynamics movement of the proposed quadrotor. The physical parameters of our model used in the present paper can be found in (Mohammed et al., 2014). A PID controller was designed based on equation (13) and integrated into Matlab. The altitude controller is designed to stabilize the vertical position of the platform at 25 m to minimize the influence of the error on the rotational angles. The control structure is described by equations (24) of the form (Koszewnik, 2014, Bouabdalth and Noth, 2004):

$$\begin{aligned} \dot{e}_Z &= \dot{Z}; \\ e_Z &= Z_{SET} - Z; \\ u_Z &= K_p * e_Z + K_i * e_Z - K_d * \dot{e}_Z. \end{aligned} \quad (25)$$

The variations on the Z, X, and Y directions over times are presented in Fig. 5, Fig. 6, and Fig. 7 respectively, the curves show that the quadrotor is attempted the desire position which prove the accuracy of the present controller. The corresponding errors are depicted in Fig. 8, Fig. 9, and Fig. 10 confirms the precision of the present model.

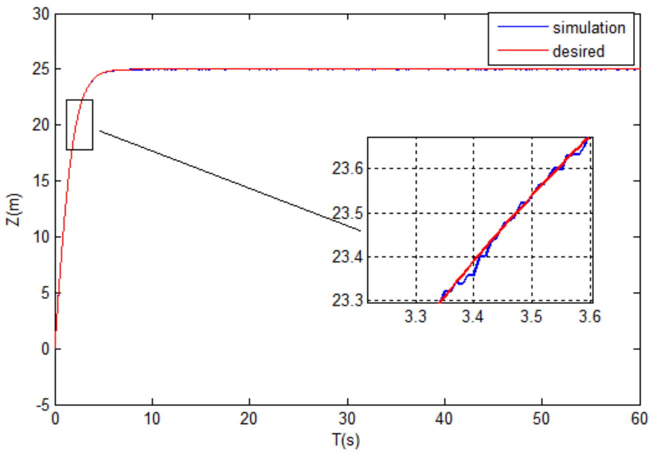

Fig. 5 The Z position path relative to the reference of UAV.

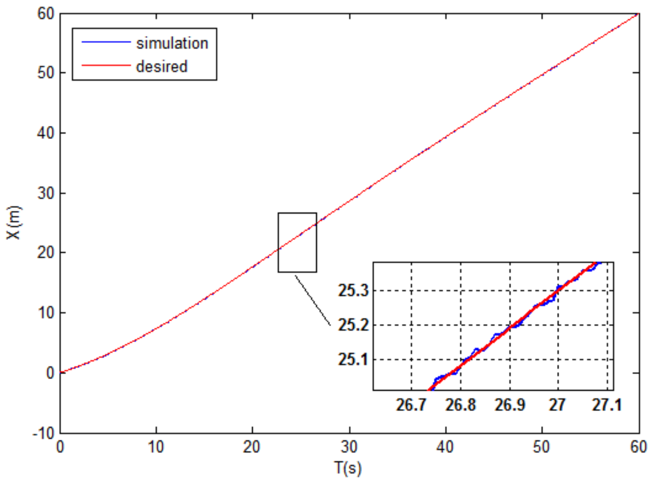

Fig. 6 The X position path relative to the reference of UAV.

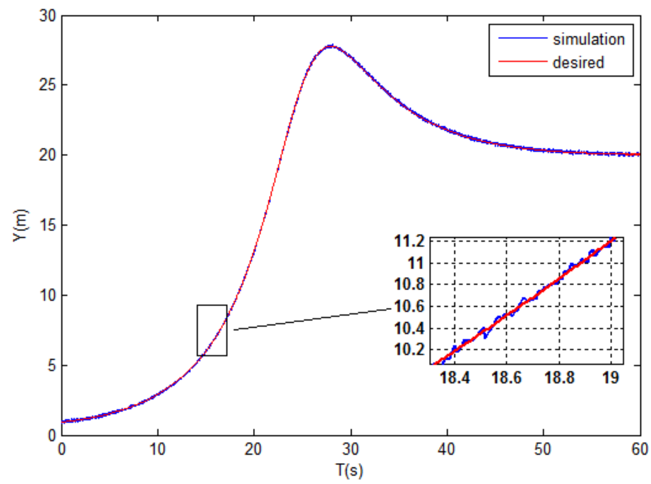

Fig. 7 The Y position path relative to the reference of UAV.

Fig. 11, Fig. 12, and Fig. 13 shows the roll, the pitch, and the yaw angles variations during the motion of the present model. The errors of the rotational angles are presented in the (Fig. 14), its shows that the model is stable with an acceptable error when it's reach $Z = 25$ m.

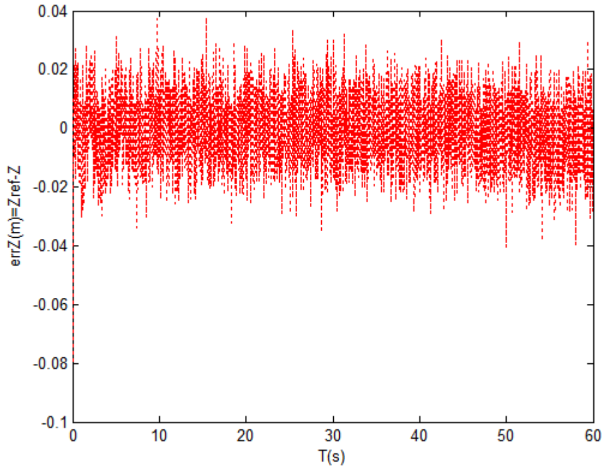

Fig. 8 Error path in altitude Z.

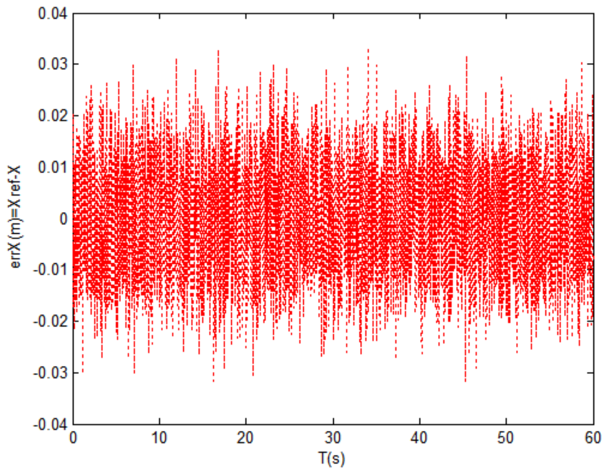

Fig. 9 Error trajectory in X position.

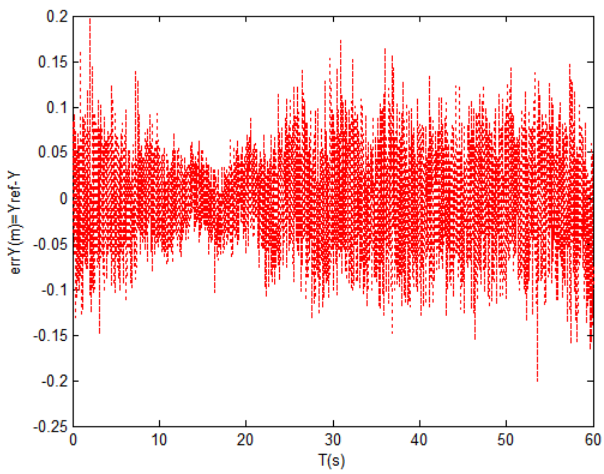

Fig. 10 Error trajectory in Y direction.

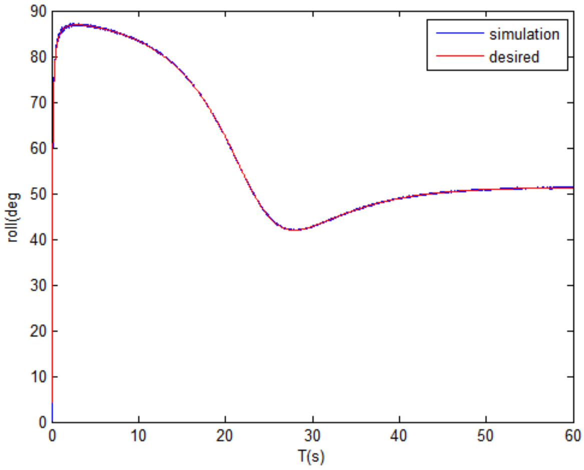

Fig. 11 Simulation results of trajectories along the Roll.

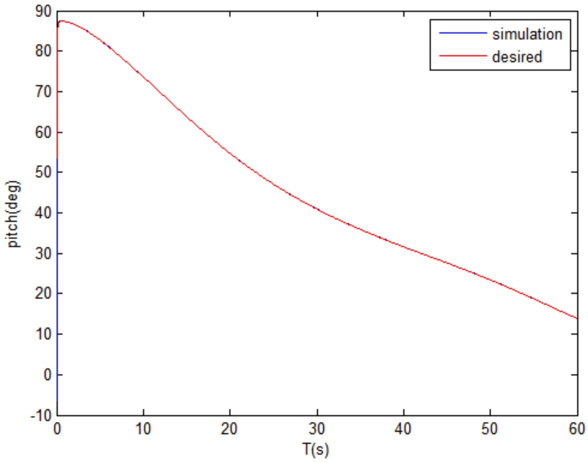

Fig. 12 Simulation results of trajectories along the Pitch.

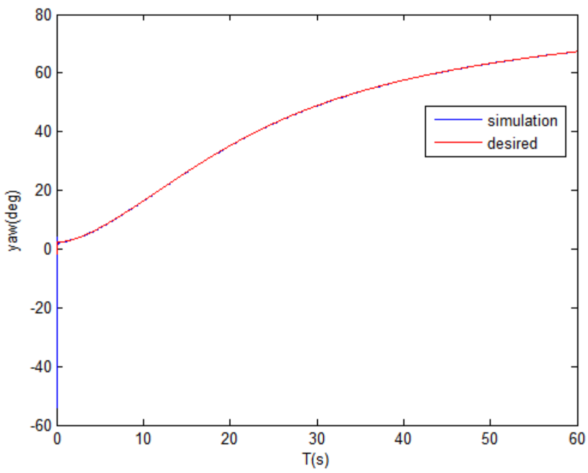

Fig. 13 Simulation results of trajectories along the Yaw.

Fig. 15 illustrates the 3D trajectory of quadrotor during the flight. It shows a good robustness towards stability and tracking for desired trajectory. This explains the efficiency and stabilizing of control strategy developed in this paper. Simulation results presented at the end, confirm that the proposed stabilization and control strategy could be succefully used UAV. The PID controller proved to be well adapted to the quadrotor when flying and hovering.

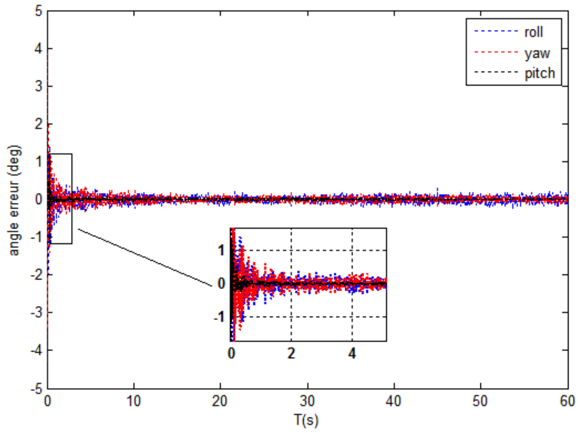

Fig. 14 Error of rotational angles (ϕ, θ, ψ).

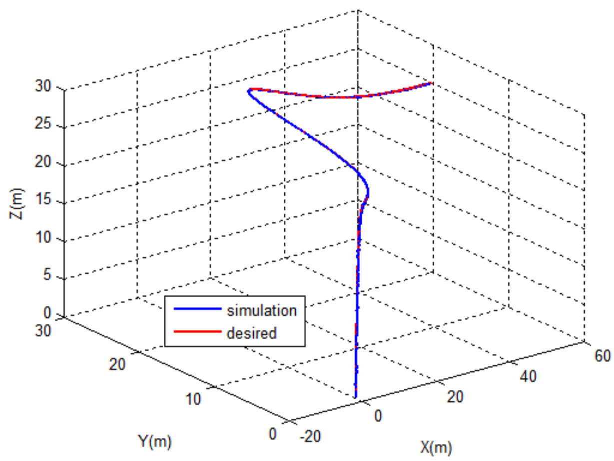

Fig. 15 The final simulation results of global trajectories in 3D.

5 Conclusions

This paper investigates the mathematical modeling, stabilization and control of a small quadrotor UAV. The model is developed based on Newton-Euler assumption. The obtained model is coupled with a PID control algorithm. The resulting system is converted to a Matlab algorithm to study the performance of the present model. The simulation results prove that the adopted process of modeling and control is uncomplicated, prompt and effective for the trajectory guiding. The error between the desired and simulation trajectory is very low for the three control angles, and the altitude which proves the robustness towards stability and tracking of the proposed model.

References

Altuğ, E. (2003). Vision based control of unmanned aerial vehicles with applications to an autonomous four rotor helicopter, quadrotor. *PhD thesis, University of Pennsylvania.* http://repository.upenn.edu/dissertations/AAI3109146/

Al-Younes, Y. (2009). Establishing Autonomous AUS-Quadrotor. *Thesis at the American University of Sharjah.* https://dspace.aus.edu/xmlui/bitstream/handle/11073/131/Final%20Thesis.pdf?sequence=1

Al-Younes, Y. M., Al-Jarrah, M. A., Jhemi, A. A. (2010). Linear vs. Nonlinear Control Techniques for Quadrotor Vehicle. In: *7th Symposium of Mechatronics and its Applications*. Sharajah, United Arab Emirates, Apr. 20-22, 2010. http://ieeexplore.ieee.org/abstract/document/5478427/

Bouabdallah, S., Noth, A., Siegwart, R. (2004). PID vs LQ Control Techniques Applied to Indoor Micro Quadrotor. *IROS.* http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.149.6169&rep=rep1&type=pdf

Çamlıca, F. B. (2004). Demonstration of a stabilized hovering platform for undergraduate laboratory. *A Thesis Submitted to the Graduate School of Natural and Applied Sciences of Middle East Technical University.* http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.633.4925&rep=rep1&type=pdf

Cooke, A. K., Fitzpatrick, E. W. H. (2002). Helicopter Test and Evaluation. *Black well and QinetiQ, Oxford, England.* https://www.amazon.com/Helicopter-Test-Evaluation-AIAA-Education/dp/1563475782

Ghaffar, A. A., Richardson, T. (2015). Model Reference Adaptive Control and LQR Control for Quadrotor with Parametric Uncertainties. *World Academy of Science, Engineering and Technology International Journal of Mechanical, Aerospace, Industrial, Mechatronic and Manufacturing Engineering.* 9(2), pp. 244-250. http://waset.org/publications/10000466/model-reference-adaptive-control-and-lqr-control-for-quadrotor-with-parametric-uncertainties

Jamkhandi, A. G., Tulpule, S., Chaturvedi, A., Charvet, J.-N. (2012). Controlling the Position and Velocity in Space of the Quad-Rotor UAV AR.Drone Using Predictive Functional Control and Image Processing in Open CV. *IPCSIT (2012) © IACSIT Press, Singapore.* 58, pp. 14-18. https://doi.org/10.7763/IPCSIT.2012.V58.3. http://www.ipcsit.com/vol58/003-ICSPS2012-P043.pdf

Khatoon, S., Shahid, M., Ibraheem, Chaudhary, H. (2014). Dynamic modeling and stabilization of quadrotorusing PID controller. In: *Advances in Computing, Communications and Informatics. ICACCI, 2014 International Conference on IEEE.* New Delhi, India, Sept. 24-27, 2014. pp.746-750. https://doi.org/10.1109/ICACCI.2014.6968383

Koszewnik, A. (2014). The Parrot UAV Controlled by PID Controllers. *Acta Mechanica et Automatica.* 8(2), https://doi.org/10.2478/ama-2014-0011

Mahony, R., Kumar, V., Corke, P. (2012). Multirotor Aerial Vehicles: Modeling, Estimation, and Control of Quadrotor. *IEEE Robotics and Automation Magazine.* 19(3), pp. 20-32. https://doi.org/10.1109/MRA.2012.2206474

Mian, A. A., Daobo, W. (2008). Nonlinear Flight Control Strategy for an Underactuated Quadrotor Aerial Robot. In: *IEEE International Conference on Networking, Sensing, and Control.* Sanya, China, Apr. 6-8, 2008. https://doi.org/10.1109/ICNSC.2008.4525351

Mohammed, M. J., Rashid, M. T., Ali, A. A. (2014). Design Optimal PID Controller for Quad Rotor System.*International Journal of Computer Applications.* (0975 – 8887) 106(3), pp. 15-20. http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.800.3073&rep=rep1&type=pdf

Salih, A. L., Moghavvemi, M., Mohamed, H. A. F., Gaeid, K. S. (2010). Flight PID controller design for a UAV quadrotor. *Scientific Research and Essays.* 5(23), pp. 3660-3667. http://www.academicjournals.org/journal/SRE/article-abstract/4EFF99440127