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RESEARCH ARTICLE

Control design for road-friendly suspension systems using an optimal weighting of LQ theorem

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Abstract

The paper comprises the control design of a road-friendly suspension system and the optimal choice of linear quadratic (LQ) weighting. By designing the influence of the dynamic loads of wheels it is possible to reduce the loads and stresses of roads and increase their lifespan. In the vehicle model the dynamics of the sprung and unsprung masses with their suspension system is considered. For road-friendly control design it is necessary to guarantee the reduction of the dynamic load of wheels. Besides, it is also important to guarantee passenger comfort and road stability of the vehicle. Considering these performances an active suspension system based on the LQ control theory is designed. Taking into consideration the effects of the variations in the weighting values of quality performances the properties of controlled system are analyzed in terms of road-friendliness and traveling comfort. The used LQ theorem and optimal weighting method are practicable for both active and semi-active suspension systems.

Keywords

active/semi-active suspension · road-friendly vehicle · passenger comfort · optimal control · weighting functions

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1 Introduction

In the control design of active suspension systems it is necessary to meet several quality performances. The movements of the chassis and the wheels determine the road stability of the vehicle, the traveling comfort and also the mechanical strengths of vehicle structures and those of roads. In case of stability (road holding) the dynamic tyre load and the compression of suspension are the most important physical parameters. For traveling comfort the vertical acceleration and the velocity of the chassis are significant. In every control problem it is necessary to minimize the actuator energy to save energy. In terms of roads the tyre load is the most important parameter, which must be minimized. This list shows that the different quality performances are not independent; the improvement of one influences the others. Therefore in active suspension design it is necessary to create a trade-off between these performance demands.

In this paper a road-friendly active suspension control design is analyzed. Road-friendly quality means that in the design of suspension the interaction between the road and vehicle is considered in terms of the road. In case of heavy vehicles the dynamic tyre load is high and it causes proportionally higher strain on the road. The protection of roads is an important requirement in every country because of the high costs of the road maintenance. Bad road conditions (e.g. potholes, wear of roads) put additional strains on the vehicle and the dynamic tyre load; the process is self-excited. The physical background of road-friendliness is the minimization of tyre load. Static tyre load can be decreased by using lightweight structures and material-saving design. The minimization of dynamic tyre load is possible by using active suspension with suitable control.

The subject of road-friendly suspension design has been considered in several papers. Cebon analyzed the effect of suspension systems using several simulation examples and measured signals (see [1, 2]). Valasek analyzed the semi-active suspension control theorems in the aspect of road-friendliness, see [12]. Gillespie and Pacejka also dealt with dynamic tyre forces [5, 8]. For the control design of active suspension system further methods and theorems are shown in [7, 13, 14].

This paper is organized as follows: Section 2 introduces the control oriented vehicle model for active suspension design. Section 3 contains the method of LQ control design and the choice of optimal weighting. Section 4 shows performances for control design e.g. road-friendliness and traveling comfort. Section 5 is on the extension of control design for semi-active suspension systems. In Section 6 are the detailed the simulation results of the controlled active suspension system. The conclusion summarizes the most important achievements of this paper.

2 Control-oriented vehicle model

For a road-friendly control design it is necessary to formalize a control-oriented vehicle model. Here a five-mass linear vehicle model is used. The five masses are the unsprung masses (the four wheels and suspension), and the sprung mass (vehicle chassis).

The position of the chassis is defined by the vertical displacement of the center of gravity (z_s), the pitch angle (Θ), the roll angle (φ) (Fig. 1). The elements in the suspension system are the springs. Their characteristics are approximated by linear functions. The active actuators can add energy to the system and also dissipate it.

l_f (l_r) are distances between the front (rear) axle and the car chassis at the center of gravity, h_f and h_r are distances between the left (right) wheel and the car chassis at the center of gravity and h_s is the arm of the roll moment. Furthermore, k_{1ij}, k_{2ij} are stiffnesses of the suspension and the tyres, c_{1ij}, c_{2ij} are damping coefficients of the suspension and the tyres.

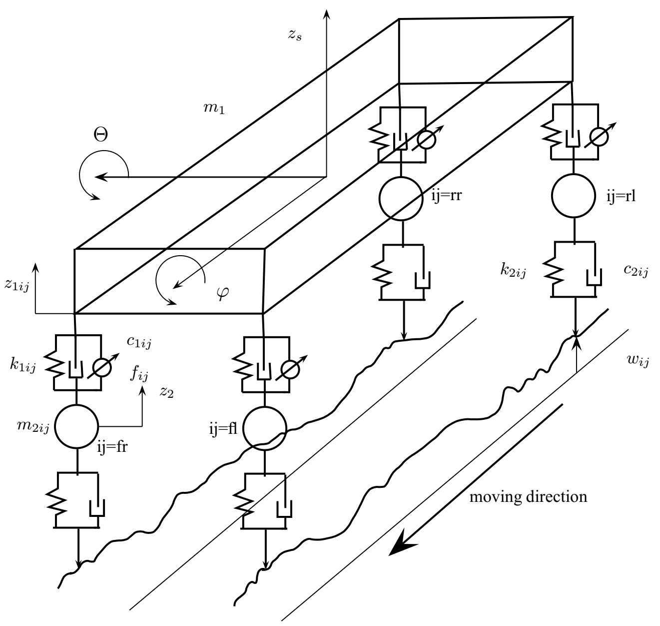

Fig. 1. Suspension model

First the differential equations of vehicle dynamics are formalized, such as the pitch torques, roll torques and vertical forces.

$$I_\Theta \ddot{\Theta} - l_f(F_{1fl} + F_{1fr}) + l_r(F_{1rl} + F_{1rr}) = 0 \tag{1}$$

$$I_\varphi \ddot{\varphi} + h_f(F_{1fl} - F_{1fr}) + h_r(F_{1rl} - F_{1rr}) - m_s g h_s \varphi = 0 \tag{2}$$

$$m_s \ddot{z_s} + \sum f_{1ij} = 0 \tag{3}$$

Then the vertical forces are formalized on wheels and in the suspension systems:

$$m_{2ij} \ddot{z_{2ij}} - F_{1ij} + F_{2ij} = 0, \tag{4}$$

$$F_{1il} = k_{1il}(z_{1il} - z_{2il}) + c_{1il}(\dot{z_{1il}} - \dot{z_{2il}}) - f_{il} \tag{5}$$

$$F_{1ir} = k_{1ir}(z_{1ir} - z_{2ir}) + c_{1ir}(\dot{z_{1ir}} - \dot{z_{2ir}}) - f_{ir} \tag{6}$$

Restoring forces on tyre:

$$F_{2ij} = k_{2ij}(z_{2ij} - w_{ij}) + c_{2ij}(\dot{z_{2ij}} - \dot{w_{ij}}) \tag{7}$$

Suspension compressions depend on the vertical displacement and its rate, and the roll and pitch of the chassis.

3 Design of the optimal weighting of LQ controller

In the suspension design several performance requirements must be met. In this section the harmonization of the requirements of road-friendliness and traveling comfort is analyzed. Road friendliness requires the reduction of the dynamic tyre load and traveling comfort is achieved by reducing the vertical acceleration of the chassis. The optimal controller which fulfils both requirements in an active suspension system is designed by using a linear quadratic control theorem. The LQ controller can efficiently consider simultaneously the different quality criteria using appropriate weighting factors. In the LQ control theorem a cost function, which contains quality performances and input powers, is determined:

$$J = \lim_{T\to\infty} \frac{1}{2}\int_0^T (x^T Q_c x + u^T R_c u + 2x^T N_c u)dt = \\ = \lim_{T\to\infty} \frac{1}{2}\int_0^T \sum q_i k_i^2 dt \tag{8}$$

During the minimizing the quality performances and input powers are taken into consideration. By using weights Q_c, N_c and R_c a balance between the different performance specifications is achieved.

For the evaluation of designed controllers the so-called Dynamic road stress factor (see (12)) and the mean of vertical acceleration of the chassis are computed. By changing the LQ weighting factors controllers with different properties according to the variation of the poles of the controlled system are designed.

By changing the weighting it is possible to design controllers with different properties. By using high weight for a quality performance it is possible to minimize them. However, in this case other quality performances may affected. In Section 1 it has been explained that an active suspension must meet different quality performances. By using high weight at a predefined performance it can be minimized without changing the others. It is not possible to minimize all performances at the same time, but it is necessary to choose a controller with which it is possible to achieve the best minimization of performances.

The properties of controller depend on the LQ cost function weighting. Suppose the different weights are: $q_1, q_2, ..., q_i, ...q_n$. If $n-1$ weights are fixed and the n^{th} weight is increased only one performance is focused on. In the simulations it is possible to compute the values of quality performances; a vector can be assigned to each controller (and simulation case) which contains those values. The vectors of controllers determine an n dimensional curve, which can be written in parametric form [10]. Applying this method to all weights n curves are yielded, on which it is possible to fit an n dimensional surface. By using Gaussian parameters the points of the surface are definitely determined by the weights of performances.

In the knowledge of the surface it is possible to find a controller with the optimal weighting. This method is demonstrated in Fig. 2.

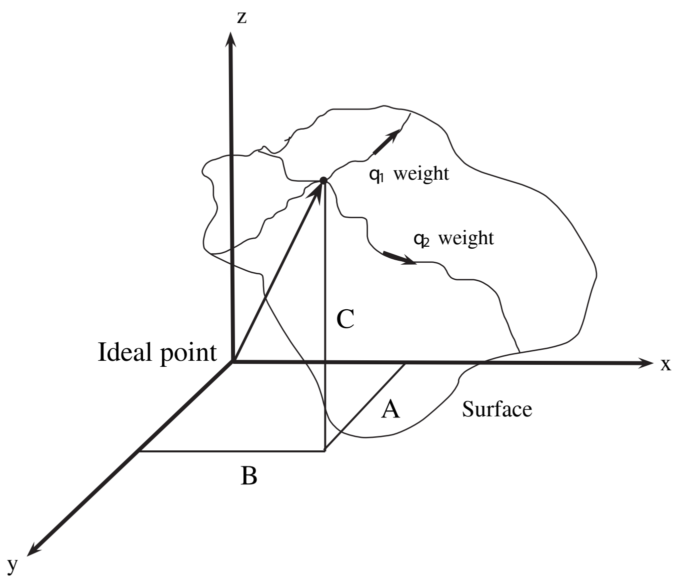

Fig. 2. 3D example of weighting exercise

The exercise is to find the point of the surface which is nearest to the vector of an ideal point (e.g. center of coordinate system). The distance between the center and a point of the surface is:

$$R = \sqrt{A^2 + B^2 + ... + I^2 + ...N^2} \tag{9}$$

If the function of surface $f(A, B, ..., I, ...N)$ is known it is possible to compute the minimum of R by derivation. Then the nearest point can be determined from the Gaussian form of surface and it is possible to obtain the optimal weighting.

In engineering practice, fitting a surface on the curves and computing the derivation of distance R is a hard exercise for reasons of numerical problems and the too complex equation of fitted surface. Instead of fitting and derivation changing the value of weights and computing the distance from center point in every simulation case is recommended. After the computations it is possible to select the smallest distance value, and the weighting of the point. This solution is not the theoretical minimum (optimal weighting), but approximates it very well. This method is similar to the finite element method.

4 Formalization of quality performances of LQ controller

In order to analyze the road-friendly properties of the vehicle several factors are used. While tyre load is randomly distributed in space, statistical methods, e.g. Root Mean Square value of the dynamic tyre force, are used to measure the dynamic tire force. This value alone does not provide sufficient information about road friendliness, therefore the so-called Dynamic Load Coefficient is formalized [1]:

$$DLC = \frac{RMS(F_{dyn})}{F_{stat}} \tag{10}$$

where F_{dyn} is the dynamic tyre force and F_{stat} is the static tire force. Under normal operating conditions DLC is between 0.1 and 0.3 [11].

In 1975, Eisenmann derived a quantity, the road stress factor, see [4]:

$$\Phi = E[F(t)^4] = (1 + 6s^2 + 3s^4)F_{stat}^4 \tag{11}$$

where F(t) is the instantaneous tyre load at the time, s is the coefficient of the variation of dynamic tyre load (essentially DLC), E[ ] means expectation operator. This assumption means that road damage depends on the fourth power of dynamic tyre load. The dynamic road stress factor comes from Eq. (11):

$$\nu = 1 + 6DLC^2 + 3DLC^4 \tag{12}$$

For typical highway conditions of roughness and speed, the dynamic road stress factor is between 1.11 and 1.46 [11].

Eisenmann also proposed a modified version of Eq. (11) which accounted for the effects of wheel configuration and tire pressures using constants, see [3]: η_I accounts for tire configuration (single or dual tires), and η_{II} accounts for tire contact pressures:

$$\Phi' = \nu(\eta_I \eta_{II} F_{stat})^4 \tag{13}$$

To consider the traveling comfort and the road friendliness of the vehicle it is necessary to describe them using a cost function in the LQ design. Traveling comfort is formalized in regulation ISO 2630, which computes a value using the vertical acceleration of the vehicle:

$$D_{\ddot{z}_e} = \sqrt{\sum a_i^2 D_{\ddot{z}_i}^2} \tag{14}$$

In Eq. (14) a_i constant depends on the mean frequency of a frequency range, $D_{\ddot{z}_i}^2$ is the variance of vertical acceleration of the chassis in the frequency range.

In this paper frequency-dependence is ignored, and for characterization of traveling comfort the mean of the absolute value of vertical acceleration is used:

$$E[|a_z|] \tag{15}$$

In order to improve traveling comfort it is necessary to minimize vertical acceleration.

To decrease the dynamic road stress factor it is necessary to minimize dynamic tyre load. Using the notation of Fig. 1 it is described by the following equation:

$$F_{dyn} = k_2 \cdot (z_2 - w) \tag{16}$$

At LQ weighting q_1 means the weight of vertical acceleration and q_2 is the weight of dynamic tyre load.

5 Road-friendly design of semi-active suspension

In the previous sections a road-friendly active suspension has been designed. Nowadays the significant majority of automobiles have passive suspension for financial reasons. In this case there is not any actuator in the suspension system, the properties of suspension are determined by spring and damper. In semi-active suspension system the damping coefficient of suspension is changeable electrically: the damper have different characteristics and it is possible to switch between them (Fig. 3). The most common physical systems are magneto-rheological and solenoid valve dampers [6].

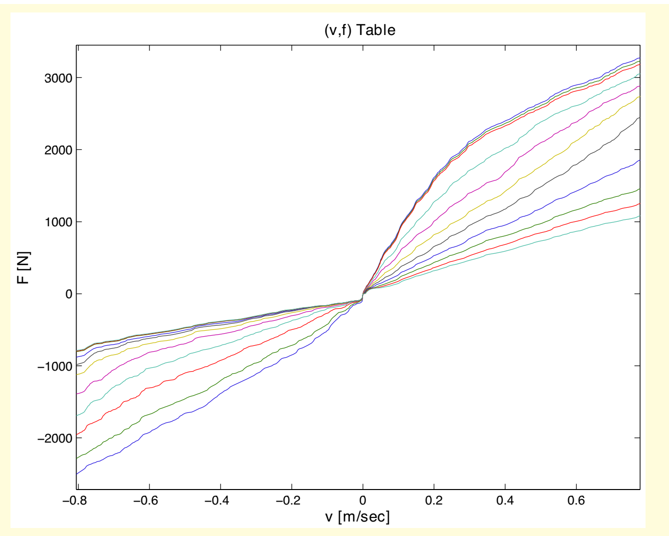

Fig. 3. Velocity-damping force table

The most significant difference between active and semi-active suspensions is their energy actuation. While active suspension can both add to and take energy out of the system, semi-active can only do the latter; this makes the control design more difficult.

In the realization the semi-active suspension controller can compute some actuator forces, which are not possible to realize, because the damper can only dissipate energy, and it can be done between some limits (see Fig. 3). A possible way to solve this problem is to add force when it is possible and to rely on the passive solution when no force can be added [9]. There are several control design methods, see e.g. [12].

In case of a LQ controller the previous model can be used (see Section 2). In the semi-active case the required active force is necessary to convert the damping coefficient such that the damping force is closest to the desired one. The damping coefficient has minimum and maximum limits and the number of achievable damping characteristics is finite. The method of finding an optimal weighting of a semi-active LQ controller is the same as in Section 3. The optimal weighting of an active suspension LQ controller can be different from semi-active, because of the constrains of the damping coefficient.

6 Simulation results

In the simulation examples according to the stochastic road surface the road excitations under each wheel differ. The simulations are run with several active and semi-active suspension controllers. In all cases the dynamic road stress factor of the front left wheel (Eq. (12)) and the mean of absolute value of vertical acceleration (Eq. (15)) are computed. Considering that the minimal dynamic road stress value is 1 and for vertical acceleration this is 0, these values determine the ideal point. The coherent $\nu - a_z$ values with the ideal point are plotted in one diagram for both active and semi-active suspensions (see in Fig. 4).

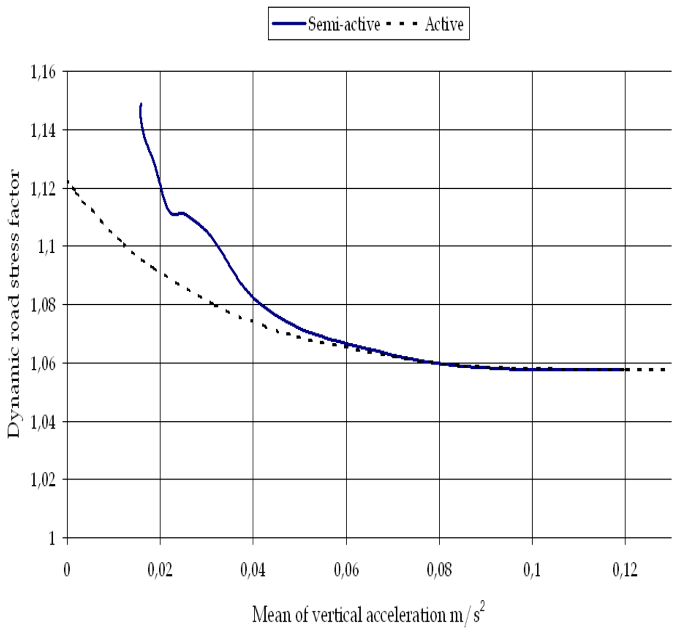

Fig. 4. Ideal point (center of coordinate system) and $\nu - a_z$ values

It can be seen that the curve for active and semi-active suspension differ from each other: the values of the active are better than those of the semi-active. This result clearly shows that both energy addition and dissipation meets quality performance requirements better. Hereafter the results of active suspension controller are detailed. According to Section 3 a curve can be fitted to the points:

$$\nu = -3859.2a_z^5 + 1676.9a_z^4 - 302.49a_z^3 + 31.378a_z^2 - 2.0679a_z + 1.122 \quad (17)$$

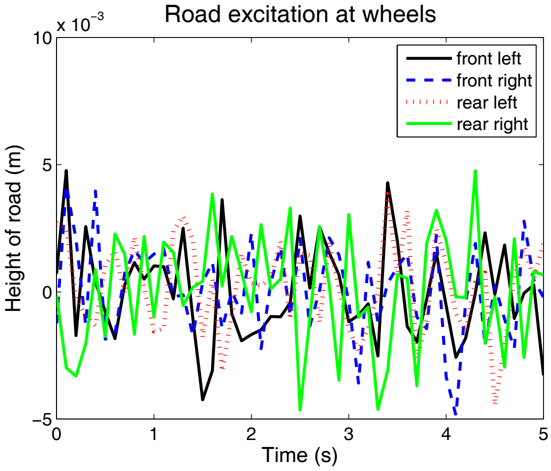

Fig. 5. Road excitation

Based on computations the nearest point to the ideal point corresponds to $q_1 = 2 \cdot 10^5$ weighting value. It means that a controller using this q_1 weighting value guarantees the best balance between passenger comfort and road-friendliness. Using $q_1 = 2 \cdot 10^5$ and $q_2 = 1$ weights the active suspension model is simulated. The dynamic road damage factor is $\nu = 1.0814$.

The road excitation is different on the wheels and they are stochastic signals with 5 mm maximal amplitude, see Fig. 5. In the simulation the vehicle moves straight at a constant velocity, which results in the dynamic tyre loads. The dynamic tyre load of the front-left wheel is in Fig. 6. In order to increase road-friendliness it is necessary to decrease this value.

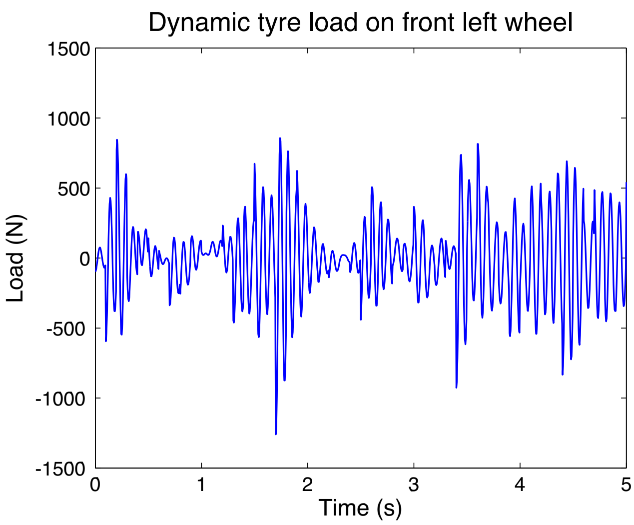

Fig. 6. Dynamic tyre load

Fig. 7 shows the compression of the front-left suspension. This value gives information about the roll of the vehicle: in the simulation its maximal value is 10 mm.

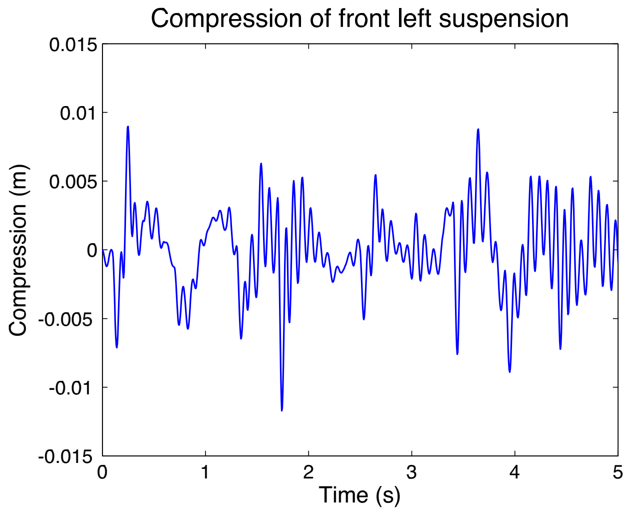

Fig. 7. Suspension compression

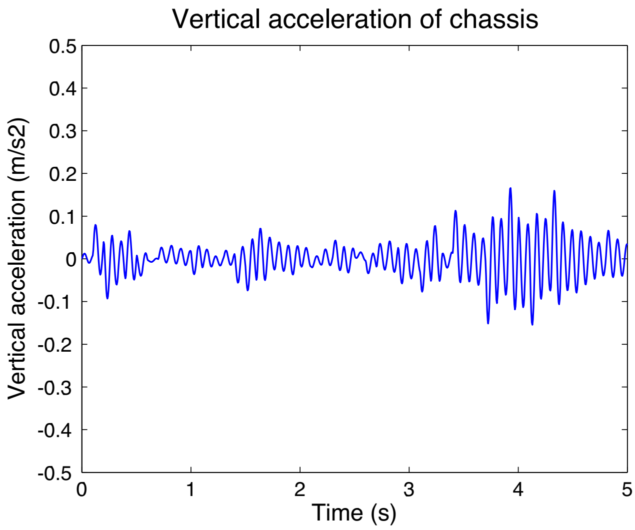

Fig. 8. Vertical acceleration of chassis

A purpose of finding optimal weighting is to minimize vertical acceleration while maintaining road-friendliness. Fig. 8 shows the vertical acceleration of the chassis, the minimization is realized well and the maximal acceleration is 0.2 m/s^2. It means 2 % of gravitation acceleration.

7 Conclusion

In this paper a road-friendly control design of an active suspension using LQ control theorem is presented. The control design is based on a linear model of the full vehicle and the controller actuates at each wheel. In order to meet the different performance requirements for vehicle suspension it is necessary to choose appropriate weighting factors. To find the optimum a special strategy using several simulations is used. By assigning an ideal point the optimal weighting between traveling comfort and road-friendliness is achieved. The method of finding the optimal weighting is also practicable for semi-active suspension control design.

References

1 **Cebon D**, *Handbook of vehicle road interaction*, Swets and Zeitlinger B.V., 1999.

2 **D. J. Cole, D. Cebon**, *Predicting vertical dynamic tyre forces of heavy trucks*, Proc. Engineering Foundation Conference on Vehicle-Road Interaction, Santa Barbara, 1992.

3 **Eisenmann J**, *Beurteilung der Strassen – Beanspuchung*, Strasse und Autobahn **3**.

4 ______, *Dynamic wheel load fluctuations - road stressg*, Strasse und Autobahn **4**.

5 **T.D. Gillespie**, *Fundamentals of vehicle dynamics*, Society of Automotive Engineers Inc., 1992.

6 **Giua A, Melas M, Seatzu C, Usai G**, *Design of a predictive semi-active suspension system*, Vehicle System Dynamics **41**.

7 **Mehra R K, Amin J N, Hedrick K J, Osorio C, Gopalasamy**, *Active suspension using preview information and model predictive control*, Proceedings of IEEE International Conference on Control Applications, Hartford, 1997.

8 **Pacejka H B**, *Tyre and vehicle dynamics*, Butterworth-Heinemann, 2006.

9 **Poussot-Vassal C, Sename O, Dugard L, Gáspár P, Szabó Z, Bokor J**, *A new semi-active suspension control strategy through LPV technique*, Control Engineering Practice.

10 **Rade, Lennart, Westergren, Bertil**, *Mathematics Handbook for Science and Engineering*, Springer Verlag, Berlin, 2004.

11 **Sweatman P F**, *A study of dynamic wheel forces in axle group suspensions of heavy vehicles*, Australian Road Research Board, Special Report SR27, 1983.

12 **Valasek M, Kortüm W, Sika Z, Magdolen L, Vaculin O**, *Development of semi-active road-friendly truck suspensions*, Control Engineering Practice, 1998, pp. 735–744.

13 **Wenger L, Borelli F**, *Predicting vertical dynamic tyre forces of heavy trucks*, Proceedings of the 41st IEEE Conference on Decision and Control, 2002, pp. 881–886.

14 **Zin A, Sesame O, Gáspár P, Dugard L, Bokor J**, *An* LPV/H_∞ *active suspension control for global chassis technology: Design and performance analysis*, American Control Conference, 2006, pp. 2945–2950.