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RESEARCH ARTICLE

The mathematical modeling of reversible lane system

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Abstract

We examine the modeling of reversible lane system configured on a road part network. The functions of the each network's elements and contacts between its each element cease in the course of a change and new contacts and new function elements are activated instead of them. This opens the door to a new principled optimal control, which happens to the dynamic change of the structure of the network graph. In the model, as in reality, the geometry elements do not disappear naturally, but create a variable network as a result of their new function and their connection system. The article presents the mathematical modeling of the problem. Points out the fundamental questions of the structure change and exemplifies the above on a simple example.

Keywords

Reversible lane · road traffic control · mathematical modeling

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1 Introduction

The application of reversible lane systems – RLS is an interesting special method, which favors maximally the change (part of the day, seasonally etc.) of primary current on the available road surface [1–9]. A number of estimations justifies the reduction of the travel time by about 40%, the waiting time by about 50% and the number of the stops by circa 40% on those part networks, where this was introduced comparing to the original data. The effect of these traffic data reduces the accident indirectly, too, but it is necessary to analyze the risk factors, which occur in the course of the application of RLS [2].

2 The mathematical model applied to this problem's solution

Our method defines a dynamic contact structure consisting of n parts inner sections and m part exterior sections. Our network model position in province, which is delimited with a closed curve. In this case the vehicle densities in inner network are the state parameters of the system: $x = [x_1(t), x_2(t), x_3(t), \ldots, x_n(t)]^T$. The vehicle densities in the exterior part network of network (which have a direct connection with some sections) are marked $s = [s_1(t), s_2(t), \ldots, s_m(t)]^T$, that we measure [10], [11], [12].

This mathematical model leads to the examination of the positive, non-linear (NL) dynamic system. The model functionally is a macroscopic model [13], [14], [15].

$$x' = \langle L \rangle^{-1} [K(x,s)x + K_{input}(x,s)s] \qquad (1)$$

Where: $\langle L \rangle^{-1}$ diagonal matrix contains the reciprocal of inner section lengths. $K(x,s)$ constructed matrix, which was constituted of $K_{\text{inner}}(x,s)$ and $K_{\text{output}}(x,s)$ matrix. The elements of $K(x,s)$ and $K_{\text{input}}(x,s)$ connection matrices are the connection functions, which depend on the density states. The physical meaning of the matrix's elements is the passing speed. The system is a positive system.

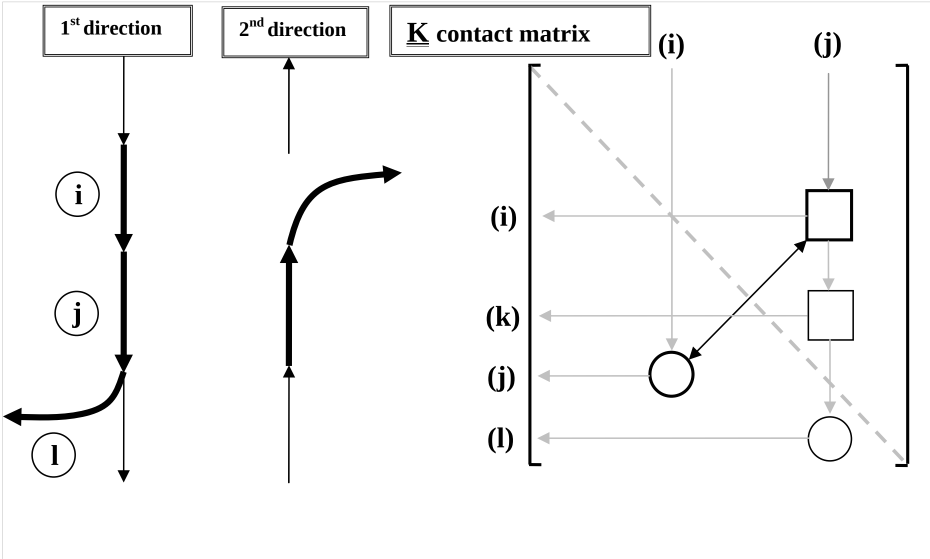

Fig. 1. Two traffic directions and the contact matrix

3 The contact matrix of Reversible Lanes System, control signs and state parameters

There are two traffic directions on the left side of Fig. 1, where we consider the contacts of the sections nominated thickly. We labeled the examined sections with a circle in case of 1. direction and with a square in case of 2. direction. We may establish, that two kinds of contact forms exist:

I Constant geometry contact: so contact of i and j. In this case the geometry contact remains in all traffic direction change, but what changes is the direction of the passing (in case of 1. direction: $i \rightarrow j$, in case of 2. direction: $j \rightarrow i$).

II Contact depending on direction: so in case of the 1. direction contact of j and l, in case of the 2. direction contact of j and k. In this case the geometry contact depends on the traffic direction (in case of 1. direction: $j \rightarrow l$, in case of 2. direction: $j \rightarrow k$).

Two things must be enhanced: the contact between i and j elements and the direction of this passing. (Ex.: $K_{i,j}$ element, if not equivalently 0, shows that there is a contact and that j works onto $i : j \rightarrow i$).

- All such contacts remain invariant in the contact matrix, which does not affect the direction change!
- The contacts, which are affected by the direction change, the contacts connected to 1. and 2. direction close one another off! So, in the contact matrix, the contact may appear, which is marked only with a square or with a circle in a certain time.
- In case of a constant geometry contact: The contact is reflected onto a main diagonal as a result of the direction change (i,j)↔(j,i).
- There is no reflection in case of the contact connected to only a single direction. This contact appears in only one direction. Ex. (j→k).

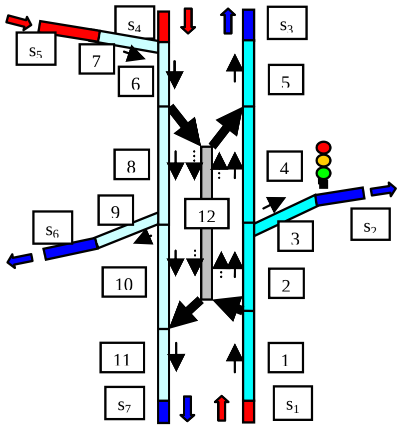

Fig. 2. Sample model, on model of reversible lanes system

- Finally this is very important, that the contact change does not happen at the same time in the contact matrix. The dissociation of the contacts happens in two steps.

4 Example the model of Reversible Lanes System

Our model is NL positive system with 12 degrees of freedom, 7 exterior contacts where:

- s_1, s_4, s_5 are measured inputs, s_2, s_3, s_6, s_7 are measured outputs, with p_1, p_2,...,p_7 section lengths.
- 1,2,...,11 are network sections characterized with x_1, x_2,...,x_{11} state parameters and with l_1, l_2,..., l_{11} section lengths.
- 12 is reversible lane in both directions marked with x_{12} state parameter and with l_{12} section length.

Let us assume that:

– A morning peak is evolved in the section directed upwards from below. Section 3 often gets a red light as a result of the crossing railway traffic, because of this the section 2 and 1get blocked.

– An afternoon peak is evolved in the section directed upside-down. At this time the vehicle density of section s_6 is overloaded, because of this section 9 often gets blocked as a result of this 8, too, and this influences 6 and onto 7!

We mark obstruction β_{ij} with passing through j to i ($j \to i$). Obstruction appears between the sections below, at the right side sections in the model:

$$\beta_{2,1} : (1 \to 2), \beta_{3,2} : (2 \to 3), \beta_{4,2} : (2 \to 4), \beta_{5,4} : (4 \to 5)$$

and at the left side sections:

$$\beta_{8,6} : (6 \to 8), \beta_{9,8} :$$

$$(8 \to 9), \beta_{10,8} : (8 \to 10), \beta_{11,10} : (10 \to 11).$$

We mark distribution rate α_{ij} with passing through j to i ($j \to i$). A distribution rate appears between the sections below, at the right side sections in the model:

$$\alpha_{12,1} : (1 \to 12),\ \alpha_{2,1} : (1 \to 2), \text{where} : \quad \alpha_{2,1} = 1 - \alpha_{12,1}$$

If we do not use section 12 $\alpha_{12,1}$=0, and $\alpha_{2,1}$= 1, if we use it we have established a $\alpha_{12,1}$=0.45 rate, and at this time $\alpha_{2,1}$= 0.55.

The survey of the rate is interestingly formed with the 2→3 transition. Let this be α_{32}: (2→3), if we do not use section 12, so this means, that being in the section 2 wish to turn right in the above ratio. These drivers wish to turn right here when the section 12 works, because this is their route in the morning peak. The number of drivers in the section 2 decrease by working of section 12, according to a $\alpha_{2,1}$factor.

Because of this transition from 2 onto 3: it is necessary to count with a $\frac{\alpha_{3,2}}{\alpha_{2,1}}$: (2→3) rate, respectively, if we count with $\alpha_{12,}1$ in all cases, the transition from 2 onto 4: $\frac{\alpha_{3,2}}{1-\alpha_{12,1}}$: (2→3) and $1 - \frac{\alpha_{3,2}}{1-\alpha_{12,1}}$: (2→4).

The same happens to the inscription of the distribution rates with the vehicles travelling from the other direction.

5 Computational results onto the model of Reversible Lane System

At first we did the examinations on the breakfast and the afternoon peaks without section 12. In the second case we did the examinations with an already operating reversible lane. The results on the figures below are received on a 20 minute time interval. We see the percental growth rate of the number of all crossing vehicles until timepoint t, and the same with activated reversible lane on Figs. 3 and 4.

The results show, that the growths are 20% in the morning peak and 30% in the afternoon peak for the full model's case. This resulted 26% growth for the whole day (Fig. 4).

In the morning peak the vehicle density decreases with activated reversible lane (left side). The speed increment in m/sec

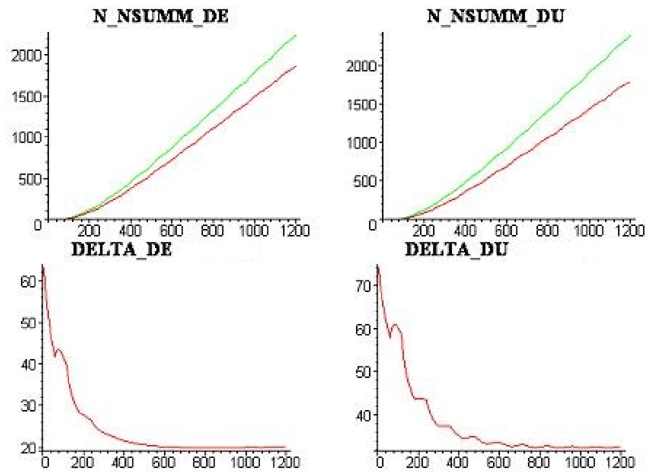

Fig. 3. The number of passing vehicles separated for the morning (DE) and for the afternoon (DU) peaks including percentile growths, with the reversible lane being activated
[the horizontal axis contains the time t [sec], the vertical axis contains the number of vehicles, the lower diagrams are in percent [%]]

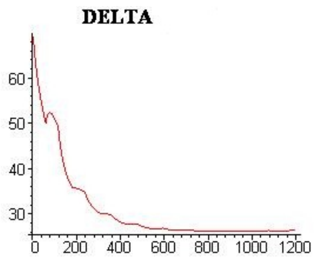

Fig. 4. Band accomplished (morning and afternoon) percental growths, with the reversible lane being activated
[horizontal axis contains the time t [sec], the vertical axis is percent [%]]

(right side) and the percentile improvements can be seen on sections 1,2, and 4 of Figs. 1-3.

These functions were critical in the original system. It it noticeable, that the traffic density on these sections reduced with 25%, 33%, 87 %, though the speed of the traffic has grown with 50%, 48%, 12 % on the same sections.

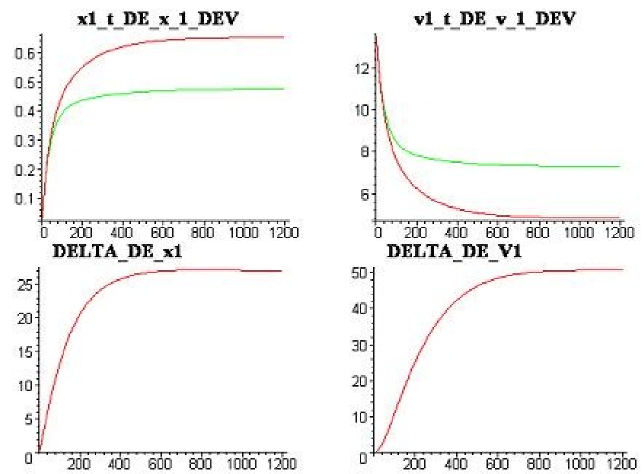

Fig. 5. The percental improvement of vehicle density caused decrease in the morning peak (on the left side) and speed increment (right side) with the reversible lane being activated on the 1 section.

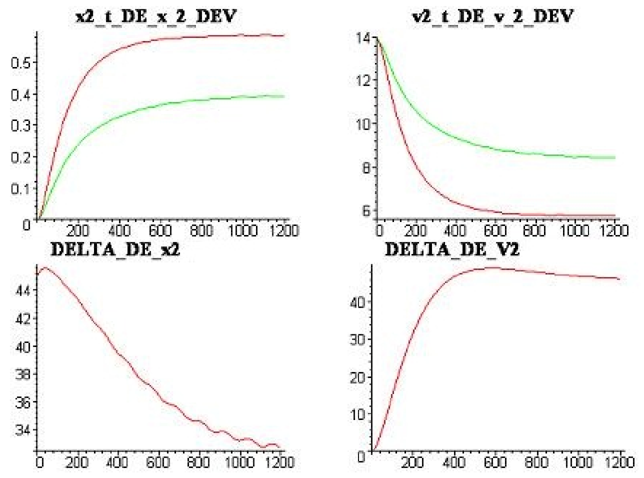

Fig. 6. The percental improvement of vehicle density caused decrease in the morning peak (on the left side) and speed increment (right side) with the reversible lane being activated on the 2 section.

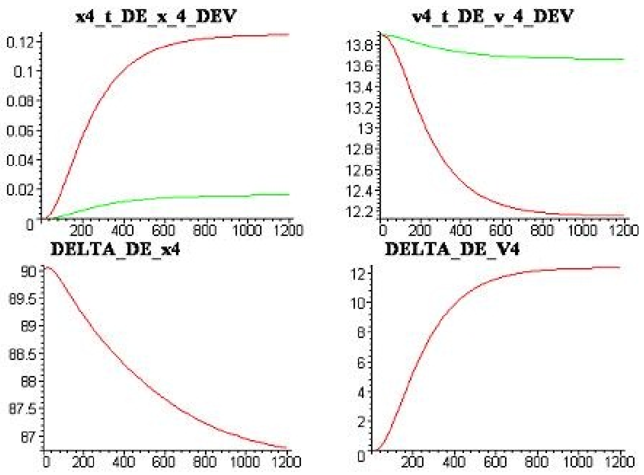

Fig. 7. The percental improvement of vehicle density caused decrease in the morning peak (on the left side) and speed increment (right side) with the reversible lane being activated on the 4 section.

6 Summary

We examined a general mathematical model describing the Reversible Lane System. Our descriptive mathematical network model is a positive non-linear dynamic system, and also important that it is a macroscopic model. The function of every element and the contacts between the elements cease in case of direction change in any part of the network, then new contacts and new functional elements are activated. We examined the availability of the optimal control in a sample network depending on the traffic density, using a new principle, which responses to the dynamic change of the structure of the network graph. It can be shown, that the results from our model are in harmony with the real traffic values based on measurements made in road traffic systems working with Reversible Lane System, included in our literature references.

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