Periodica Polytechnica
Transportation Engineering

47(1), pp. 1-5, 2019
https://doi.org/10.3311/PPtr.11653

RESEARCH ARTICLE

Bank Angle Estimation Using Radar Data

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Received 28 October 2017; accepted 22 November 2017

Abstract

The procedure designer's toolbox calculating nominal track and protection areas is not entirely appropriate for determining the standard deviation of the trajectories, when designing a new departure procedure. The nominal track provides only a theoretical mean values to be shown on the chart but the aircraft cannot be expected to adhere to this route in a turn. Whereas protection areas are too wide and conservative estimates as they serve the purpose of providing adequate distance from ground obstacles under the worst conditions. This is why a new, radar based solution is needed to tackle this problem. The first step into this direction is to investigate the part of the route which infers the biggest uncertainty, the turn.

Keywords

instrument flight procedure design, radar data analysis, bank angle, SID, Radius-to-Fix, PBN

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1 Introduction

Track deviation along the turns is important from several aspects. Trajectory prediction of appropriate accuracy is a vital element of future innovations (Ghasemi Hamed et al. 2013). Further, when sequencing methods based on the difference of remaining track miles are to be used, it is essential that the shortening of the route due to a-priori undetermined curves can be accurately estimated (Madácsi et al. 2015).

Noise mitigation is also an important motivating factor for designing performance based navigation (PBN) routes (Jensen et al. 2017), thus, as much as it is possible, nominal tracks are not positioned over populated areas. However, due to the deviation of the trajectories, noise mitigation is not always surely achieved by procedure design: although it affects the parameters of the turn (by the indicated airspeed constraint), the remaining uncertainty/variance is still too large to exactly determine the actual routes.

A possible solution to this problem is applying radius to fix (RF) legs which are conceived to guarantee separation between aircraft on closely positioned routes including turns (Polischuck, 2016). As track guidance is available to the aircraft also along the curve, the populated areas can be evited with great confidence (Geister et al, 2013). Nonetheless, the ratio of RF capable aircraft does not yet reach the threshold from where it would be worthwhile designing procedures using this technique. Hence, the present paper does not look into RF legs or its influencing parameters. Rather, it shows how the bank angle can be estimated in conventional turns as based on measured radar data and what implications this has on traditional protection area calculations.

The paper is further structured as follows. Section 2 explains the rationale behind the proposed methodology, while Section 3 explains the technique itself. Section 4 includes the results as based on real, measured radar data collected around the vicinity of Budapest Liszt Ferenc International Airport. Finally, Section 5 offers the relevant conclusions.

2 Rationale

One of the most complex tasks of instrument procedure design is calculating the protection area of the turn. The main input parameter to this is naturally the radius of the turn. To calculate this radius, the altitude dependent true airspeed, the bank angle and the rate of turn have to be taken into account. The exact calculation is the following:

True airspeed of 210 kts IAS at 2500 feet with ISA+15°C:

$$\begin{aligned}\mathrm{TAS} &= \mathrm{IAS}*171233*\frac{\sqrt[2]{\left(288\pm\mathrm{VAR}\right)-0.00198\mathrm{H}}}{\left(288-0.00198\mathrm{H}\right)^{2.628}}\\ &= 210*171233*\frac{\sqrt[2]{\left(288\pm15°\mathrm{C}\right)-0.00198*2500}}{\left(288-0.00198*2500\right)^{2.628}}\\ &= 223.5913\mathrm{kts}\end{aligned} \tag{1}$$

Rate of turn with bank angle of 15°:

$$R=\frac{3431*\tan\alpha}{TAS*\pi}=\frac{3431*\tan 15°}{223.59*\pi}=1.3088°\,/\,sec \tag{2}$$

Radius of turn:

$$r=\frac{TAS}{20*\pi*R}=\frac{223.59}{20*\pi*1.31}=2.7164NM \tag{3}$$

Rate of turn with bank angle of 25°:

$$R=\frac{3431*\tan\alpha}{TAS*\pi}=\frac{3431*\tan 25°}{223.59*\pi}=2.2777°\,/\,\mathrm{sec} \tag{4}$$

Radius of turn:

$$r=\frac{TAS}{20*\pi*R}=\frac{223.59}{20*\pi*2.28}=1.5608NM \tag{5}$$

The equations show how much, all other parameters being equal, the bank angle influences the expected radius of the turn.

The essence of procedure design is basically to determine the protection areas ensuring the safe separation from ground obstacles. Hence, the most conservative solution, providing the highest level of safety is to use a big turn radius (the bank angle is then small). This entails a large protection area, thus also increasing the number of obstacles to be investigated.

If the aim is to determine the expected flight track, this method, due to its understandable conservatism, will yield a misleading result. Instead, in these calculations, it would be important to use parameters as close to reality as possible. ICAO Doc 8168 (hereafter referred to as PANS-OPS) on the rules of flight procedure design includes values to be used for the nominal track determination of the departure procedures. However, it is questionable how accurate these estimations may be.

The aim of the present paper is to propose a method to accurately estimate bank angle values as based on radar data to provide procedure design with a methodology that can enhance the accuracy of trajectory estimations in the course of procedure design above those of classic protection area and nominal track calculations.

3 Methodology

To estimate the bank angle, first, the turn radius is to be determined from the curves obtained from radar data. To do so, using the least square method, a circle is fitted to the curves where, as based on radar data, the aircraft has passed.

3.1 Circle fitting on radar plots

Based on Bullock (2017). Center of the circle: u_c and v_c. The task is to minimize S.

$$\begin{aligned}S &= \sum_{i=1}^{n}(g\left(u_i,v_i\right))^2 \text{ where } g\left(u,v\right)\\ &= \left(u-u_c\right)^2+\left(v-v_c\right)^2-\alpha \text{ and } \alpha=r^2 \text{ and}\\ u_i &= x_i-\frac{1}{n}\sum_{i=i}^{n}x_i;\; v_i=y_i-\frac{1}{n}\sum_{i=i}^{n}y_i\end{aligned} \tag{6}$$

$$\frac{\partial S}{\partial\alpha}=2\sum_{i=1}^{n}g\left(u_i,v_i\right)\frac{\partial g}{\partial\alpha}\left(u_i,v_i\right)=-2\sum_{i=1}^{n}g\left(u_i,v_i\right) \tag{7}$$

$$\frac{\partial S}{\partial\alpha}=0 \textit{ iff } \sum_{i=1}^{n}g\left(u_i,v_i\right)=0 \tag{8}$$

$$\begin{aligned}\frac{\partial S}{\partial u_c} &= 2\sum_{i=1}^{n}g\left(u_i,v_i\right)\frac{\partial g}{\partial u_c}\left(u_i,v_i\right)\\ &= 2\sum_{i=1}^{n}g\left(u_i,v_i\right)2\left(u_i-u_c\right)\left(-1\right)\\ &= -4\sum_{i=i}^{n}\left(u_i-u_c\right)g\left(u_i,v_i\right)\\ &= -4\sum_{i=i}^{n}u_i g\left(u_i,v_i\right)+4u_c\sum_{i=1}^{n}g\left(u_i,v_i\right)\\ &= -4\sum_{i=i}^{n}u_i g\left(u_i,v_i\right)\end{aligned} \tag{9}$$

$$\frac{\partial S}{\partial u_c}=0 \textit{ iff } \sum_{i=i}^{n}u_i g\left(u_i,v_i\right)=0 \tag{10}$$

$$\frac{\partial S}{\partial v_c}=0 \textit{ iff } \sum_{i=i}^{n}v_i g\left(u_i,v_i\right)=0 \tag{11}$$

$$\begin{aligned}\sum_{i=i}^{n}u_i g\left(u_i,v_i\right) &= \sum_{i=i}^{n}u_i\left[u_i^2-2u_iu_c+u_c^2+v_i^2-2v_iv_c+v_c^2-\alpha\right]\\ &= \sum_{i=1}^{n}u_i^3-2u_c\sum_{i=1}^{n}u_i^2+u_c^2\sum_{i=1}^{n}u_i+\sum_{i=1}^{n}u_iv_i^2\\ &\quad -2v_c\sum_{i=i}^{n}u_iv_i+v_c^2\sum_{i=1}^{n}u_i-\alpha\sum_{i=1}^{n}u_i\\ &= \sum_{i=1}^{n}u_i^3-2u_c\sum_{i=1}^{n}u_i^2+\sum_{i=1}^{n}u_iv_i^2-2v_c\sum_{i=1}^{n}u_iv_i=0\end{aligned} \tag{12}$$

$$\sum_{i=i}^{n} v_i g(u_i,v_i) = \sum_{i=i}^{n} v_i\left[u_i^2 - 2u_iu_c + u_c^2 + v_i^2 - 2v_iv_c + v_c^2 - \alpha\right]$$
$$= \sum_{i=1}^{n} v_iu_i^2 - 2u_c\sum_{i=1}^{n} v_iu_i + u_c^2\sum_{i=1}^{n} v_i + \sum_{i=1}^{n} v_i^3$$
$$-2v_c\sum_{i=i}^{n} v_i^2 + v_c^2\sum_{i=1}^{n} v_i - \alpha\sum_{i=1}^{n} v_i$$
$$= \sum_{i=1}^{n} v_iu_i^2 - 2u_c\sum_{i=1}^{n} v_iu_i + \sum_{i=1}^{n} v_i^3 - 2v_c\sum_{i=1}^{n} v_i^2 = 0 \tag{13}$$

$$v_c = \frac{\dfrac{\sum_{i=1}^{n} u_i^3 + \sum_{i=1}^{n} u_iv_i^2}{2\sum_{i=1}^{n} u_i^2} - \dfrac{\sum_{i=1}^{n} v_i^3 + \sum_{i=1}^{n} v_iu_i^2}{2\sum_{i=1}^{n} u_iv_i}}{\dfrac{\sum_{i=i}^{n} u_iv_i}{\sum_{i=i}^{n} u_i^2} - \dfrac{\sum_{i=i}^{n} v_i^2}{\sum_{i=i}^{n} u_iv_i}} \tag{14}$$

$$u_c = \frac{\dfrac{\sum_{i=1}^{n} u_i^3 + \sum_{i=1}^{n} u_iv_i^2}{2} - v_c\sum_{i=i}^{n} u_iv_i}{\sum_{i=i}^{n} u_i^2} \tag{15}$$

$$x_c = u_c + \bar{x} \tag{16}$$

$$y_c = v_c + \bar{y} \tag{17}$$

$$r = \sqrt{u_c^2 + v_c^2 + \frac{\sum_{i=1}^{n} u_i^2 + \sum_{i=1}^{n} v_i^2}{n}} \tag{18}$$

3.2 Filtering radar plots to fit the circle on

Having filtered the flights to be investigated, radar data, to be used for circle fitting, have to be elicited from the surveillance processing system. Should radar data be also included which does not belong to a turn, the fitting will become corrupted. Fig. 1 shows a match leading to incorrect results.

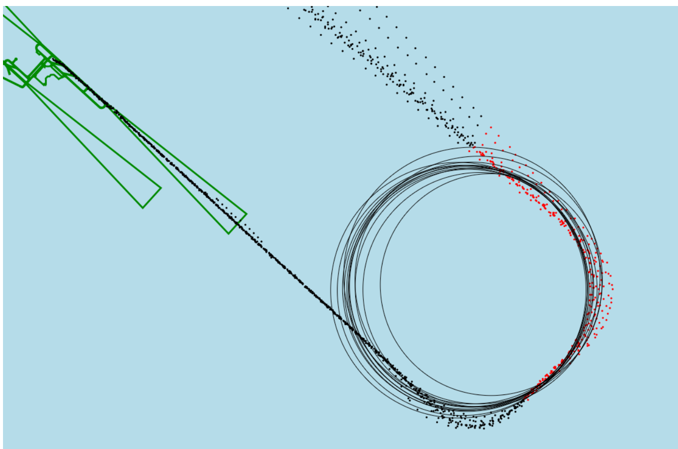

Fig. 1 Fitting leading to incorrect results

Whereas, Fig. 2 shows a fitting yielding accurate results.

The heading parameter as measured by the radar seems to be the obvious choice for selecting the radar data to be included into the fitting process. As long as the heading does not change, the plot series may be regarded as a straight line, while, else it may be taken to be a curve. Nonetheless, due to the small alternations of the radar headings as measured by the radar, this simple method is not applicable.

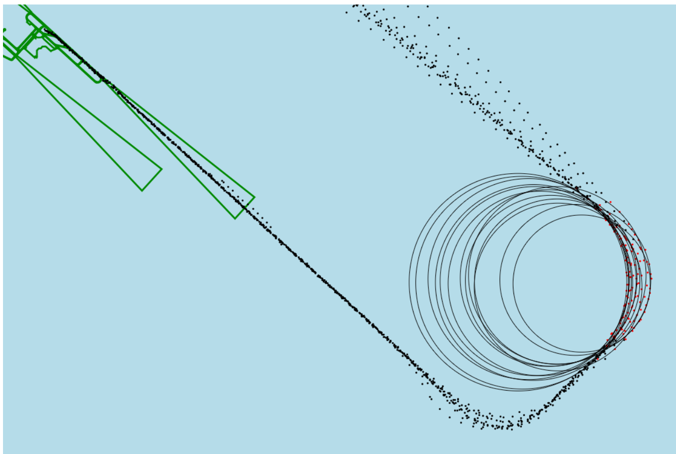

Fig. 2 Fitting with accurate results

To work around this problem, the differences of the headings may be added up and compared to a limit value. In case of straight flight, even if there are small variations in the headings measured by the radar, these differences even out. When the cumulative sum of the differences reaches a certain threshold, the turn has started, and then, when it decreases again below a given value, the turn ends.

Naturally, the turn radius to be investigated will significantly influence the order of magnitude of the cumulative sum of heading differences. This means, that the limit value is dependent exactly on the parameter to be investigated, so, even the simple version of this methodology will not be useable.

The turn starts when the following equation (Eq. (19)) becomes true:

$$\sum_{i+1}^{n}\left(HDG_i - HDG_{i-1}\right) > limit \tag{19}$$

However, the aim of the investigation, circle fitting, may at the same time be used to filter out radar data of aircraft in the turn. The limit value shall be different for flights following different turn radiuses. Nevertheless, brute force algorithms may be sufficient to determine the limit value, where the total difference of distances between the radar plots and the fitted circle is minimal. Using the resulting value as optimized for the given flight will then provide adequate results when applied with the cumulated heading distances methodology.

The start of the turn of the jth flight can then be determined by Eq. (20).

$$\sum_{i+1}^{n}\left(HDG(j)_i - HDG(j)_{i-1}\right) > limit_j \tag{20}$$

3.3 Estimating bank angle

The data used are one month of arrivals for RWY31R via right downwind.

To calculate the bank angle, the true airspeed would have to be known. However, this information is not available, only ground speed is recorded. The difference is due to the wind measured at the given geographic location and the given flight level. Applying the notion of omnidirectional wind often utilized in procedure design, it can be presumed that the direction of the wind is of standard deviation at the point of measurement. This means, that even though the ground speed, including the effect of the wind, is not suitable to accurately calculate the value of the bank angle, the average of its calculated value can provide a good estimate for the bank angle in the given phase of flight.

Thus, rearranging (2) and (3), an estimated bank angle value can be calculated using the ground speed from radar data used for circle fitting and determined in the previous section.

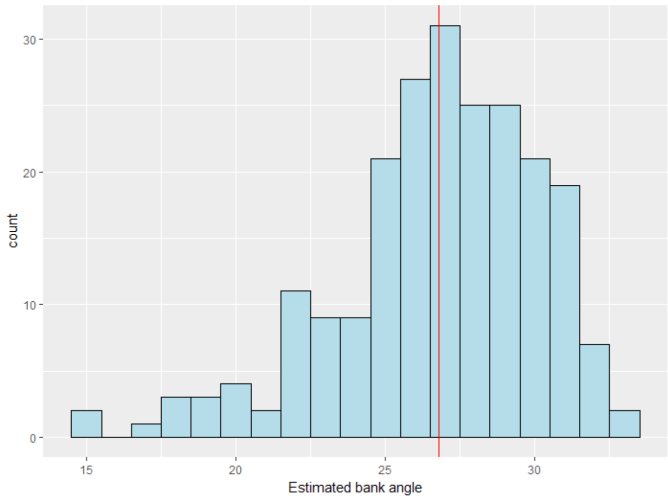

Fig. 3 Estimated bank angle of arrivals

The resulting average is 26.8°, which is well in line with the 25°[1] to be used in the arrival phase according to procedure design rules. The distribution is unimodal. In case of the Boeing 737 fleet, under LNAV operations the bank angle is maximized in 30° above 200 feet AGL. So, the maximum error of the calculation can be well estimated by the highest measured (33°) minus the maximum permitted (30°), being 3°.

The average difference of the measured and calculated turn radiuses is -0.13. The right skew and the non-zero mode present in the distribution may be explained by the fact that the measurements took place in the direction of runway 31. On the downwind leg, tailwind can be expected with a higher probability and so the average of the real true airspeeds is lower than the average of the general ground speed, and, regarding turn radiuses, calculating with the latter will yield an overestimate as compared to the real value.

For arriving traffic, the method provided the results as laid down in by PANS-OPS, thus, the measurements can be regarded as the validation of the methodology.

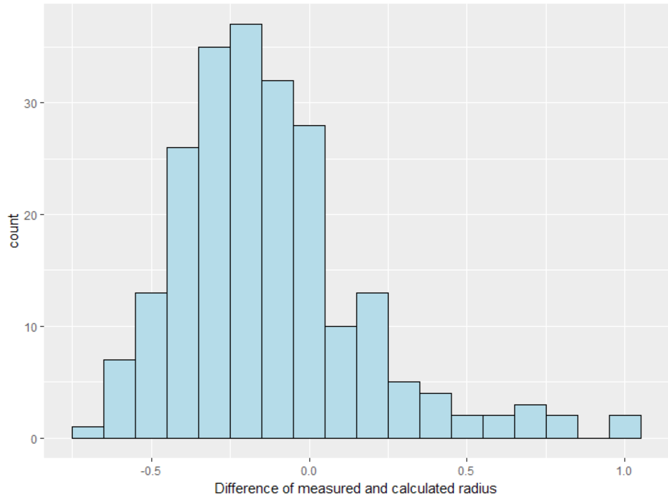

Fig. 4 Difference of measured and calculated radius of arrivals

4 Results

Leaving LHBP airport runway 31, starting the turn after the first point in the departure procedure, the average altitude of the departing aircraft is 3500 feet. As based on the values included in Fig. 5., the bank angle matching this is 25°. However, to design the protection area, 15° is to be calculated with.

*Bank angle***
For turn area calculation: 15° For establishment of the average flight path only: 15° until 305 m (1 000 ft) 20° between 305 m (1 000 ft) and 915 m (3 000 ft) 25° above 915 m (3 000 ft)

Fig. 5 Table of bank angles to be used for departures (PANS-OPS)

Estimating the bank angle with the methodology described in the previous section for traffic departing from runway 31L to the west yielded the results shown in Fig. 6.

The distribution is visibly bimodal. The characteristic values are grouped around 17° and 25°, which seem to correspond to the numbers included in PANS-OPS to use for calculating separation from obstacles and the nominal track.

1 PANS-OPS, *III-2-1-3,* 1.2.4.1 For approach phases, the bank angle is 25°/(or 3°/s), except in the missed approach phase where a 15° bank angle is assumed.

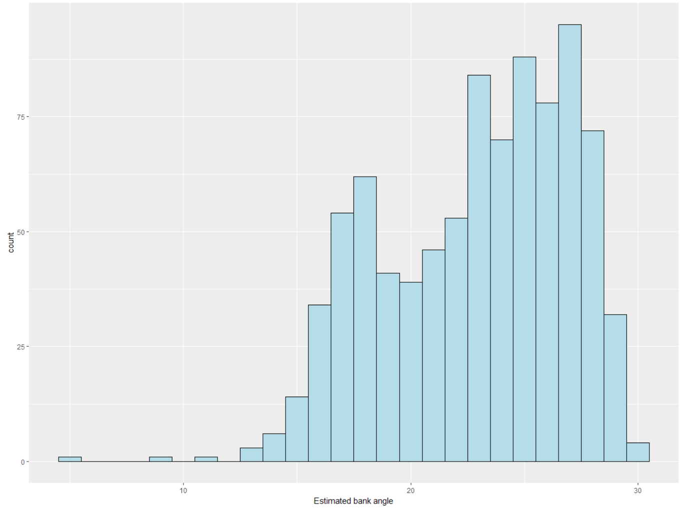

Fig. 6 Estimated bank angles of departures

5 Conclusion

5.1 Departure track estimations

It is a widespread belief that the dispersion of flights in the west turn of departing aircraft from RWY31 is due solely to the differences in speed. This approach, however, may also give ideas as to how the number of affected areas can be reduced. If, as procedure design standards enable to a certain extent, the maximum speed in the turn were to be limited, this dispersion would decrease. However, this decrease would not come about universally, only on the track belonging to the given modus, as the effect of bank angle is in total greater. Thus, the affected areas could be expected to be reduced only minimally. Nonetheless, to seize a better estimate of track dispersion, it would be important to investigate what leads to different bank angle values when the altitude and speed values are not varying.

5.2 Track guidance in the turn

Regarding arriving traffic, when there is no track guidance, the distance from the fitted circle should increase proportionately to the time spent in the turn. Statistically speaking, in theory, heteroscedasticity of the residuals is expected. Despite this and apart from some exceptions, as based on the measurements, heteroscedasticity is not seen, meaning that the aircraft tries to adhere to the curve of the fly-by turn. This means that a radius to fix like turn is taking place with an unknown and not fix centre point.

This fact, if it can be further proven, questions the applicability of the wind spiral method used at present to calculate the protection areas in the turns. The starting point of the wind spiral method is the presumption that, in the turn, the wind diverts the aircraft from its track proportionately with the time elapsed. The resulting protection area is too large, conservative, thus including obstacles which do not entail real flight safety risks. In case of a complex terrain, this might even lead to aborting the entire procedure design process.

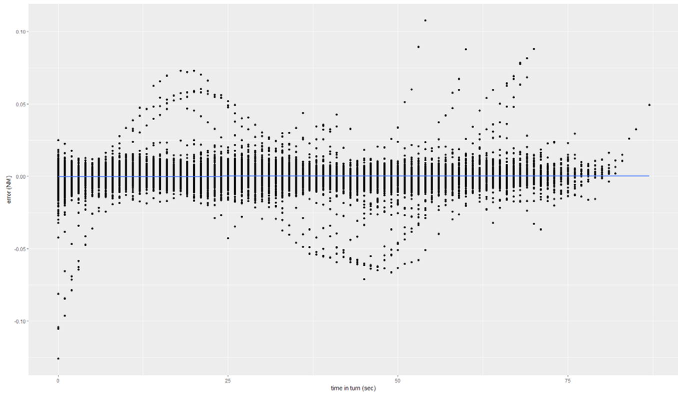

Fig. 7 Residuals as a function of time (sec) spent in turn

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