Abstract

The trigonal Wankel engine is kinematically based on the motion where a circle $p_m$ of radius $3d$, as the moving pole curve, rolls on the circle $p_r$ of radius $2d$, as the standing pole curve in the interior. Then the regular trigonal rotor with circumcircle of radius $\rho > 3d$, fixed concentrically to the moving pole circle, describes its orbit curve $c_\rho$. This orbit curve $c_\rho$ is crucial in forming the engine space.

Answering a question of István Revuczky, we prove (and animate by computer) that $c_\rho$ is a convex curve iff $\rho$ bigger or equal to $9d$. The parallel curve $c_{\rho+r}$ with distance $r$ will be the solution to the engine space if the triangle rotor touches small roller circles of radius $r$ centred in the vertices of the triangle.

All these concepts will be generalized – with animation – to a $k$-gonal rotor ($2 < k$ natural number) in a unified way.

Keywords

Wankel engine · kinematical geometry · trochoidal motion

Introduction

We continue the abstract with a few of historical remarks, for these we thank engineer professor colleagues István Emőd and Liviu Finichiu, furthermore Professor Hellmuth Stachel (Vienna University of Technology).

After studying the literature of the topic (e.g. [1], [8] and cited references there) it turned out that Wankel himself thought of this generalization and of other peculiar constructions. Although Wankel engine is not effective enough, its mechanism is attractive still nowadays.

About 1970 the envelopes of trochoids under trochoidal motion were intensively studied by Austrian and German geometers. Here we cite only [3], [6], [9] and the references there. The phenomenon of convex engine space seems to be new in this paper, and may be the simple short derivation of different possibilities by mathematical tools cause some enjoyments for the interested reader.

Optimization of such type of Wankel engines seems to be an open problem yet!

1 Formulation of the problem, results

Let a regular $k$-gon, with centre $Q$ and circumcircle of radius $\rho$, glide on an orbit curve $c_\rho$ of centre $O$ with similarity parameter $OQ=d$ constant. Then any vertex $P_i$ moves by time $t$ as follows:

$$\overrightarrow{OP_i(t)} = \overrightarrow{OQ(t)} + \overrightarrow{Q(t)P_i(t)} = d \overrightarrow{e(kt)} + \rho \overrightarrow{e(t)}.$$  \hspace{1cm} (1)

Here we assume unit angular velocity for unit vector $\overrightarrow{e(t)}$ of angle $t$ (with $1 = \overrightarrow{e(0)}$). $\overrightarrow{Q(t)} = d \overrightarrow{e(kt)}$ is then a good choice for the requirements that $P_i$ describes the same curve $c_\rho$ for $i = 1, 2, \ldots, k$. Indeed, the substitutions

$$t' \mapsto t + \frac{2\pi i}{k}, \quad i = 1, 2, \ldots, k$$  \hspace{1cm} (2)

lead to the same geometric description in Eq. (1). In the or-
thornormal coordinate system \((O; \mathbf{i}, \mathbf{j})\) the Eq. \((1)\) will have the form for the vectors \(c_p(t)\) pointing to the points of the so-called trochoid curve \(c_p\):

\[
\begin{align*}
c_p(t) : x &= d \cos kt + \rho \cos t, \\
y &= d \sin kt + \rho \sin t, \\
\end{align*}
\]

and \(\dot{c}_p(t) : \dot{x} = -kd \sin kt - \rho \sin t, \quad \dot{y} = kd \cos kt + \rho \cos t; \quad \ddot{c}_p(t) : \ddot{x} = -k^2d \cos kt - \rho \cos t, \\
\ddot{y} &= -k^2d \sin kt - \rho \sin t
\]

express the velocity and acceleration vectors of \(c_p(t)\), respectively. Thus the absolute value of the velocity is

\[
|\dot{c}_p(t)| = \sqrt{k^2d^2 + \rho^2 + 2kd \rho \cos (k - 1)t} > 0,
\]

for any \(t\), if \(\rho > kd\).

Therefore the parallel curve \(c_{p+r}\) can be given by coordinates

\[
X = \frac{1}{|\dot{c}_p(t)|} \left[ d \left[ |\ddot{c}_p(t)| + rk \right] \cos kt + \rho \left[ |\ddot{c}_p(t)| + r \right] \cos t \right],
\]

\[
Y = \frac{1}{|\dot{c}_p(t)|} \left[ d \left[ |\ddot{c}_p(t)| + rk \right] \sin kt + \rho \left[ |\ddot{c}_p(t)| + r \right] \sin t \right].
\]

This \(c_{p+r}\) and \(c_p\) as well will be of non-negative signed curvature, and both will be convex by Eq. \((3)\), iff

\[
\rho \geq k^2d.
\]

Thus, for \(k = 3, \ d = 1, \ \rho = 9\), e.g. \(r = 1\), we shall have an ellipsis like motor, called \(\text{elliptor}\) by István Revuczky (Fig. \(\text{2}\)).

\[
a := \rho + d + r = 11, \quad b := \rho - d + r = 9
\]

will be the half axes of the elliptor that contains the corresponding ellipsis.

This last fact does not fit Revuczky’s imagination, but convexity may have advantages for constructing a new type of engine. The profile of rotor can also be varied so that the “explosion” from a fire point near \(F\), e.g. in Fig. \(\text{2}\) be optimal by its effect.

The equation of the engine profile in Eq. \((??)\) is very general by its parameters \(k, d, \rho, r\). The exact computations for the time variable \(t\) are hopeless without computer. Animation of the movements is still timely problem, \([4]\) is a first step, only.

2 The pole curves

To describe the motion of any point \(X\) of the moving regular \(k\)-gonal rotor (in Fig. \(3, \ k = 3\)), we introduce the fixed coordinate system \((O; e_1, e_2)\) and the moving coordinate system \((Q; a_1, a_2)\) in the sense of Section 1:

\[
\overrightarrow{OQ} = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \begin{pmatrix} d \cos kt \\ d \sin kt \end{pmatrix},
\]

\[
\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix},
\]

\[
\overrightarrow{OX} = \begin{pmatrix} x^1 \\ x^2 \end{pmatrix},
\]

\[
\overrightarrow{QX} = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} \begin{pmatrix} y^1 \\ y^2 \end{pmatrix}.
\]

Here \((x^1, x^2)^T\) is fixed, but \((y^1(t), y^2(t))^T\) depends on the time, because \(\overrightarrow{OQ}\) and \((a_1, a_2)\) move. We use row-column multiplication, lower-upper index convention and transpose (by upper \(T\)) for saving place.
For an instantaneous pole \( X(x^1, x^2)^T \), its velocity in the fixed frame is zero, \( \dot{y} := \frac{d}{dt}O \dot{X} = 0 \) yields from the last eq. (6):

\[
\begin{pmatrix}
0 \\
0
\end{pmatrix}
= \begin{pmatrix}
\dot{y}^1(t) \\
\dot{y}^2(t)
\end{pmatrix}
= \begin{pmatrix}
-kd \sin kt \\
k d \cos kt
\end{pmatrix}
+ \begin{pmatrix}
-\sin t & -\cos t \\
\cos t & -\sin t
\end{pmatrix}
\begin{pmatrix}
x^1 \\
x^2
\end{pmatrix}.
\]

In a fixed moment \( t = \tau \) this serves us the pole \((x^1(\tau), x^2(\tau))^T\) as a solution of Eq. (9). Finally we get

\[
\begin{pmatrix}
x^1(\tau) \\
x^2(\tau)
\end{pmatrix}
= -kd \begin{pmatrix}
\cos (k - 1)\tau \\
\sin (k - 1)\tau
\end{pmatrix}
\]

the moving pole curve \( p_m \)

(10)

in the moving frame \((Q, a_1, a_2)\). Formula (10) describes \( p_m \) as a moving circle of radius \( kd \) with centre \( Q \). Substituting (10) into the last equation of (9), we obtain

\[
\begin{pmatrix}
x^1(\tau) \\
x^2(\tau)
\end{pmatrix}
= -(k - 1)d \begin{pmatrix}
\cos k\tau \\
\sin k\tau
\end{pmatrix}
\]

the fixed pole curve \( p_s \)

(11)

in the standing frame \((O, e_1, e_2)\). This is a circle of radius \((k - 1)d\) with centre \( O \).

Indeed, the moving pole curve \( p_m \) rolls on the standing pole curve \( p_s \). Meanwhile, the rotor, fixed to the moving pole curve \( p_m \), can be studied (Fig. 4), animated (Home pages 24-25), etc.

Thus the profile of the engine space, the rotor and fire point \( F \) can be formed optimally by experiments or by other theoretical tools (?), as we sketched it for \( k = 3 \), e.g. in Fig. 3.

### 3 Concluding remarks for k-gonal rotors, \( k > 3 \)

Our Fig. 4 (see also Fig. 5) indicates the situation for \( k = 4, d = 1 = r, \rho = kd = 4 \). The moving pole curve \( p_m \) is a circle of the critical radius \( kd = 4 \), with centre \( Q \). The fixed pole curve \( p_s \) is of radius \((k - 1)d = 3 \). The regular \( k(= 4) \)-gonal rotor is fixed to the moving pole curve \( p_m \). The \( k - 1(= 3) \)-fold rotatory symmetry of any orbit is obvious. For \( \rho \geq k^2d \) (= 16) we shall have convex orbit \( c_\rho \), so convex motor space (Fig. 5). The parallel curve \( c_{\rho + r} \) of distance \( r \) will be formed for a \( k (= 4) \)-gonal rotor with rolling circles of radius \( r (= 1) \) in the vertices. Our animation [4] shows the motion.
References