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RESEARCH ARTICLE

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## Abstract

The article is concerned with optimization of the winter maintenance of the selected stretch of roads within the Strakonice district by applying the graph theory.

The graph theory and the Chinese postman problem are applied in case of the winter maintenance of the specific selected section of roads. The article also includes the evaluation of performing the winter road maintenance so far. The results of optimization are compared with the present state and economically evaluated at the end.

## Keywords

winter maintenance, winter maintenance technology, optimization, graph theory, Eulerian path, Chinese postman problem, rank practicability

## 1 Introduction

Optimization of the winter maintenance route is, basically, the traffic problem, and, hence, the roads need to be transformed into the form of the continuous flow chart (transport network) defined as a final set of nodes and stretches, in which case it is necessary to meet the basic prerequisite for the transport network, i.e. to be continuous so that there is at least one path for each pair of nodes connecting both nodes. Also, the structure is allocated to each stretch – e.g. the price or length expressed in suitable units – e.g. in the units of length, price for the use of the stretch, etc., whereby the network becomes evaluated.

“The graph in which all edges (nodes) are evaluated is referred to as “edge (node) evaluated graph” (Cejka, 2016).

## 2 Solution of Winter Maintenance Issue using the Graph Theory

For the needs of the winter maintenance optimization, the so-called Eulerian path and Eulerian graph can be used (Polya et al., 2010). In graph theory, the Eulerian path is a trail which contains every edge exactly once (Fig. 1). This term was introduced by the Swiss mathematician Leonhard Euler (1707-1783). He solved the problem if it is possible to go across seven bridges over the Pregel River in Königsberg (Russia), each only once, and to return to the initial point (Chovancova and Klapita, 2017; Simkova et al., 2015).

“The graph which can be covered by one closed stroke is called Eulerian path” (Yu et al., 2017; Lu et al., 2017).

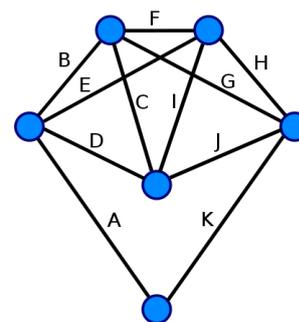


Fig. 1 Eulerian path, Source: Authors

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Based on the graph theory, Euler reformulated the problem and proved that the Eulerian path does not exist in the graph created upon the map of Königsberg city as Fig. 2 shows. Only the Eulerian graphs can be drawn at one stroke. If the seven bridges of the city of Königsberg do not form the Eulerian graph, it proves that the bridges cannot be crossed this way.

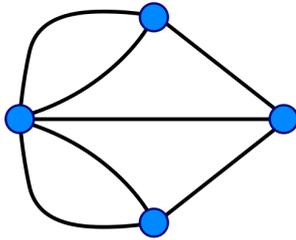


Fig. 2 Königsberg Bridges graph. Source: Authors

### 2.1 Chinese postman problem

The Chinese postman problem is, basically, a problem of any postman anywhere in the world. The thing is that the postman must go through all the streets of his district and return to the point from where he started and to walk as least kilometres as possible. In other words, we have a non-oriented continuous graph with the edges evaluated by positive numbers and it is necessary to find the shortest closed sequence which would contain all graph edges (Cejka et al., 2016; Bartuska et al., 2015).

If the graph is Eulerian, i.e. if there is a closed Eulerian path in it, the task is solved by this Eulerian path (Kawarabayashi and Kobayashi, 2015).

If there is no Eulerian path in the graph, the sequence must go through some edges twice or more times to visit all edges. It is, however, possible to construct the shortest sequence which would go through each edge only once or twice at most.

The repeatedly gone-through edges in the shortest sequence which contains all graph edges forms a system of paths connecting always two vertices of an odd degree. Such paths are disjoint, i.e. no edge of the graph lies on the two of such paths (Smetanova, 2015; Zitricky et al., 2015).

### 3 Route Optimization

Fig. 3 shows the highlighted risk stretches of the route. The points on the roads with inclination where the traffic situation often gets worse in winter or even the transport stops due to the trucks stuck in snow or due to icing are marked in red. The next stretches – marked in blue – are the stretches of roads in

an open landscape where the wind forms the snow banks on the snow coverage.

Fig. 4 Shows the selected stretch of the winter maintenance. For the route optimization, it needs to be transformed into the evaluated flow chart.

Vertices are marked  $v_1, v_2, v_3, \dots, v_{11}$  and are evaluated as per the prioritisation. Graph edges are evaluated as per the length of the edge between two vertices (Di Matteo et al., 2016). The Table 1 shows the distances between the vertices in metres (Cejka, 2016; Kampf et al., 2012).

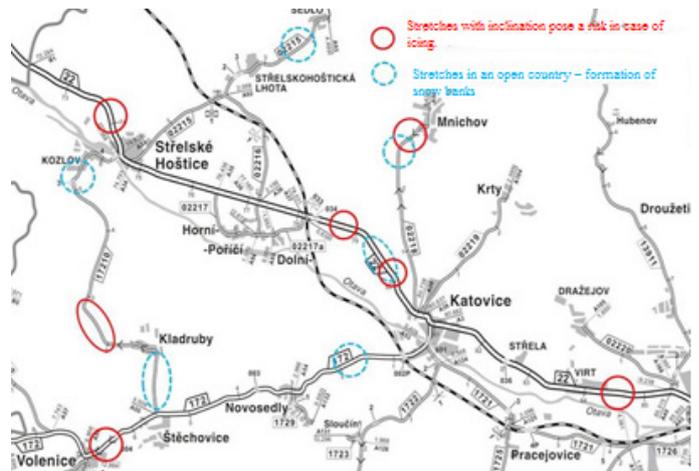


Fig. 3 Risk stretches on the maintenance route. Source: Authors



Fig. 4 Route of the selected stretch. Source: Authors

As Fig. 5 shows, the flow chart contains also the multiple edges which represent the roads I/22 and II/172 and which need to be treated in both directions as to their width. The road I/22 needs to be treated as a priority since it is of the 1st priority and needs to be treated within three hours. The road II/172 is of the

Table 1 Length of edges between individual vertices in m

1	2	3	4	5	6	7	8	9	10	11	12	13
$v_{10}v_{11}$	$v_5v_6$	$v_2v_3$	$v_1v_2$	$v_4v_{11}$	$v_7v_8$	$v_4v_{10}$	$v_5v_9$	$v_2v_4$	$v_4v_7$	$v_7v_5$	$v_4v_5$	$v_{11}v_1$
2 061	2 313	2 340	2 386	2 550	2 725	3 422	4 488	4 560	5 561	6 388	6 395	9 496

Source: Authors

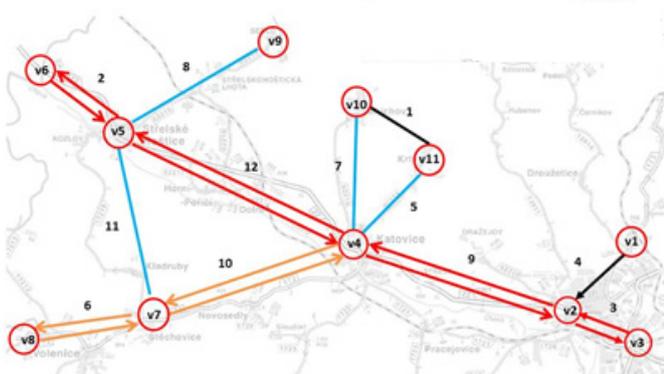
**Table 2** Length of edges between vertices – travelled distance

v2v4	v4v5	v5v6	v6v5	v5v4	v4v7	v7v8	v8v7	v7v4	v4v2	v2v3	v3v2
4 560	6 395	2 313	2 313	6 395	5 561	2 725	2 725	5 561	4 560	2 340	2 340

Source: Authors

2nd priority with the maintenance time-limit within 6 hours, but it is merged with the road I/22 into one circuit. The length of these roads is 47,788 m, the average speed of the maintenance mechanism is approx. 30 km/h and the time of maintenance is approximately 1 hour 36 minutes. The distances between individual vertices and the distance travelled are shown in.

Local road between Mnichov and Krty



Stretch of 1st priority, maintenance in both directions

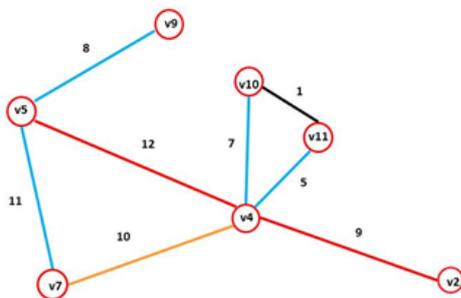
Stretch of 2nd priority, maintenance in both directions

Stretches of 3rd priority, only road spreading (snowploughing by the plough share – ensured by suppliers)

**Fig. 5** Evaluated flow chart. Source: Authors

Now, the roads of the 3rd category, namely III/17210, III/02215, III/02218 and III/02219 are left to be treated. The flow chart in Fig. 6 shows all edges usable for passing these roads.

Local road between Mnichov and Krty.



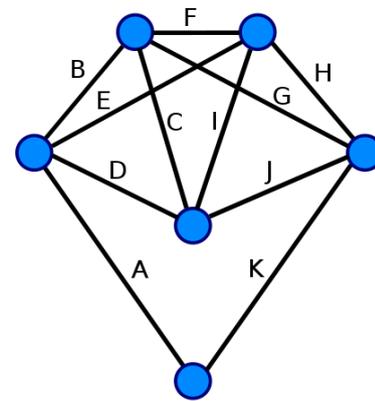
Stretches of 3rd priority, only road spreading (snowploughing by the plough share – ensured by suppliers)

**Fig. 6** 3rd category roads. Source: Authors

#### 4 Optimized Route

The shortened local route between the villages of Mnichov and Krty-Hradec is suitable for optimization – the non-technological part of the stretch will be further shortened.

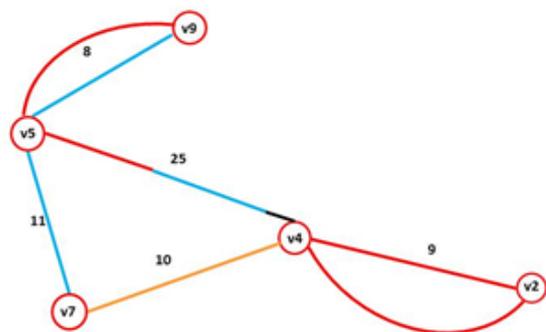
The graph in Fig. 7 shows that the graph has 4 vertices of an odd degree – v2, v4, v5 and v9 and there are only two edges left for pairing as interconnecting of vertices v10 and v11 forms a loop which can be merged with the edge v4v5 (Stopka et al., 2015; Kampf et al., 2012; Klapita, 2012).



**Fig. 7** Optimization variant. Source: Authors

The shortest pairing is pairing of the edges v2v4 and v5v9.

After inserting the multiplied edges in the original graph, we will obtain the closed Eulerian path as shown in Fig. 8.



**Fig. 8** Closed Eulerian Path. Source: Authors

By inserting the distances and searching for the matrix (Table 2), we will obtain the shortest passage of the route of this variant, namely from v2→v4→v5→v9→v2.

**Table 3** Matrix of distances between vertices

	v2	v4	v5	v9
v2		<b>4 560</b>	16 509	23 476
v4	4 560		<b>11 949</b>	16 437
v5	16 509	11 949		<b>4 488</b>
v9	<b>23 476</b>	16 437	4 488	

Source: Authors

#### 4.1 Proposal of Stretch Route Optimization

The variant proposed for optimizing the route of the selected stretch is the one which shortens the non-technological travel of the salt spreader most, it is most continuously passable and provides a good access to the above-mentioned stretch on the road III/17210.

#### 5 Evaluation

Evaluation of the selected stretch optimization concerns only non-technological travels of the salt spreader trucks from the viewpoint of the distance travelled, time and economic impact. The costs of one kilometre of travel amount to 50.70 CZK (The value is taken from the internal accounting of the Road Administration & Maintenance of the South Bohemia Region, Strakonice plant, and includes the costs of material, fuels, wages, repairs and maintenance and the overhead costs).

Comparison of the original and optimized routes calculate for one salt spreader as to the saving of distance, time and costs. After optimizing, the total route of the salt spreader will be reduced by 11,228 m and the non-technological travel by 4,736 m, i.e. saving of 240.12 CZK per one departure of the salt spreader. Expressed as a percentage: optimization will save 10.25 % and 21.32 % of costs of the non-technological travel of the salt spreader.

#### 6 Conclusion

The analysis shows that during maintenance of the selected stretch as per the Plan of Winter Maintenance, the salt spreader travels more “empty kilometres” of non-technological travels than it is necessary.

The route optimization by applying the Chinese postman problem shortened the route by 11,228 m in total and the non-technological travel by 4,736 m, which generated the saving in costs per one salt spreader in the amount of 240.12 CZK, i.e. a saving of 21.32 % as compared with the original route.

The saving for the average winter season with 27.5 travels of the salt spreader would amount to 130.24 km of non-technological travels and 6,603.17 CZK in costs.

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