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György Kovács<sup>1\*</sup>, Miklós Gubán<sup>2</sup>

RESEARCH ARTICLE

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## Abstract

*The selection of the petrol station for refuel and the amount of refilled fuel are not determined centrally at most transport companies, but depend on the individual decision of the driver, so that the total cost of the burned fuel is not minimal. The goal of this study is to elaborate a precise and reliable mathematical model and method for the determination of the optimal refuel points and the amount of loaded fuel to fulfil specific transport tasks. The model is a mixed valued non-linear programming model, which can be handled by optimization procedures. Based on the elaborated model and method, a decision-supporting software was developed, which provides the required information that is necessary for the economical fulfilment of transport trips.*

## Keywords

*transport trip, petrol station, refueling, optimization, software development*

## 1 Introduction of international road transport

The volume of the transport activities connected to production and service is ever increasing due to the growth of the efficiency of the economy and the long international supply chains.

In Europe the ratio of road transport in the total transport volume is 78 % (data from (Fraunhofer, 2014)), furthermore this ratio is constantly increasing. The remaining 22 % breaks up to railway (7 %), waterway (3 %), sea (8 %), pipeline (4 %), while air freight transport is almost negligible (8 million tons per year).

Road transport is economical mainly for regional range, but due to its many advantages it is also used in case of long-distance transport as well. Its high ratio compared to the other transport modes is originated from its high density of road network, short transport time, high level of adaptability to the customer's demands and provides door-to-door service.

Approximately 30 % of the cost of the whole supply chain comes from transportation, therefore every production and service companies put large emphasis on the optimization of transportation.

The increase of the efficiency of international transport routes and the decrease of the greenhouse gas emission can be realized by the following ways:

- modernization of the vehicle fleet, the use of engines with lower fuel consumption and greenhouse gas emission (Zöldy and Török, 2015),
- development of the drivers' driving skills,
- application of multimodal transport methods, where road, rail, and water transportation modes are combined,
- *optimization of the transport task:*
  - integration of multiple transportation tasks within one transport trip,
  - elimination of idle runs,
  - maximization of the load efficiency of the vehicle,
  - optimal formation of transport routes,
  - refueling the optimal amount of fuel at the ideal petrol station.

<sup>1</sup>Institute of Logistics,  
Faculty of Mechanical Engineering and Informatics,  
University of Miskolc,  
H-3515 Miskolc, Miskolc-Egyetemváros, Hungary

<sup>2</sup>Department of Management and Information Technology of Economics,  
Zalaegerszeg Faculty of Business Administration,  
Budapest Business School  
H-8900 Zalaegerszeg, Gasparich u. 18/a, Hungary

\*Corresponding author, e-mail: [altkovac@uni-miskolc.hu](mailto:altkovac@uni-miskolc.hu)

Goal of this research is the optimization of the road freight transport activity by the selection of optimal petrol station (depending on the fuel price and distance to the original transport way), and by the determination of the optimal amount of refilled fuel (only the required amount of fuel, not higher). The result of wrong decisions is that the cost of the burnt fuel is not minimal. This is true in case of inland, but it has even more significance in case of international road freight transport routes, which case high amount of fuel is burnt.

Another goal is to develop a software based on the elaborated theoretical method that would plan the optimal fuel supply, therefore the total cost of the transportation activity would be reduced.

## 2 Literature review

Before the elaboration of the optimization method for refueling, the national and international literature of the topic was reviewed.

A wide range of literature is available that introduces the general logistics costs (e.g. Ross, 2015), and logistics literature often deals with the introduction of cost components of road transportation (e.g. Birge and Linetsky, 2007; Anbuudayasankar et al., 2014).

Vehicle route planning (VRP) is an important solution to reduce the fuel consumption of delivery vehicles. VRP aims at planning the routes of a fleet of vehicles on a given freight transport network to serve a set of clients under constraints. Furthermore a reasonable route plan could effectively reduce the transportation costs and energy consumption. (Zhifeng et al., 2014).

The literature often discusses route optimization and optimization of transport trips and networks (Birge and Linetsky, 2007; Caramia and Dell’Olmo, 2008; Ehmke, 2012; Anbuudayasankar et al., 2014; Bohács et al., 2016).

There are several available mathematical optimization algorithms to solve VRP problems, e.g. heuristic algorithms, such as genetic algorithm (Ombuki et al., 2006) or ant colony algorithm (Yu et al., 2009; Zhifeng et al., 2014).

There are very few literatures that deal with refueling policies of transport trips, e.g. Lin et al. (2007) show a linear-time greedy algorithm for finding optimal refueling strategies in a fixed route. Lin (2012) later improved the previous study and proved the efficiency of the best algorithm to minimize fuel costs in case of a multi route problem. Tegelberg (2015) optimized the fuel expenses of long international trips by introducing a solution of refueling at cheap gas stations and refueling less but more often, thus keeping the fuel consumption low. Dynamic programming was used to solve this optimization problem.

There is gap in the literature in the field of fuel optimization that takes into consideration the optimal route, the fuel prices at the different stations, the fuel consumption of different vehicles on different routes and the optimal amount of refilled fuel. The

fuel optimization task to be solved has not been covered before, therefore this topic is absolutely actual and unique.

In order to organize road transportation three different categories have to be defined: simple trips, shuttle trips and round trips (Simchi-Levi et al., 2014; Anbuudayasankar et al., 2014). An important characteristic of road freight transportation is that it consists of transport trips. The main goal of the organization of the transport routes is to minimize the cost and idle runs, namely the unit cost of transportation per product (Birge and Linetsky, 2007; Ehmke, 2012).

Every transport trip departure from the depot of the company, then goes to the first product dispatch point, then it proceeds to the first discharge station, then continues to the following dispatch and discharge stations etc. (Fig. 1). After the last discharge station the vehicle finishes its route at the depot. The number of dispatch/discharge stations can be arbitrary chosen in a transport route, furthermore any dispatch station can be a discharge station as well. (Kovács and Cselényi, 2006, Kovács et al., 2007).

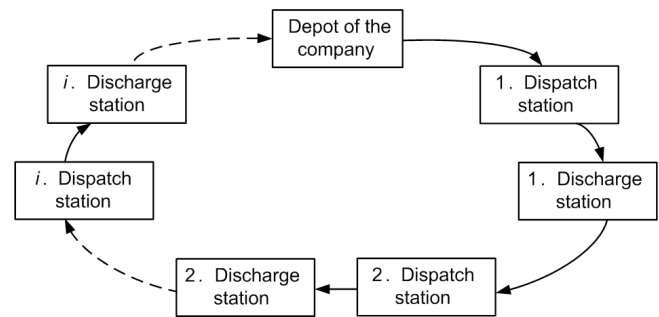


Fig. 1 Structure of road transport trips

## 3 Problem formulation

In case of transport trips the selection of the petrol station for refuel and the amount of refilled fuel are not determined centrally at most transport companies, but depend on the individual decision of the driver. Therefore many times the refueling does not take place at the ideal petrol station, and not the optimal amount of fuel is refilled or in some cases the driver refuels unnecessarily. The result of these wrong decisions is that the cost of the burnt fuel is not optimal.

This is true in case of inland transportation tasks, but it has even more significance in case of international routes, where high amount of fuel is used. There are many available petrol stations, but the difference between the unit price of the fuel can be 50-70 HUF/liter between the different petrol stations, which results in 35,000 HUF difference in case of the refill of only 500 liters.

Another common mistake is that the driver refuels an unnecessarily high amount of fuel in the end of a transport route before arriving to the depot of the company, sometimes at an expensive petrol station. This is a big loss, since most transportation companies have the opportunity to refill at the depot as well with more reasonably priced fuel.

The above examples justify the aim of the authors to determine an optimization method for refueling that is essential for transportation companies for reduction of the transport costs.

The goal of this study is to elaborate a mathematical model that would eliminate these wastes, and would reduce the losses.

The task is to determine the most efficient petrol station amongst the preferred one by the company ( $t_i$ ), in case of a transportation task from station  $F_{i+1}$  to destination  $L_{i+1}$  defined by GPS coordinates, furthermore to determine the amount of refilled fuel with a known initial level of fuel (Fig. 2).

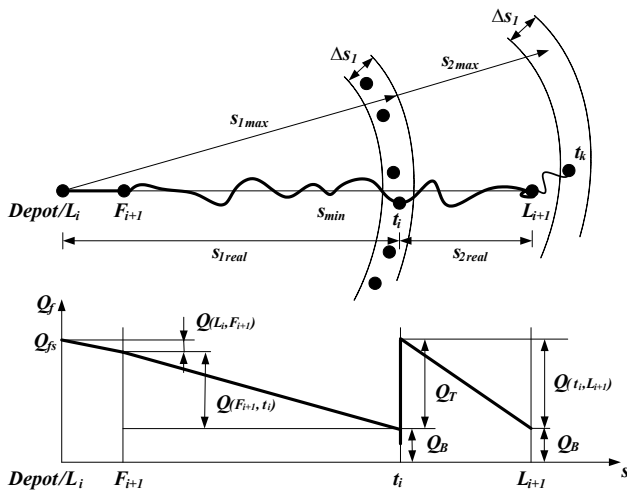


Fig. 2 Determination of possible petrol stations, and the change of fuel level on the road

Selection of petrol stations and the optimal amount of refilled fuel during long road transport trips is depends on the individual decision of the camion driver, not defined by a central decision making process at forwarding companies. Therefore the total cost of the burned fuel is not minimal.

The goal of the study is to elaborate a mathematical model that allows the elimination of lost arising from the above mentioned problems. Therefore the task is to define the location of the optimal petrol station from the preferred ( $t_i$ ) petrol stations if it is required in case of a transport task from point  $F_{i+1}$  (dispatch station) to point  $L_{i+1}$  (discharge station, Fig. 2), and to define the volume of fuel to be refilled ( $Q_r$ ).

Therefore the goal is to determine whether the vehicle is able to fulfil the whole transport task with the known initial level of fuel, or it has to be refilled between the dispatch and discharge stations. In case of refueling is necessary, the ideal refill point and the amount of fuel has to be determined.

Most of the transport companies – due to the huge amount of fuel consumption – can buy fuel on a discounted price at a contracted fuel selling company. Transport companies prefer those fuel suppliers which have a world-wide petrol station network.

The contracted fuel selling company provides the GPS coordinates of the available petrol stations and the actual fuel price at

the individual stations. Accordingly the GPS coordinates of the petrol stations can always be defined. Digital maps convolute the road network to short, straight sections (Gubán and Gubán, 2001). This allows the determination of the length of the shortest way between any two chosen points (between dispatch/discharge station and petrol station, between the dispatch and the discharge station, etc.). This can be achieved by the use of an A\* algorithm (Russel, 2010; Bazaraa, 2007; Zhang and Yap, 2001), or by the application of roadmap sheets (data sheets) (Gubán and Gubán, 2001). Furthermore topographical information is also provided, which is also necessary since fuel consumption is different on flat roads than uphill or downhill conditions.

In the following sections the possible petrol stations will be chosen from every possible petrol stations, which can contribute to solution of the task. This significantly decreases the number of variables in the final model.

During the calculations it is advantageous to divide the transport trips into sections, which means the road between a dispatch station and a discharge station. This is necessary because the value of each cost category varies due to the different carried loads, road conditions, etc.

The total cost of a transport trip is the sum of costs of sections:

$$C_\alpha = \sum_{\beta} s_{\alpha\beta} \cdot c_{\alpha\beta} \quad [\text{euro}], \quad (1)$$

where:  $s_{\alpha\beta}$  - the distance of  $\beta$ -th section of  $\alpha$ -th transport trip [km];  $c_{\alpha\beta}$  - specific cost of the  $\beta$  -th section of  $\alpha$ -th transport trip [euro/km];  $\beta$  - section identifier,  $\alpha$  - transport trip identifier.

The fuel consumption of a vehicle depends on the consumption of vehicle without useful freight load (general characteristics of the engine), the weight of the transported freight load and the topography. These above mentioned influencing factors can be calculated or estimated, and as a consequence they can be taken into consideration during the elaboration of our model. There are further factors, which also have influence on the fuel consumption, e.g. the traffic situation on the road, the weather conditions, the road conditions, etc. These factors are absolutely stochastic, therefore it is not easy taking into consideration. These stochastic effects are handled in our model by the application of an emergency amount of fuel ( $Q_B$ ), which covers the extra consumption resulting from these non-predictable conditions (Fig. 2). Different alternatives for different refilling petrol stations – because of the high density of petrol station network – can not result in high differences in transport distances and transport times especially in case of long international transport trips. We assume that the selection of optimal petrol station does not have any influence on the time constraint of the transportation, so the allowed time window of the transport tasks can be satisfied.

The specific cost can be calculated by Eq. (2).

$$c_{\alpha\beta} = p_{\alpha\beta} \left( f_f + f_f \cdot \varepsilon_{\alpha\beta}^D + \varepsilon^T \cdot q_{\alpha\beta} \right) \left[ \frac{\text{euro}}{\text{km}} \right] \quad (2)$$

where:

$p_{\alpha\beta}$  - price of fuel  $\left[ \frac{\text{euro}}{\text{liter}} \right]$ ;

$f_f$  - specific fuel consumption in case of empty vehicle  $\left[ \frac{\text{liter}}{\text{km}} \right]$ ;

$\varepsilon_{\alpha\beta}^D$  - correction factor for fuel consumption depending on topography, varies between 0 (flatland), 0.3 (downy), 0.6 (mountain);

$\varepsilon^T$  - correction factor for different loading conditions (every additional tons of useful load results 0.5 liter extra fuel consumption)  $\left[ \frac{\text{liter}}{\text{ton} \cdot \text{km}} \right]$ ;

$q_{\alpha\beta}$  - transported useful load [ton].

The determination of correction factors is based on a previous research of authors of these paper (Kovács and Cselényi, 2006, Kovács et al., 2007).

The fuel level of the vehicle is known at starting point ( $Q_{fs}$ ), the fuel consumption can be calculated continuously between dispatch- and discharge stations (Fig. 2).

There is a constraint relating to the emergency amount of fuel capacity ( $Q_B$ ) which has to be available.  $Q_B$  emergency amount covers the extra consumption resulting from traffic jam, diversion, missed way, etc. between points  $L_i$  and  $t_i$ .  $Q_B$  emergency amount at the end of the section also covers the before mentioned stochastic effects and the fuel consumption of the vehicle to find the next petrol station after the discharge station  $L_{i+1}$ . This amount of fuel ensures that the vehicle can refuel on the end point or the nearest ideal petrol station to the end point.

The maximal distance ( $s_{1max}$ ) which can be completed by a vehicle is calculated as follows (Fig. 2):

$$s_{1max}^{\alpha\beta} = \frac{Q_{fs} - Q_B}{f_f + f_f \cdot \varepsilon_{\alpha\beta}^D + \varepsilon^T \cdot q_{\alpha\beta}} \quad (3)$$

This radius arch ( $s_{1max} - \Delta s_1$ ) defines the coordinates of the  $t_i$  possible petrol stations, from which the ideal one should be selected. The value of  $\Delta s_1$  can be freely defined, and it refers to the distance from which a suitable petrol station has to be selected before running out of fuel.

#### 4 Determination of the optimal petrol station and the amount of refilled fuel

The optimal petrol stations will be selected from the ideal petrol stations. Although the model does not require previous selection, this selection has some practical values, since it reduces the model size.

The previously defined variables will be specified for the model.

A specific road plan includes some stations that must be crossed to dispatch and/or discharge the load. Their order is predetermined and used to identify each dispatch and discharge station.  $N$  notes the number of dispatch and discharge stations

including the departure and final destination stations as well (which can be the same in case of transport trips, but in this case they are noted with different indexes). Each dispatch and discharge station is noted by a natural number between 1 and  $N$ . From this point on the dispatch and discharge stations will not be differentiated (both of them are called interchanges). Dispatch and discharge stations will differ by weight of the load during the road-section departing from the given interchange (0 in case of discharge, and higher than 0 in case of dispatch).

#### 4.1 Definitions of used terms

*Road-section*: the set of roads between the given dispatch or discharge station and the next interchange according to the planned transport route is called a road-section. Usually there are more alternative directions to the next interchange, which will be defined at the petrol stations in this model. Road-sections will be noted by  $i$  indexes. Therefore the road-section  $\beta$  of trip  $\alpha$  can be easily identified by the interchanges of the trip and the identities of the assigned road-sections (road-sections will be noted by  $k$  index). This identification is more appropriate in case of computerized solutions.

*Set of fuel stations, probable road-section*:  $T_i$  stands for the set of fuel stations between departure station  $i$  and destination station  $i+1$ . In this case  $\tau_i = |T_i|$  notes the number of petrol stations on road-section  $i$ . A specific road of the road-section belonging to the fuel stations is called a probable road-section. Probable road-sections are noted by  $i, k$  indexes. Probable road-sections must include only one petrol station. If there are more than one, the model can be changed to a new model in which there is only one petrol station on each road-section (see also Remark 4).

*Notation of petrol stations*:  $t_{ik} \in T_i$  notes petrol station  $k$  (petrol station that belongs to probable road-section  $k$ ) on road-section  $i$ .

*Maximum number of petrol stations on a given road-section*:  $H$  ( $H = \max_i \tau_i$ ) notes the maximum number of the petrol stations on a road-section. If road-section  $i$  includes fewer than  $M$  petrol stations, then fictive probable road-sections are to be defined between  $\tau_i + 1$  and  $H$ . The length of the probable road-section should be high enough to ensure that it will not be chosen during the optimization process. Road 0 (with 0 index) is a road-section without petrol refill, which means the vehicle does not stop on this probable road-section.

*Topography*: There will be two topography-related variables assigned to the probable road-sections in this model: the first one is the topography factor assigned to the section from the departure station to the petrol station; the second one is assigned to the section from the petrol station to the destination.

- $\varepsilon_{ik}^{DL}$  notes the topography factor in relation  $i, i+1$  of the section to the petrol station  $k$ .
- $\varepsilon_{ik}^{DM}$  notes the topography factor in relation  $i, i+1$  of the section from petrol station  $k$  to the destination.

*Correction factor for different loading conditions:*  $\varepsilon^T$  notes the correction factor for different loading conditions (See also (2)).

*Unit price matrix:*  $p_{ik}$  notes the unit price of the fuel at the petrol station  $t_{ik}$  ( $P = [p_{ik}]_{N \times H+1}$ ).  $p_{i0} = 0$  ( $i = 1; \dots; N$ ).

*Distance matrices:* Distance matrices – similarly to topography factors – are divided into two parts: the first one is assigned to the way of the section from the departure station to the petrol station, and the second one is assigned to the way of the section from the petrol station to the destination.

- $l_{ik}$  notes the length of the road-section from the departure station  $i$  (with destination station  $i + 1$ ) to the petrol station  $k$  ( $L = [l_{ik}]_{N \times H+1}$ ).
- $m_{ik}$  notes the length of the road-section between departure station  $i$  to destination station  $i + 1$  from the petrol station  $k$  to the destination station ( $M = [m_{ik}]_{N \times H+1}$ ).
- The definition of the total length of road-section  $i$  is also necessary:  $s_{ik} = l_{ik} + m_{ik}$  ( $S = [s_{ik}]_{N \times H+1}$ ).

Vector of the amount of load  $q_i$  notes the amount of load in the relation  $i, i + 1$ . It is 0 in case of empty vehicle. This way the loaded and idle transfer trips can be handled uniformly ( $q = [q_i]_{N-1}$ ).

*Specific fuel consumption:*  $f_j$  notes specific fuel consumption (liter/km) see also at (2).

*Capacity of fuel tank:*  $Q_{max}$  notes the maximum capacity of the fuel tank.

### Remarks

1. In case of **L**, **M**, **S**, **P** matrices – due to computational reasons – indexes starts from 0 and ends at  $H$ .
2. Because of practical reasons in case of road 0 the total length assigned to  $l$  is  $m = 0$  ( $l_{i0} = s_{i0}, m_{i0} = 0$ ).
3. When it is possible to refill fuel on the highway as well, which is road 0, then this road must be defined again, but in this case with  $l$  and  $m$  indexes assigned to the petrol station. This probable road-section appears twice, but the section with index different than 0 is a probable road-section where fuel is refilled. In case of there are more than one petrol stations on the highway, the procedure described by Statement should be executed.
4. There are two possible ways to proceed from one intersection to the next one:
  - a) With free choice, that means it is allowed to choose freely from the probable road-sections.
  - b) Mandatory proceeding, which means that if we arrive from probable road-section  $k$  then the next intersection also must be approached on road-section  $k$ . The significance of it lies in the following Statement.

In the following the given mathematical model, for the sake of simplicity, can take into account only one petrol station per road section. Although, practically it is possible to have more petrol station within one section. Therefore a statement is given which prove that any given practical task (which contains more than one petrol stations on a probable road section) can be modelled in a way that allows only one petrol station on every probable road sections.

**Statement.** The original problem can always be converted into a task, in which there is only one petrol station between two intersections.

The convolution of a probable road-section is necessary when there are more petrol stations on the same probable road-section. In the original problem each intersection allows free choice (Fig. 3).

**Case 1.** Let us convolute a probable road-section into as many probable road-sections, as the number of the petrol stations. This convolution is complete when all road-section fulfil the requirement that the distance between the previous probable petrol station (or the departure station in case of the first road-section) and the petrol station of the particular road-section can be reached with one full tank without intermittent refill, furthermore the petrol station of the next road section (or the destination station in case of the last road-section) can also be reached with the same conditions (Fig. 4).

Two petrol stations are on the same possible road section according to conditions of Case 1 (Fig. 3)

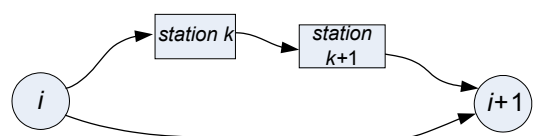


Fig. 3 Case 1. Two stations are in a probable road-section between nodes  $i, i + 1$ . Base case.

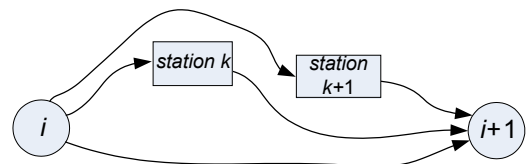


Fig. 4 The probable road section is substituted two probably road sessions as described in Case 1.

**Case 2.** When Case 1. cannot be fulfilled, then a fictive intersection that only exists in the model has to be inserted into road-section  $i$  at the initial phase, so that it stands between the two petrol stations of the probable road-section and it ensures the fulfilment of the criterion for the probable road-sections (Fig. 5). Then in the same manner fictive intersections must be defined

in case of each road-sections of the  $i, i + 1$  relation. This fictive station could be any point of the relation except – due to computational reasons – the point of the actual petrol station of the probable road-section. The convoluted road-sections must be indexed so that the  $k$  index of the road section that arrives to the virtual intersection is the same as of the outgoing index of the convoluted road-section. Then the inserted intersection must be defined as an intersection with mandatory proceeding (Fig. 6). If the criterion still cannot be fulfilled, then Case 2. must be repeated until it is fulfilled, or a new road-section is created on which there are not any petrol stations and the road-section cannot be solved (but at the same time the task can be completed).

Two petrol stations are on the same possible road section according to conditions of Case 2. (Fig. 5).

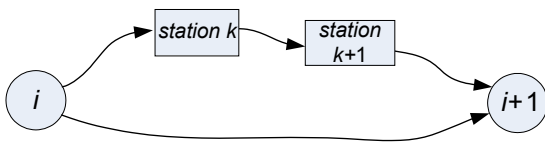


Fig. 5 Two stations are in a probable road-section between nodes  $i, i + 1$  according to Case 2. Base case.

**Remark.** Fig. 3 and 5 are same forms, but the distance of station  $k$  and  $k + 1$  in the Fig. 3 is less or equal than the distance of a full tank can be done, as in the Fig. 5 is greater.

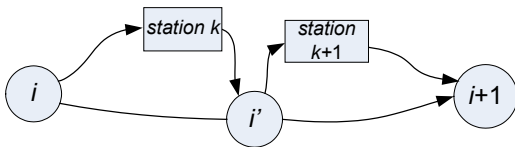


Fig. 6 Node  $i'$  is inserted between nodes  $i, i + 1$ . Node  $i'$  becomes an intersection with mandatory proceeding

## 5 Constraints and objective function of the optimization process for the determination of the ideal refueling points and the amount of fuel to be refilled

The following matrix contains the first group of the variables. The elements of it define the amount of refilled fuel at each road-section. It is noted by:

$$\mathbf{X} = \begin{bmatrix} x_{ik} \end{bmatrix}_{N \times H} \geq 0, \quad (4)$$

the amount of refilled fuel at petrol station  $k$  that is between departure station  $i$  and destination station  $i + 1$ . In case of  $k = 0$  no fuel loading takes place on the given road-section, so that the amount is a fictive quantity  $x_{i0} = 1$ . This does not cause any problems, since this 0 variable will not appear in the cost of the fuel consumption.

## 5.1 Constraints

### 5.1.1 Constraint for the determination of the probable route

Note 1 if the probable road-section  $ik$  is chosen, 0 otherwise. If the driver refuels in probable road-section  $ik$  then  $x_{ik} > 0$ . Then  $\text{sgn}(x_{ik}) = 1$  so the probable road-section  $ik$  is chosen. One probable road-section has to be chosen. This leads to the following:

$$\sum_{k=0}^{\tau_i} \text{sgn}(x_{ik}) = 1, \quad (5)$$

as a consequence one of the road-sections has to be chosen.

### 5.1.2 Constraint for the total length of the transport way

Based on formulas (5), the total length of the road can be determined. The model does not contain the total length of road, but it is very important other calculations (for example rest period, calculating of required number of drivers, etc.).

$$\sum_{k=1}^H \text{sgn}(x_{ik}) \cdot s_{ik} = \sum_{k=1}^H \text{sgn}(x_{ik}) \cdot (l_{ik} + m_{ik}) \quad (6)$$

means the length of the road-section between  $i$  and  $i+1$ , if we refill at petrol station  $k$ . Based on these and taking into account the bypasses necessary for refueling, the total length of the drive is:

$$s = \sum_{i=1}^n \sum_{k=1}^H \text{sgn}(x_{ik}) \cdot s_{ik} \quad (7)$$

### 5.1.3 Constraint for the minimal amount of fuel for the proceeding of the vehicle from a dispatch or discharge station

$Q_i$  denotes the amount of fuel in the vehicle at intersection  $i$ .

$$\mathbf{Q} = [Q_i]_N \quad (8)$$

where  $Q_i$  notes the amount of fuel in the tank of the vehicle at intersection  $i$ .

Then the amount of fuel after the refueling intersection can be defined as:

$$Q_{i+1} = Q_i + \sum_{k=0}^H \text{sgn}(x_{ik}) \cdot \left[ -(l_{ik} + m_{ik}) \cdot x_{ik}, \right. \\ \left. (f_f + f_f \cdot \varepsilon_{ik}^{DM} + \varepsilon^T \cdot q_i) \right] \quad (9)$$

which means to subtract the consumed amount of fuel (depending on the load) from the available amount of fuel at the previous intersection and adding the amount of refilled fuel.

#### 5.1.4 Constraint for the amount of remaining fuel upon arrival to the last discharge station or depot after the fulfilment of the transport task

An important criterion is that the amount of fuel cannot drop under a minimal limiting value. The tank of the vehicle must be filled with this minimal amount of fuel when arriving to the petrol station:

$$Q_i - \sum_{k=0}^H \text{sgn}(x_{ik}) \cdot l_{ik} \cdot (f_f + f_f \cdot \varepsilon_{ik}^{DL} + \varepsilon^T \cdot q_i) \geq Q_B \quad (10)$$

#### 5.1.5 Constraint for the maximum amount of loaded fuel

The amount of refilled fuel ( $x_{ik}$ ) cannot exceed the capacity of the tank:

$$x_{ik} \leq Q_{max} - Q_i + l_{ik} \cdot (f_f + f_f \cdot \varepsilon_{ik}^{DL} + \varepsilon^T \cdot q_i). \quad (11)$$

#### 5.1.6 Constraint for the obligatory proceeding direction in case of a round trip

$O$  notes the set of the indexes of mandatory intersections. Then the following criterion belongs to the mandatory intersections:

$$\text{sgn}(x_{ik}) = \text{sgn}(x_{i+1,k}), \quad i \in O, \quad k = 0, \dots, H. \quad (12)$$

#### 5.1.7 Constraint for the amount of remaining fuel upon arrival to the depot after the fulfilment of the transport task

Upon arrival to the destination ( $i = N$ ) a certain amount of fuel ( $Q_Z$ ) has to remain in the vehicle in order to prepare for the departure of the next route:

$$Q_N \geq Q_Z. \quad (13)$$

$C_t$  notes the cost of loaded fuel at the relation (0 in case there were no refueling), furthermore

$$C_T = \sum_{i=1}^{N-1} C_{t_i}. \quad (14)$$

The cost of refueling at the given fuel station is:

$$C_{t_i} = \sum_{k=0}^H p_{ik} \cdot x_{ik}. \quad (15)$$

## 5.2 Cost objective function

Objective function means the cost of the initial loading of the fuel tank (fix cost), and the cost of the refueling during the drive decreased by the cost of the remaining fuel (variable cost):

$$C = C(\mathbf{X}, \mathbf{Q}) = C_{fs} + C_T - C_r = C_{fs} + \sum_{i=1}^{N-1} \sum_{k=1}^H p_{ik} \cdot x_{ik} - \sum_{k=1}^H \text{sgn}(x_{N-1,k}) \cdot p_{N-1,k} \cdot Q_N \quad (16)$$

The previous criterion means that the cost of the remaining fuel has to be calculated with the unit price of the last fuel station.

The goal:

$$C = C(\mathbf{X}, \mathbf{Q}) \rightarrow \min. \quad (17)$$

The task is a mathematical programming task.

*Note:*  $C_{fs}$  can be neglected during the optimization, since it is constant so it does not affect the optimum. Then the following equation can be used:

$$C' = C'(\mathbf{X}, \mathbf{Q}) = \sum_{i=1}^{N-1} C_{T_i} - C_r. \quad (18)$$

## 6 Summary of constraints and objective function

$C_{fs}$  can be neglected during the optimization, since it is constant so it does not affect the optimum.

The model can be simplified according to (Gubán and Kovács, 2016). The final model is the following:

$$x_{ik} \geq 0, \quad i = 1, \dots, N-1; k = 0, \dots, H \quad (19)$$

$$y_{ik} \in \{0; 1\}, \quad i = 1, \dots, N-1; k = 0, \dots, H \quad (20)$$

$$\sum_{k=0}^H y_{ik} = 1, \quad i = 1, \dots, N-1 \quad (21)$$

$$Q_1 = Q_{fs} \quad (22)$$

$$Q_i - \sum_{k=0}^H y_{ik} \cdot l_{ik} \cdot (f_{\bar{u}} + f_{\bar{u}} \cdot \varepsilon_{ik}^{DL} + \varepsilon^T \cdot q_i) \geq Q_B, \quad i = 1, \dots, N-1 \quad (23)$$

$$x_{ik} - y_{ik} \geq 0, \quad i = 1, \dots, N-1; k = 0, \dots, \tau_i \quad (24)$$

$$x_{ik} - \left[ Q_{max} - Q_i + l_{ik} \cdot (f_f + f_f \cdot \varepsilon_{ik}^{DL} + \varepsilon^T \cdot q_i) \right] \cdot y_{ik} \leq 0, \quad i = 1, \dots, N-1; k = 0, \dots, \tau_i \quad (25)$$

$$\sum_{k=0}^H y_{ik} \cdot \left[ -(l_{ik} + m_{ik}) \cdot (f_f + f_f \cdot \varepsilon_{ik}^{DM} + \varepsilon^T \cdot q_i) \right] + \sum_{k=1}^H x_{ik} + Q_i - Q_{i+1} = 0, \quad i = 1, \dots, N-1 \quad (26)$$

$$y_{ik} - y_{i+1,k} = 0, \quad i \in O, \quad k = 0, \dots, H \quad (27)$$

$$Q_N \geq Q_Z \quad (28)$$

$$C' = C'(\mathbf{X}, \mathbf{Y}, \mathbf{Q}) = C_T - C_r = \sum_{i=1}^{N-1} \sum_{k=1}^H p_{ik} \cdot x_{ik} - \sum_{k=1}^H y_{N-1,k} \cdot p_{N-1,k} \cdot Q_N \rightarrow \min. \quad (29)$$

## 7 Software application for determination of the optimal petrol station and the amount of fuel to be refilled

Based on the elaborated theoretical concept, a software was developed for optimization of refueling process by the contribution of Norbert Cziczter, an engineering student.

The software was written in C# programming language and the Microsoft Visual Studio was used to develop the software (Cziczter, 2016).

The developed software has two menu points which are the “Definition of new data” and “Optimization”.

### 7.1 Menu “Definition of new data”

- In the menu “Definition of new data” we can define:
- new petrol stations (Fig. 7),
- new dispatch stations and discharge stations (Fig. 8), and
- new transport vehicles (Fig. 9).

Fig. 7 Definition of new petrol stations

Fig. 8 Definition of new dispatch stations and discharge stations

Fig. 9 Definition of new transport vehicles

### 7.2 Menu “Optimization”

Part 10.a of Fig. 10 shows the possibility of selection of a transport vehicle for a given transport task. In this menu the actual fuel level of the vehicle [liter] at the beginning of the transport task and the fuel price [HUF/liter] of this existing fuel also can be defined.

Part 10.b of Fig. 10 provides the possibility of determination of the transport trip, the dispatch stations and discharge stations and the loading conditions [ton] (transported weight and the changing of loading conditions at the dispatch- or discharge stations).

Part 10.c of Fig. 10 shows the results of the optimization. The total cost of the road transport trip [HUF], the total fuel consumption of the transport way [liter] and the remaining fuel volume [liter] at the end of the way are listed.

Part 10.d of Fig. 10 describes the graphical map of the transport way and the location of the optimal petrol station where the driver has to refill.

Part 10.e of Fig. 10 shows the name and location of the optimal petrol station and the volume of fuel [liter] to be refilled. The actual fuel price [HUF/liter] of the ideal station and the total cost [HUF] of the fuel to be refilled also listed.

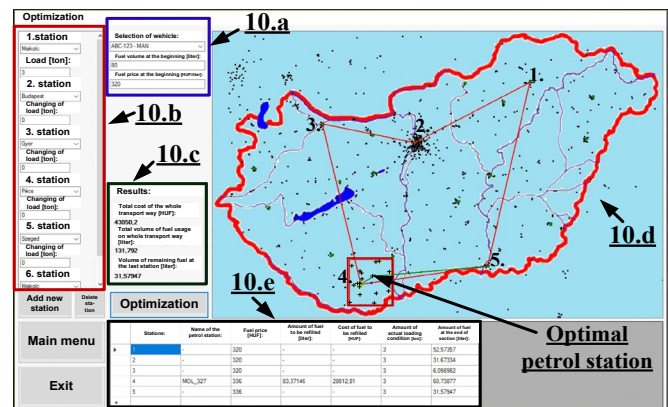


Fig. 10 Screen of “Optimization” menu, result of the case study

### 7.3 Case study – application of the software

In our case study the transport trip is the following (see Fig. 11):

Station 1.: Miskolc, +3 ton;

Station 2.: Budapest, -3 ton +3 ton;

Station 3.: Győr, -3 ton +3 ton;

Station 4.: Pécs, -3 ton +3 ton;

Station 5.: Szeged, -3 ton +3 ton;

Station 6.: Miskolc, -3 ton.

Distances between the dispatch stations and discharge stations can be seen in Fig. 11.

The selected vehicle is a light truck, fuel consumption is 14 liter/100km, the maximal loading capacity is 3,5 ton, the maximal fuel tank capacity is 150 liter.

The correction factor for different loading conditions is 0.3 (every additional tons of additional freight load results 0.3 liter extra fuel consumption).



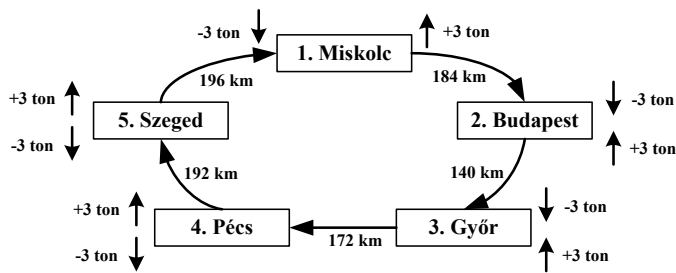


Fig. 11 Case study

The fuel volume at the beginning of the transport trip is 80 liter (see part 10.a of Fig. 10), the stations of the transport way and the loading conditions can be defined in part 10.b of Fig. 10.

As it can be seen on part 10.d of Fig. 10, the volume of the fuel will be under the given limited value before the station 5, so the driver has to refuel the vehicle between station 4. and station 5. The software defines the optimal fuel station in the searching area (red rectangle in part 10.d of Fig. 10) after the calculation. The identifier of the optimal refueling station is MOL\_327 (where the fuel price is minimal) and the amount of fuel (83.4 liter) to be refilled are listed in the part 10.e.

The final result of the fueling optimization can be seen in part 10.c of Fig. 10. The total cost of the whole transport way of the case study is 43050 HUF, the total volume of fuel usage on the whole transport way is 131.8 liter, the volume of remaining fuel at the last station is only 31.6 liter.

The developed software is capable for selection of the optimal petrol station and the determination of the optimal amount of refilled fuel during a long transport trip. Based on this information the camion drivers can be supported by the required refueling information and the total cost of the transport trip can be minimized.

There are lot of possible petrol stations during the transport way and in the practice the selection of the petrol station where the driver refill the vehicle is absolutely depending on the individual decision of the driver. Therefore the total cost of the burned fuel is not optimal, because the fuel prices are different at different petrol stations. In Hungary for example the difference of fuel prices at the cheapest and at the most expensive petrol stations can be 50-70 HUF/liter. It can be concluded that if the decision making of the driver could be supported by our software the total cost of the transport way can be reduced significantly. This reduction is 13.5-19% in case of one liter of fuel. It can be calculated that in case of a long transport way the cost reduction can be significant.

Recently the software is under testing, but we hope that in the near future more and more companies will apply it and the transport cost can be reduced in this way.

## 8 Conclusion

The main goal of the study was to elaborate a precise and reliable mathematical model and method for the determination of optimal fuel refill locations and the amount of loaded fuel for the execution of specific transportation tasks. There have not been any available solutions in the literature for the complex handling of our refueling problem. This motivated the authors to create a model and method that is able to generally handle the refueling problem and to also give a general solution for it.

In the first step the concepts were clarified and disambiguated. This created a dictionary that made the creation of model possible. In the following step the probable (ideal) petrol stations were chosen, where it is worth refueling. It is necessary, since countless petrol stations are available during a longer (international) transport trip. A method was also given which determines the length of the way between each intersections or petrol stations based on their GPS coordinates and digital map parameters. After choosing the probable refuel points the mathematical model of the problem was created taking into account the conditions and constraints arising from practical tasks. A software was also developed for the optimization of refueling problem, which helped to demonstrate the applicability of the model on a case study.

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