Metro Passenger Flow Forecast with a Novel Markov-Grey Model

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Abstract

Accurate forecasts of passenger flow entering and leaving metro stations is an important work for Metro operation management, such as for the automatic adjustment of train operation diagrams or station passenger crowd regulation planning measures. In this study, Grey theory is introduced to develop a time series GM (1, 1) model for total passenger forecasting. Two modification factors determined by two minimum mean square error principles are proposed to decrease the discreteness of input data and thus improve the forecast accuracy. Moreover, the Markov chain approach is further used to optimize the residual error series. Passenger flow data entering and leaving the Xiaozhai station of Xi'an Metro Line 2 from September 1-30, 2015, were utilized to verify the effectiveness of the proposed method; the forecast results show that this novel Markov-Grey model performs well in terms of forecast accuracy with smaller SMSE and MAPE values. To this effect, the proposed method is especially well-suited to smooth passenger flow forecasting compared to other forecast techniques.

Keywords

passenger flow forecast, metro station, GM (1, 1) model, modification factor, Markov chain

1 Introduction

Passenger flow forecasts of mass rail transit (MRT) serve to evaluate and determine the volume of passenger flow at peak hours or within a total day up to a year in the future. The information returned by these forecasts can be utilized for a wide array of applications related to public transit (Lalinská et al., 2017; Zhang et al., 2017). At the planning stage, more importantly, the choice of subway, light rail transit, or streetcar mainly depends on the volume of passenger flow; this affects the preliminary decisions on planning networks as well as the location and design of individual stations and the process of constructing them, including land allocation and supply investment. During the operation period, the passenger's choice of MRT, bus, or other modes of traffic is affected by a variety of uncertain factors including land use type, fees, and convenience of arrival / departure. All of these factors can vary over space and time (Ding et al., 2018; Ni et al., 2017). In practice, then, it is impossible – or at least very difficult – for forecasters and planners to accurately and comprehensively ascertain the future passenger demands of public transit.

The passenger flow volume of Guangzhou Metro Line 1 was predicted to reach 290,000 in 1998 according to an official assessment, but from its opening day on June 28, 1997, through the end of 1998, the subway carried 175,000 ~ 180,000 individuals daily – only about 60 % of the expected value. Its total amount of passengers then reached 66,291,300 in 2002, representing annual growth of 0.45 %. When the Metro Line 2 opened on June 28, 2003, however, the volume of passenger flow exceeded 1 million, marking an increase of 20 % over the previous year. This figure reached 164 million in 2004. In Xi'an City, the 20.5 km first section of Metro Line 2 from the North Railway station to the Qujiang International Conference and Exhibition Center was officially opened at midday on September 16, 2011, with services operating at 9.5 min headways from 6:30 to 21:30; the route's average daily passenger flow volume grew from 151,000 in 2011 to 196,300 before the opening of Metro Line 1 on September 15, 2013. The line now accommodates a passenger volume of approximately 600,000 daily, with a peak of 742,900 passengers on October 1, 2015. Such differences between the

forecasted and actual values of passenger flow represent serious challenges (e.g., spacious or crowded platforms and carriages) to the operating management, thus necessitating more reliable and more accurate approaches to passenger flow forecasting (Li, X. et al., 2018; Zhang et al., 2017).

Previous researchers have established several approaches to passenger flow forecasting, but from a more general perspective (Zhao et al., 2011). For example, data sequences are generally used and mathematical modeling is generally the focus in developing these techniques (Leng et al., 2013; Li, L. et al., 2018), but they are lacking in accuracy overall. Grey system theory is a multidisciplinary theory that was firstly proposed in early 1980s by Deng to describe systems such as passenger flow (Deng, 1982); the theory has indeed proven superior to conventional statistical models in dealing with systems that have both partially known and partially unknown parameters (Liu et al., 2011). Over recent decades, Grey system theory has garnered research attention across a variety of fields and has been successfully applied to systems such as economy, finance, transportation, mechanics, geology, military, and medical care (Carmona Benítez et al., 2013; Yin et al., 2017; Zhou and He, 2013). The GM (1, 1) model is the main forecasting model applicable to Grey theory, and has achieved satisfactory results in field use though its forecasting precision and applicability merit further improvement.

Two of the deficiencies in the methods described above constituted the primary motivation of this paper. First, explored an approach to improving the accuracy of the general GM (1, 1) model in forecasting the passenger flow of mass rail transit; and second, we attempted to combine the Markov process with the proposed Grey model to enhance forecasting efficiency. The remainder of this paper is structured as follows: Section 2 describes our process in designing the proposed Markov-Grey method, and Section 3 presents its application in forecasting the passenger flow of Xiaozhai station of Xi'an Metro Line 2. Concluding remarks are provided in Section 4.

2 Methodology
2.1 Grey forecast model
Let an original non-negative time series with n records be $x^{(0)} = \left[x^{(0)}(1), \ x^{(0)}(2), \ \ldots, \ x^{(0)}(n) \right]$, then the monotonically increasing sequence $x^{(1)}$ can be cumulatively generated as $x^{(1)} = \left[x^{(1)}(1), \ x^{(1)}(2), \ \ldots, \ x^{(1)}(n) \right]$, where $x^{(1)}(1) = x^{(0)}(1)$ and $x^{(1)}(k) = \sum x^{(0)}(i)$, $i = 1, 2, \ldots, k$ ($k = 2, \ldots, n$). The raw data $x^{(0)}$ are then ranked as $\sigma = \left[\sigma(2), \ \sigma(3), \ \ldots, \ \sigma(n) \right]$, where

$\sigma(k) = x^{(0)}(k-1)\big/ x^{(0)}(k)$, $k = 2, 3, \ldots, n$, and satisfy the Grey model if the sequence falls within the range 0.1345-7.389 (Deng, 1982).

According to Grey theory, the original Grey model can be expressed as $x^{(0)} = -az^{(1)}(k) + b$, $k = 2, 3, \ldots, n$, where $z^{(1)}(k) = \left[x^{(1)}(k) + x^{(1)}(k-1) \right]\big/ 2$. This differential equation is:

$$\frac{dx^{(1)}(k)}{dt} + ax^{(1)}(k) = b \tag{1}$$

with initial condition $x^{(1)}(1) = x^{(0)}(1)$; such replacement is defined as "whiteness processing" (Deng, 1982). Optimal coefficients a and b can both be estimated by the minimum least square method as $[a, b]^{\mathrm{T}} = (B^{\mathrm{T}}B)^{-1}B^{\mathrm{T}}Y$, where B is the accumulated matrix and Y is the constant term vector (Deng, 1982), and then substituted into the differential equation to obtain the approximate relation for $k = 2, 3, \ldots, n$:

$$\hat{x}^{(1)}(k) = \left[x^{(0)}(1) - \frac{b}{a} \right] e^{-a(k-1)} + \frac{b}{a} \tag{2}$$

where $\hat{x}^{(1)}(1) = x^{(0)}(1)$, for $k = 2, 3, \ldots, n$. The forecast value of the series can be restored via the following inverse accumulated generating operation:

$$\hat{x}^{(0)}(k) = \left(1 - e^a\right)\left[x^{(0)}(1) - \frac{b}{a} \right] e^{-a(k-1)} . \tag{3}$$

Precision checking was conducted to determine the residual errors $e(k)$ between actual $x^{(0)}(k)$ and predicted $\hat{x}^{(0)}(k)$, respectively, as $\left[x^{(0)}(k) - \hat{x}^{(0)}(k) \right] \times 100\ \%\big/ x^{(0)}(k)$; the precision index was calculated to be $1 - e(k)$. We utilized the Standard Mean Square Error ($SMSE$) and the Mean Absolute Percentage Error ($MAPE$) to verify the performance of residual error $e(k)$ as given by Eqs. (4), (5). Here, the acceptable $MAPE$ was set to be 0.2 for daily passenger flow forecasting and an average precision of 1-$MAPE$ exceeding 80 % was considered to indicate excellent model forecast efficiency.

$$SMSE = \sqrt{\frac{1}{n-1}\sum_{k=2}^{n}\left[x^{(0)}(k) - \hat{x}^{(0)}(k) \right]^2} \tag{4}$$

$$MAPE = \frac{1}{n-1}\sum_{k=2}^{n}\left| \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \right| \times 100\ \% . \tag{5}$$

2.2 Precision modification
According to Grey theory, forecast accuracy is affected significantly by two parameters: a and b. Their solution depends on the background value. In establishing the GM (1, 1) model, $z^{(1)}(k).z^{(1)}(k) = \alpha x^{(1)}(k) + (1-\alpha)x^{(1)}(k-1)$ was

considered as the smoothing value, where α is the weighting factor, and $\alpha = 0.5$ is chosen as a special case but may not be the optimal result (Hsu and Chen, 2003). When the original data sequence $x^{(0)}(k)$ changes smoothly, the proposed GM (1, 1) model yields more accurate forecast results (Hui et al., 2013). Therefore, the forecast data $\hat{x}^{(0)}(k)$ requires further modification. Suppose the approximate relation of $\hat{x}^{(0)}(k)$ is modified as follows:

$$\tilde{x}^{(1)}(k) = \left[x^{(0)}(1) + \delta - \frac{b}{a} \right] e^{-a(k-1)} + \frac{b}{a} \tag{6}$$

and thus reaches $\tilde{x}^{(0)}(k) = \hat{x}^{(0)}(k) + \delta\left(1 - e^a\right)e^{-a(k-1)}$, $k = 2, 3, \ldots, n$, where δ is the modification factor. Two criteria are proposed here to determine the value of δ.

CRITERIA 1: Minimum mean square error forecast of cumulative data series

$$\min_{\delta_1} \left[\tilde{x}^{(1)}(k) - x^{(1)}(k) \right]^2 . \tag{7}$$

Since $\tilde{x}^{(1)}(k) = \left[x^{(1)}(1) + \delta_1 - \frac{b}{a} \right] e^{-a(k-1)} + \frac{b}{a}$, then

$$\varphi_1 = \sum_{k=2}^{n} \left\{ \left[x^{(k)}(1) + \delta_1 - \frac{b}{a} \right] e^{-a(k-1)} + \frac{b}{a} - x^{(1)}(k) \right\}^2 . \tag{8}$$

If $\dfrac{\partial \varphi_1}{\partial \delta_1} = 0$, then

$$\left[x^{(0)}(1) - \frac{b}{a} \right] \sum_{k-2}^{n} e^{-2a(k-1)} + \delta_1 \sum_{k=2}^{n} e^{-2a(k-1)} - \sum_{k=2}^{n} x^{(1)}(k) e^{-a(k-1)}$$
$$= 0 . \tag{9}$$

It is clear that $\displaystyle\sum_{k=2}^{n} e^{-2a(k-1)} = \frac{1 - e^{-2na}}{1 - e^{-2a}}$, hence, by Eq. (9):

$$\delta_1 = \frac{\displaystyle\sum_{k=2}^{n} x^{(1)}(k) e^{-a(k-1)}}{\dfrac{1 - e^{-2na}}{1 - e^{-2a}}} - \frac{b}{a} \cdot \frac{\dfrac{1 - e^{-na}}{1 - e^{-a}}}{\dfrac{1 - e^{-2na}}{1 - e^{-2a}}} \left[x^{(0)}(1) - \frac{b}{a} \right] . \tag{10}$$

CRITERIA 2: Minimum mean square error forecast of original data series

$$\min_{\delta_2} \left[\tilde{x}^{(0)}(k) - x^{(0)}(k) \right]^2 . \tag{11}$$

Inserting $\tilde{x}^{(0)}(k) = \left(1 - e^a\right)\left[x^{(0)}(1) + \delta_2 - \frac{b}{a} \right] e^{-a(k-1)}$ into Eq. (11) yields:

$$\varphi_2 = \sum_{k=2}^{n} \left\{ \left(1 - e^a\right)\left[x^{(0)}(1) + \delta_2 - \frac{b}{a} \right] e^{-a(k-1)} - x^{(0)}(k) \right\}^2 . \tag{12}$$

Let $\dfrac{\partial \varphi_2}{\partial \delta_2} = 0$, then

$$\left(1 - e^a\right) \cdot \left[x^{(0)}(1) - \frac{b}{a} \right] \sum_{k=2}^{n} e^{-2a(k-1)} + \delta_2 \left(1 - e^a\right) \sum_{k=2}^{n} e^{-2a(k-1)}$$
$$- \sum_{k=2}^{n} x^{(0)}(k) e^{-a(k-1)} = 0 . \tag{13}$$

Because $\displaystyle\sum_{k=2}^{n} e^{-2a(k-1)} = \frac{1 - e^{-2na}}{1 - e^{-2a}}$, δ_2 can be rewritten as:

$$\delta_2 = \frac{\displaystyle\sum_{k=2}^{n} x^{(0)}(k) e^{-a(k-1)}}{\left(1 - e^a\right)\dfrac{1 - e^{-2na}}{1 - e^{-2a}}} - \left[x^{(0)}(1) - \frac{b}{a} \right] . \tag{14}$$

Using δ_1 and δ_2, therefore, we obtained two improved GM (1, 1) models by plus $\delta_i\left(1 - e^a\right)e^{-a(k-1)}$ ($k = 2, 3, \ldots, n$), respectively. The optimal model (with relatively fewer errors) was then used for the remainder of the forecasting process.

2.3 Markov process

To reduce forecast errors, Markov approach is integrated into the model described above to process the forecast error series $\{e(k), k = 2, 3, \ldots, n\}$. Let $e(k)$ be divided into q states $(R_1, R_2, \ldots, R_q)$ by appealing to the equiprobability principle, and E_{li} and E_{ui} represent the lower and upper bounds of each state, respectively. Let current state be R_i and another state be R_j, then the transition probability P_{ij} from state R_i to R_j after w steps is written as $P_{ij}^{(w)} = m_{ij}^{(w)} / M_i$, $i, j = 1, 2, \ldots, q$, where $m_{ij}^{(w)}$ is the frequency of transition from state R_i to R_j in w steps and M_i is the value of state R_i (Samet and Mojallal, 2014). Thus, the transition probability matrix can be expressed as:

$$P^{(w)} = \begin{bmatrix} p_{11}^{(w)} & p_{12}^{(w)} & \cdots & p_{1q}^{(w)} \\ p_{21}^{(w)} & p_{22}^{(w)} & \cdots & p_{2q}^{(w)} \\ \vdots & \vdots & \vdots & \vdots \\ p_{q1}^{(w)} & p_{q2}^{(w)} & \cdots & p_{qq}^{(w)} \end{bmatrix} . \tag{15}$$

Considering $w = 1$, if the residual errors $e(k)$ falls in the ith state, then check the ith row of transition probability matrix $P^{(1)}$. If $\text{Max}P_{ij}^{(1)} = P_{iL}$, then state L is likely to be the next transition of forecast state (Samet and Mojallal, 2014). The forecast residual error value $e(k)$ can be modified into $e(k) - (A_k + C_k)/2$ by the lower and upper bounds of A_L and C_L. Subsequently, the forecast value can be appropriately modified.

3 Illustrative Example and Analysis

The first and second phases of Xi'an Metro Line 2 were officially opened on September 16, 2011, and June 16, 2014, respectively. The line runs between the new North Railway station and Weiqunan along a straight north-south corridor passing directly through the city centre, covering the Administrative Centre, Bell Tower commercial area, Xiaozhai, Qujiang International Conference and Exhibition Center, and other important passenger distribution centres. It is currently 26.8 km long with 21 stations and is known to provide a safe, comfortable, convenient, and swift transportation service for about 600,000 passengers from 6:00 a.m. to 11:10 p.m. daily.

As discussed in the Introduction, passenger flow volume forecasting has long been regarded as a critical requirement for mass rail transit scheduling and operations management (Leng et al., 2013; Li, L. et al., 2018). We selected Xiaozhai station, a major transfer station of Metro Line 2, as a case study and gathered its daily passenger flow data for

September, 2015. Each record in the data set provided here refers to the number of passengers entering and leaving this station. The sample data 20150901-20150922 were used for model fitting, as shown in Table 1, and 20150923-20150930 were reserved for forecasting and accuracy testing.

The primary data sequence $x^{(0)}$ is given as [38745, 43633, …, 42124], and the one-order cumulative sequence is generated as $x^{(1)}$ = [38745, 82378, …, 1069172]. The σ series is then [0.8880, 0.9911, …, 1.4551], with values ranging between 0.1345 and 7.389; this indicates that the selected sample data are suitable for constructing the Grey forecast model for passenger flow in Xiaozhai metro station.

Two parameters, a and b, were estimated by minimum least square method as −0.0057 and 45957.7945, respectively, yielding an original GM (1, 1) model expressed as $\hat{x}^{(0)}(k) = 46047.2816\, e^{0.0057(k-1)}$, $k = 2, 3, …, n$ with the initial condition $\hat{x}^{(0)}(1) = x^{(0)}(1) = 38745$. After substituting $k = 2, 3, …, n$ into Eq. (1), the forecast data of passenger flows from 20150902 to 20150922 were obtained as well

Table 1 The observed and forecasted passenger flow using three GM(1,1) models

Data	Passengers	GM(1,1)		GM(1,1) with δ_1			GM(1,1) with δ_2	
	$x^{(0)}$	$\hat{x}^{(0)}$	$e/\%$ [1]	$\tilde{x}^{(0)}$	$e/\%$ [2]	State	$\tilde{x}^{(0)}$	$e/\%$ [3]
20150901[4]	38745	38745	—	38745	—	—	38745	—
20150902	43633	46311	−6.14	44104	−1.08	3	44462	−1.90
20150903	44026	46575	−5.79	45004	−2.22	3	44716	−1.57
20150904	43497	46841	−7.69	44978	−3.40	3	44972	−3.39
20150905	54755	47109	13.96	57006	−4.11	5	45229	17.40
20150906	63344	47379	25.20	66489	−4.97	6	45488	28.19
20150907	49411	47649	3.57	52824	−6.91	5	45748	7.41
20150908	54040	47922	11.32	58236	−7.76	5	46009	14.86
20150909	37999	48196	−26.83	42101	−10.79	1	46272	−21.77
20150910	39227	48471	−23.57	43861	−11.81	2	46537	−18.63
20150911	41868	48748	−16.43	47144	−12.60	2	46803	−11.79
20150912	49486	49027	0.93	55778	−12.71	4	47070	4.88
20150913	60949	49307	19.10	68672	−12.67	6	47339	22.33
20150914	51189	49589	3.13	58927	−15.12	4	47610	6.99
20150915	39448	49872	−26.42	46863	−18.80	1	47882	−21.38
20150916	40840	50158	−22.82	48891	−19.71	2	48156	−17.91
20150917	43009	50444	−17.29	51801	−20.44	2	48431	−12.61
20150918	44328	50733	−14.45	53820	−21.41	3	48708	−9.88
20150919	54049	51023	5.60	65178	−20.59	4	48986	9.37
20150920	71911	51314	28.64	85798	−19.31	6	49266	31.49
20150921	61294	51608	15.80	74847	−22.11	5	49548	19.16
20150922	42124	51903	−23.21	54060	−28.34	2	49831	−18.30

[1] SMSE = 8913.42, MAPE = 15.14 %, [2] SMSE = 7568.37, MAPE = 13.18 %, [3] SMSE = 9128.64, MAPE = 14.34 %

[4] Volume of passengers entering and leaving Xiaozhai Station of Metro Line 2 on September 1, 2015

as the residual errors $e(k)$ between $x^{(0)}(k)$ and $\hat{x}^{(0)}(k)$ (see Table 1). The values of $e(k)$ vary between −26.83 % and 28.64 %, and the corresponding *SMSE* and *MAPE* are 8913.42 and 15.14 %, respectively.

Modification factor δ was introduced to improve the forecasting performance of the original GM (1, 1) model. Criteria 1 and 2 yield δ_1 = 520646.1540 and δ_2 = −323352.0908, respectively, thus the proposed GM (1, 1) model is improved into $\tilde{x}^{(0)}(k)$ = 49006.5228 $e^{0.0057(k-1)}$ and $\tilde{x}^{(0)}(k)$ = 44209.4176 $e^{0.0057(k-1)}$ for k = 2, 3, …, n. We obtained the data sequence of $\tilde{x}^{(0)}(k)$ by substituting k = 2, 3, …, n into the two improved forecasting models. These results as well as the corresponding *SMSE* and *MAPE* values are presented in Table 1. We conclude, accordingly, that $\tilde{x}^{(0)}(k)$ = 49006.5228 $e^{0.0057(k-1)}$ is a well-fitted forecast model.

These error series were further divided into six states: State 1 for −∞ – −20 %, state 2 for −20 % – −10 %, state 3 for state 2 for −10 % – 0, state 4 for 0 % – 10 %, state 5 for 10 % – 20 %, and state 6 for 20 % – +∞, according to the precision requirements for forecasting. The state divisions of error series $\{-1.08, -2.22, …, -28.34\}$ are given in Table 1. The state matrix S and one-step transition probability matrix $P^{(1)}$ are:

$$S = \begin{bmatrix} 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 2 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 & 0 \end{bmatrix} \text{ and }$$

$$P^{(1)} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0.25 & 0.25 & 0 & 0 \\ 0 & 0 & 0.5 & 0.25 & 0.25 & 0 \\ 0.33 & 0 & 0 & 0 & 0 & 0.67 \\ 0.25 & 0.25 & 0 & 0 & 0.25 & 0.25 \\ 0 & 0 & 0 & 0.33 & 0.67 & 0 \end{bmatrix}.$$

The new time series of k = 23, 24, …, 30 were then substituted into the general GM (1, 1) model and novel GM (1, 1) model improved by modification factor δ_1 to obtain the forecast values and performance indicators, which were then compared against the statistical data $\{59602, 53752, 60297, 59195, 58567, 50608, 56664, 60760\}$ as shown in Table 2. The error series from the novel Grey forecast model were further processed via Markov approach. For example, the error 6.79 of 201509230 data remains at state 4 and 0.67 is the maximum element in row 4 of $P^{(1)}$. So, this error is most likely to turn to state 6 associated with residual error of −5.10 %. Clearly, the Markov-Grey model exhibits remarkable forecasting accuracy with relatively small *SMSE* and *MAPE* values.

4 Conclusion

Metro passenger flow forecasting is an estimation of the future traffic demand for a certain station, metro line, or overall metro network. Quick, comprehensible forecasts are required to provide sufficient travel information for passengers, and to allow relevant personnel to adjust train operation diagrams dynamically; accurate forecast information also allows personnel to make effective emergency management decisions for stations encountering

Table 2 Performance comparison of passenger flow forecast

Data	Passengers	GM(1,1)		GM(1,1) with δ_1		Markov-GM(1,1) with δ_1	
	$x^{(0)}$	$\hat{x}^{(0)}$	$e/\%$ [1]	$\tilde{x}^{(0)}$	$e/\%$ [2]	$\tilde{x}^{(0)}$	$e/\%$ [3]
20150923	58602	52199	10.93	55554	5.20	—[4]	—
20150924	59752	52498	12.14	55871	6.49	61849	−3.51
20150925	60297	52798	12.44	56191	6.81	62220	−3.19
20150926	62195	53100	14.62	56512	9.14	62730	−0.86
20150927	58067	53403	8.03	56835	2.12	61714	−6.28
20150928	61608	53708	12.82	57160	7.22	63321	−2.78
20150929	63164	54015	14.48	57487	8.99	63802	−1.01
20150930	60986	54324	10.92	57815	5.20	63913	−4.80

[1] SMSE = 6596.64, MAPE = 10.67 %, [2] SMSE = 3539.32, MAPE = 5.53 %, [3] SMSE = 2190.62, MAPE = 3.20 %

[4] Data unavailable due to lack of four-step transition probability matrix

an outburst of passengers. Passenger flow forecasting is often largely inaccurate due to various uncertainties (Li, L. et al., 2018; Zhao et al., 2011) that merit special consideration in ATC application.

In this paper, we simultaneously propose a passenger flow forecast method comprised of a novel Markov-Grey model which includes modification factors δ_1 and δ_2, as determined by two minimum mean square error principles, to improve the forecast precision; the Markov approach is utilized to further process the residual error series. Flow data for passengers entering and leaving the Xiaozhai station of Xi'an Metro Line 2 from September 1-30, 2015, were used to construct the Grey forecast model. Our analysis results revealed that the proposed method has better forecasting performance, with smaller *SMSE* and *MAPE* values than the original method; Markov processing further decreases the residual errors.

This study arguably has some methodological limitations. For one, because the Grey forecast model is the form of exponential function, better forecasting performance would be possible if the original input $\{x^{(0)}\}$ strictly increases (or decreases) the time series data; but the data series would yield large residual errors if $\{x^{(0)}\}$ is harsh. The model proposed here also does not include the stochastic characteristics of metro passenger flow series. In future work, we plan to extend this methodology to incorporate the uncertainty of metro passenger flow forecasting using a fuzzy rule-based approach.

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