

# Traffic control designing using model predictive control in a high congestion traffic area

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## Abstract

The paper investigates a designing method for urban traffic management system. A busy traffic area was chosen for test field in the 10<sup>th</sup> district of Budapest. The control algorithm is based on model predictive control (MPC). The control aim is to relieve traffic congestion, reduce travel time and improve homogenous traffic flow. Theory and realization details of the control method are also presented.

The MPC based control strategy was implemented into the test network's management system. The applied environment contains microscopic traffic simulator, scientific mathematical software and some computational applications for the evaluation. The simulation results show that the system is able to ameliorate the network efficiency and reduce travel time. The designed MPC based traffic control strategy proves effectiveness by creating optimal flow in the network subjected to control input constraints.

## Keywords

road traffic control · model predictive control · traffic simulation

## 1 Introduction

Road traffic congestion is a well-known symptom all over the world. People have to face traffic jams day by day on freeway just like in urban transportation network. Knowing the tendency of rate of motorization, it is obvious that congestions mean a growing problem. The capacity of the roads saturates in rush hours. At the same time, the traditional control measures are getting less effective to manage the traffic flow. As a result, external costs e. g., time of delay, pollution, stress increase.

In case the distance is relatively short between several intersections with traffic lights, it is advisable to co-ordinate the operation of the intersection controller devices. The co-ordination may include public transport devices and pedestrian traffic too, besides vehicles. Where several intersections are near to each other in smaller or bigger networks, primarily in cities, the co-ordination is especially emphasized.

The development of new control strategies is a real demand of nowadays. One of the possible solutions is the practical application of modern control theory. In case of designing urban traffic control by using state space theory, it is advisable to choose a simple, possibly linear model. A model like this is the store-and-forward approach [5] which describes the queue building before the stop line. This model is also the base of Papageorgiou's urban control strategy in the TUC model [2]. The TUC system implements an LQ control algorithm which is not able to manage the upper and lower constraints of the control input. In this case these constraints come from the green time's limits. A possible solution can be the use of MPC [4] which is able to take these constraints into consideration.

The stability problem of MPC based control can be solved by correct designing [9]. MPC has been successfully applied to motorway traffic management [6].

Another problem appears in the models based on store-and-forward approach. Turning rates of the intersections influence strongly the validity of the model. These rates are usually supposed to be constant and known. Several methods are known for estimating split rates, for example, the Kalman filter [19] which is used most frequently. Our method (cMHE) [7] can take the constraints concerning the rates into consideration.

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In the paper we present an urban traffic control design based on MPC. The aim of the control is to increase capacity. To test and validate our control strategy we applied it to a real-world transportation network where the actual system is not efficient enough to manage the traffic in rush hours. The new system realizing a coordinated control for all traffic lights was designed for the test, depending on the actual traffic situation. The simulation results show the effectiveness of the procedure compared with the old system. The MPC based strategy is more suitable to optimally manage the traffic network by minimizing the number of vehicles waiting at the stop line.

## 2 Control system development of the test network

The test area is situated in the 10th district of Budapest. We chose seven neighbouring intersections that we considered a transportation network shown in Fig. 1. This area is suitable for

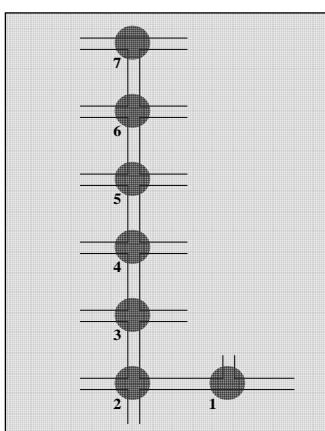


Fig. 1. Schematic representation of the test network consisting of seven junctions

testing our new control system since the included road stretches have a heavy traffic volume in rush hours, especially three streets of the network which are the most frequent routes for the transit traffic of the district. The current traffic management system is offline. The traffic lights work autonomously. Three of them use fix time signal plan and the others work by partly depending on the actual traffic situations.

### 2.1 Development of the control system

At present the seven junctions are controlled individually. The controllers are operating without taking the states of the others into consideration. Three of them are managed by fix programs. In the other four intersections detectors help the controllers. They can partly modify their fix programs but are only partly dependent on traffic volume.

The current control is effective but only in case of normal traffic flow. If the volume of vehicles increases extremely, the system cannot manage the situation and traffic becomes congested before the stop lines. The biggest problem is that the controllers work locally and independently. Our new control design, however, takes the seven junctions into consideration as a real network. The aim is to minimize the queue lengths at the

stop lines and to maximize the number of vehicles passing the intersections. The representation of the control system is shown in Fig. 2.

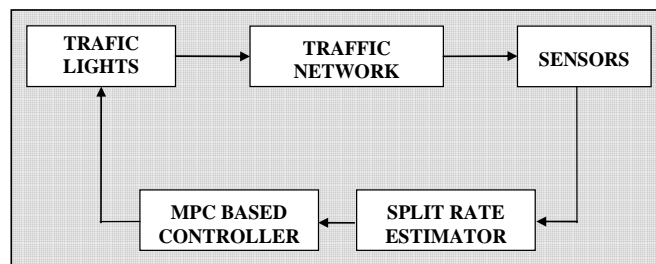


Fig. 2. Coordinated application of MATLAB and VISSIM. The control loop of the traffic control system. The actuators are the traffic lights in the junctions. Sensors mean loop detectors or cameras.

### 2.2 Development steps

The development consists of five steps. To develop and test the new traffic management strategy we need to build up the model of the control and the network model step by step. The final goal is to realize the system in practice.

- 1 First, we designed the model of the traffic network by using VISSIM [17] microscopic traffic simulator. This software is suitable for exact modelling. We reproduced the network based on the real geometry and traffic parameters.
- 2 As the second step of the development we realized the coordinated application of MATLAB [10] and VISSIM (Fig. 3). The MPC based control model was constructed in MATLAB,

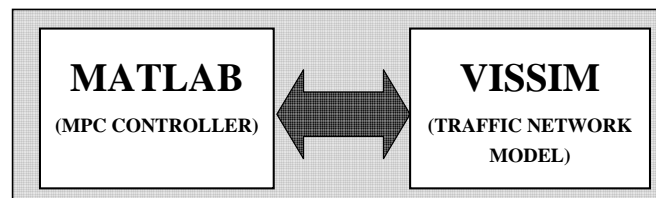


Fig. 3. Coordinated application of MATLAB and VISSIM.

due to the high computational need and complexity of the calculation. The controller and the network model communicate with each other. After MATLAB receives the measurement data of the traffic parameters, it computes the new phases for the traffic lights and returns them to VISSIM.

- 3 In the third phase we realized the communication between MATLAB and a real local traffic controller machine (Fig. 4).

In fact, this communication takes place mostly between the controller ACTROS and the PC, since MATLAB can reach files in the computer memory. ACTROS [1] includes an industrial computer running JAVA applications. In order to transfer data, a DOS application was written, which controls FTP communication between the controller and PC.

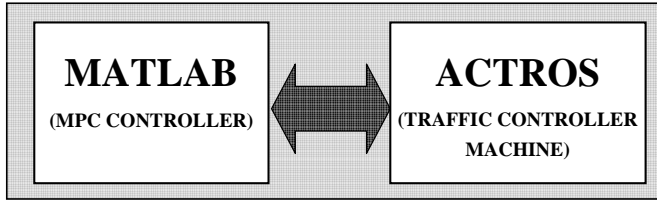


Fig. 4. The MATLAB – ACTROS system

### 3 MPC based control strategy

The aim of our research was to elaborate a control process related to networks consisting of several junctions which perform the control of all the traffic lights in its sphere of action in a coordinated way depending on the traffic. The controller must be able to dynamically make the traffic signal set of the intersections. From the point of view of realization, this means that before every period a new traffic sign must be generated regarding all the traffic lights, in harmony with the present traffic. To solve the above, one must choose a method in which it is possible to take all the constraints into consideration in course of the control input setting. Regarding the above, the procedure based on model predictive controller is suitable. In this algorithm the minimization of the number of vehicles waiting at the stop line can be the aim.

#### 3.1 The state space model

The state space traffic flow model contains the store-and-forward model (Fig. 7). It is similar to the model used in TUC [2],[3] urban traffic control strategy.

Fig. 7 shows the coherence ( $z$  link) of two neighbouring intersections ( $M, N$ ) in the transportation network where  $q_z$  is the inflow,  $h_z$  is the outflow,  $d_z$  and  $s_z$  are the demand and the exit flow. In the case of traffic networks with multiple ramifications,

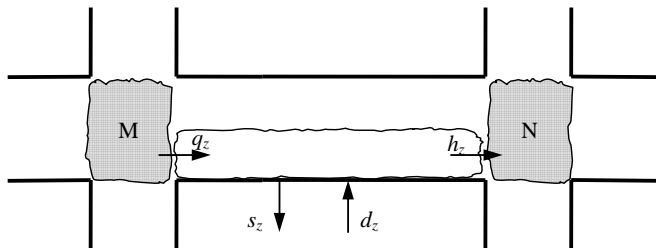


Fig. 5. Store-and-forward model

for the portrayal of vehicles movement in the space state, one proposed the following discrete time stochastic LTI state equation:

$$x_z(k+1) = x_z(k) + T \left[ (1 - \kappa_{z,0}) \sum_{w \in I_M} \alpha_{w,z} \frac{S_w \sum_{i \in v_w} g_{M,i}(k)}{C} - \frac{S_z \sum_{i \in v_z} g_{N,i}(k)}{C} \right], \quad (1)$$

where  $x(k)$  is the state vector, representing the number of vehicles standing in a certain branch of the intersection, green time

$g(k)$  is the control input,  $\alpha_{w,z}$  are the entering turning rates which have been estimated,  $\kappa$  represents the fixed and known exit rates.  $S$  denotes saturation flow,  $C$  means the cycle time,  $T$  is the control interval, and  $k = 1, 2, \dots, n$  is the discrete time index. Index  $j$  denotes the junction identifier, index  $i$  means the stage.

The cycle time is fixed, the lost time of each junction is fixed and it means that the sum of the green times is also fixed in each junction.

The demands to enter at the boundaries of the network are considered as measurable fault/noise:

$$x(k+1) = Ax(k) + Bg(k) + x_{in}(k) + w(k) \quad (2)$$

$$y(k) = Cx(k) + v(k), \quad (3)$$

where  $x_{in}$  means the number of input vehicles,  $w$  is the sum of the non-measurable fault/noise, while  $v$  is the measurement noise (zero mean random components).

#### 3.2 The controller

The control objective of TUC is the minimization and balancing of the numbers of vehicles within the streets of the controlled network. This control objective is approached through the appropriate manipulation of the green splits at urban signalized junctions, assuming given cycle times and offsets. The TUC has some alternative control laws but the main concept is based on LQ and LQI control theory.

However, a control solution had been searched for which was able to satisfy the following constraints on the control input [16]:

$$u_i \geq t_{MIN} \quad \forall i \quad (4)$$

$$u_i \leq t_{MAX} \quad \forall i \quad (5)$$

$$\sum_{i=1}^{O_j} u_i \leq t_j^{MAX} \quad j = 1 \dots J, \quad (6)$$

where  $O_j$  is the number of vehicles' columns in intersection  $j$ ,  $J$  is the number of controlled intersections.

By employing the predictive control model, the dynamic determination (per cycle) of the traffic light's period is possible either with the consideration of the natural constraints existing in the system.

A method had been elaborated for designing an MPC controller which minimizes the number of vehicles in queue. The controller ensures the control input in order to minimize the next functional whereas satisfying dynamic equation (1), measurement equations (2) and constraints (4), (5), (6):

$$J(k) = \frac{1}{2} \sum_{i=1}^{N_p} \left( x_i^T(k) Q x_i(k) + u_i^T(k) R u_i(k) \right), \quad (7)$$

where  $N_p$  is the length of the predictive horizon,  $x_i(k)$  is the state vector representing the number of vehicles standing in a certain branch of the intersection and  $u_i(k)$  is the control input (green time).

Generally speaking, Q and R are appropriately chosen tuning parameters to adjust the MPC performance.

The following appropriate dimensional and diagonal weights are applied:

$$\begin{aligned} Q &= \text{diag}(1) \\ R &= \text{diag}(0.1) \end{aligned} \quad (8)$$

The weightings reflect that the control input variation is lightly punished, compared to the state variation.

The selection of the appropriate weighting matrices ( $Q \geq 0$ ,  $R > 0$ ) is important, because this could influence (especially the end-point weight) the stability of the closed loop [8].

Different stability proofs exist for receding horizon control algorithms. However, [9],[11] offer different methodological approaches, one prefers using a predefined terminal set (based on the solution of the Algebraic Riccati Equation of a steady state LQ feedback problem) [19],[13]. The terminal set is subjected to the control input's constraints. Therefore, the solution of the finite horizon minimization can be interpreted as an optimal state feedback driving the closed loop into an invariant set.

### 3.3 Estimation of turning rates (OD)

One of the most important tasks is to define the dynamic OD (Origin Destination) matrix. Split variables are independent trials. The LTI model and its constraints are given by:

$$x_{ij}(k+1) = x_{ij}(k) + w_{ij}(k), \quad (9)$$

where  $x_{ij}(k)$  is the split rate or turning rate that shows the percentage of vehicles coming from direction  $i$  and exiting to direction  $j$ ,  $k=1,2,\dots,N$ ,  $w_{ij}(k)$  is the state noise. The random variation in the split parameter is small and  $w_{ij}(k)$  is a zero mean random component. All random components are mutually independent terms.

In the system the number of vehicle entering and leaving, is measured, the measuring equation is given by:

$$y_j(k) = \sum_{i=1}^n q_i(k)x_{ij}(k) + v_j(k) \quad (10)$$

where  $q_i(k)$  represents the traffic volume (the number of vehicles) entering the intersection from entrance  $i$ , during time interval  $k$ ,  $y_j(k)$  is the traffic volume (the number of vehicles) leaving the intersection from exit  $j$ ,  $i = 1,\dots,n$  and  $j = 1,\dots,m$ ,  $v_j(k)$  is a zero mean noise term. The input measurement is a noisy term where  $\zeta_i(k)$  is a zero mean noise term.

The problem is to observe the  $x_{ij}(k)$  states under certain conditions. The fact that constraints have to be taken into consideration makes the task difficult. In our case, two types of constraints (inequality and equality) are applied, but further constraints may be implemented. When using state estimation, constraints are

difficult to be inserted into the observer because this would lead to a constraints state estimation problem.

A possible way to estimate states is to apply a finite back stepped state observer. A class of optimal state estimation methodologies is called Moving Horizon Estimation (MHE)/Receding Horizon Estimation (RHE) method. The technique estimates the expected states values in order that functional  $\Psi_k$  should be minimized, while subjected to the dynamic equation (13) as the measurement equation (14).

$$\min_{(\bar{x}_{k-N-1}, \hat{w}_{k-N-1|k}, \dots, \hat{w}_{k-1|k})} \Psi_k \quad (11)$$

$$\begin{aligned} \Psi_k &= \hat{w}_{k-N-1|k}^T Q_0^{-1} \hat{w}_{k-N-1|k} + \\ &+ \sum_{j=k-N}^{k-1} \hat{w}_{j|k}^T Q^{-1} \hat{w}_{j|k} + \sum_{j=k-N}^{k-1} \hat{v}_{j|k}^T R^{-1} \hat{v}_{j|k} + \Psi_{k-N}^* \end{aligned} \quad (12)$$

subject to the following dynamic equality constraint:

$$\begin{aligned} \hat{x}_{j+1|k} &= A\hat{x}_{j|k} + G\hat{w}_{j|k} \\ j &= k-N-1, \dots, k \end{aligned} \quad (13)$$

and the following measurements:

$$\begin{aligned} y_j &= C\hat{x}_{j|k} + \hat{v}_{j|k} \\ j &= k-N-1, \dots, k \end{aligned} \quad (14)$$

the initial value:

$$\hat{x}_{k-N|k} = \bar{x}_{k-N} + \hat{w}_{k-N-1|k}. \quad (15)$$

One needs to note that output map  $C$  is time dependent, since the elements of  $C$  are the input measurements. Henceforth, one defines the supplementary equality and inequality constraints coming from the geometry of the intersection.

## 4 Simulation and results

VISSIM was chosen for modelling. This software is a microscopic traffic simulator for analyzing traffic operations. It is able to simulate network consisting of several intersections and allow the use of external control algorithm in the control processes. These properties make it suitable to use this software by reason of the several junctions and the control algorithm written in MATLAB. VISSIM uses a so-called psycho-physical driver behaviour model based on the car-following model of Wiedemann [18]. The model is microscopic, so it describes all the cars found in the system. The vehicles are defined by the parameters such as origin, destination, speed, driver behaviour, vehicle type etc. The simulation is an iteration process of acceleration and deceleration.

### 4.1 Software requirements

Though for VISSIM models one can write control logic program using the optional module Vehicle Actuated Programming

[17], we created the MPC based control design in MATLAB, due to the complexity of computing.

The communication does not work directly between MATLAB and VISSIM because the last one can be accessed from outside just using Component Object Model (COM) interface [14]. To control the communication a Microsoft Visual C++ [12] application was created. The program can reach VISSIM via COM and transfer simple text files for MATLAB [15].

#### 4.2 The simulation process

The created C++ application starts simulation process and controls the data transfer between the software (Fig. 6). The program gets traffic measurements via COM from VISSIM and forwards them to MATLAB, using text files. After minimizing the cost function, the new control signals get back into the traffic simulation environment on the same path.

The process runs every 60 s while VISSIM is working continuously. The computing time is not significant. The data query from VISSIM is approximately 10 s, and the computing time of the controller (in MATLAB) is 5 s.

#### 4.3 Simulation results

To prove the applicability of the MPC based control design it was compared with the current control system of the test network, which is a partly adaptive control strategy.

The same input traffic volumes were set for both simulations. We used volume data for which the traffic lights were originally designed. The simulation provided similar results for both strategies as we expected. This means the current management system is correctly designed, and can control non-extreme traffic flow with good results.

To test the effectiveness of the two systems in case of heavier traffic we generated more intensive traffic flow during the simulation. The original input volumes were increased by 10 % in the network. This simulation gave different results to the previous case. The current system could manage the traffic less effectively compared with the MPC based control system. The simulation time was 1 hour. The results are presented in Table 1. All important traffic parameters changed in a right way. The new system can provide a very effective control in the test network.

**Tab. 1.** Simulation results of the seven junctions

Parameter	Offline strategy	MPC based strategy	Change
Number of vehicles that have left the network	5278	5394	↑ 2.2 %
Total travel time per vehicle [sec]	114	96	↓ 16 %
Average speed [km/h]	20.6	24.9	↑ 21 %
Average delay time per vehicle [sec]	68	56	↓ 18 %
Average number of stops per vehicles	3.8	3.1	↓ 18 %

At the same time these simulations were run in a reduced environment. We diminished the number of junctions in the test network from seven to four. Namely the traffic lights at junctions 4, 5, 6 (see Fig. 1) work totally offline. The capacities of these locations increased apparently. So only the junctions 1, 2, 3, and 4 were kept in order to focus on the comparison of the two adaptive strategies.

**Tab. 2.** Simulation results of the four junctions with design input volumes

Parameter	Offline strategy	MPC based strategy	Change
Number of vehicles that have left the network	9532	9716	↑ 2 %
Total travel time per vehicle [sec]	105	96	↓ 9 %
Average speed [km/h]	20.5	23.5	↑ 15 %
Average delay time per vehicle [sec]	64	52	↓ 19 %
Average number of stops per vehicles	1.2	1.2	↓ 0 %

**Tab. 3.** Simulation results of the four junctions with 10% augmentation of the design input volumes

Parameter	Offline strategy	MPC based strategy	Change
Number of vehicles that have left the network	9692	9876	↑ 2 %
Total travel time per vehicle [sec]	110	96	↓ 13 %
Average speed [km/h]	18.4	23.6	↑ 28 %
Average delay time per vehicle [sec]	71	52	↓ 27 %
Average number of stops per vehicles	1.5	1.2	↓ 20 %

Alike above, the behaviour of the reduced network was analyzed with normal and heavier input traffic volumes. The results ameliorated in both cases (see Table 2 and 3). The simulation time was 2 hours.

The aim of the MPC based control is the minimization of the number of vehicles waiting at the stop line. The current system cannot adapt to the increased volume. The average queue length grew strongly during the simulations. However, the MPC strategy is able to manage heavier traffic situations real-time Fig. ?? represents unambiguously the effectiveness of our system. It shows the variation of average queue lengths in the network.

## 5 Conclusion

Novadays several alternative urban traffic management methodologies are in use. However the existing methods cannot incorporate physical constraints of the system. To solve this problem a model based and constrained predictive terminology is applied. A new traffic control system has been developed which is able to take the green time limits of traffic lights into

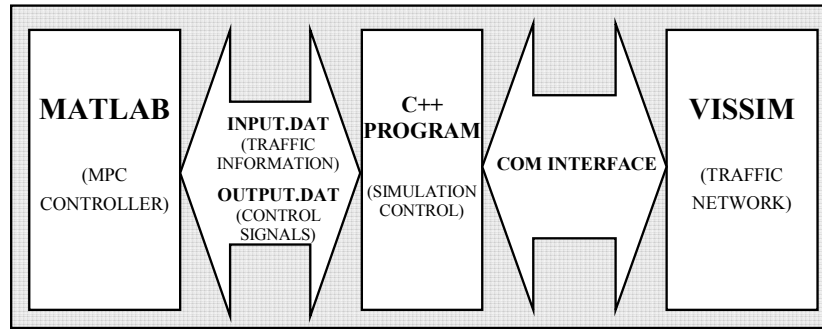


Fig. 6. The simulation process of the system model

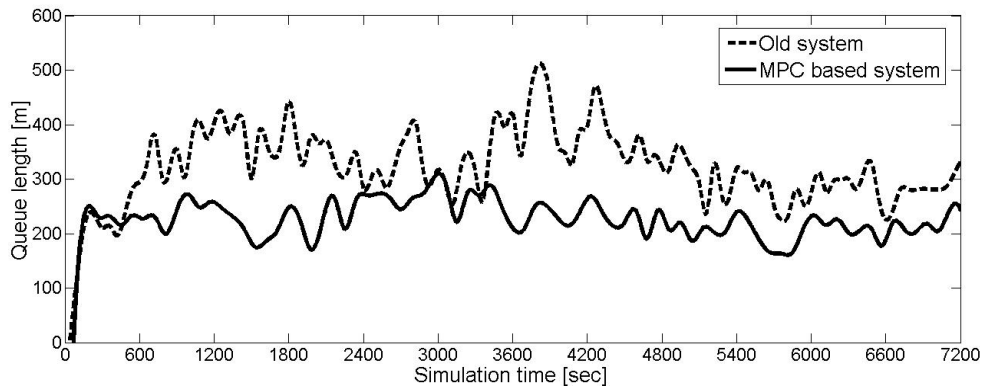


Fig. 7. The variation of average queue lengths in the two different control cases

consideration. Furthermore, the state feedback policy has been completed with dynamical and constrained state estimation.

The strategy was implemented in real-world transportation network, and validated in closed loop simulation environment using real data and parameters acquired from the test traffic network. The hardware in the loop simulation shows that the MPC based control fits perfectly to the urban traffic control problem. Two traffic volume scenarios were used for the simulations. In both cases the network's traffic became more smooth and extreme queue building could be avoided. The results represent significant efficiency especially in case of increased traffic.

Generally speaking, the designed strategy can provide effective traffic flow in traffic network, reducing the total travel times and preventing the extreme queue building at stop lines.

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