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Improving Wheelset Stability of Railway Vehicles by Using an H_∞/LPV Active Wheelset System

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Abstract

The complexity of railway vehicle structures has been part of an evolutionary process for almost two hundred years. Challenges such as increased weight, increased maintenance, higher costs and energy consumption have become common. The vision for future railway vehicles is to reduce complexity, hence enable simpler structures and reduce maintenance and cost, and of course various research challenges arise from this. In fact, a number of papers in the railway engineering literature have presented practical ways to control steering of railway vehicles to improve performance. The model of the railway wheelset is highly nonlinear, mainly due to the nature of the wheelset structure and the related wheel-rail contact forces involved during operation. In this paper, the simplest design in terms of retrofitting, the actuated solid-axle wheelset is considered, we investigate actively controlled wheelsets from a Linear Parameter Varying (LPV) control aspect. We use the grid-based LPV approach to synthesize the H_∞/LPV controller, which is self-scheduled by the forward velocity, as well as the longitudinal and lateral creep coefficients. The aim of the controller is to reduce the lateral displacement and yaw angle of the wheelset. Simulation results show that the proposed controller ensures the achievement of the above targets in the considered frequency domain up to 100 rad/s.

Keywords

railway vehicles, active wheelset, active control, H_∞ control, Linear Parameter Varying (LPV) system

1 Introduction

1.1 Context

Some of the most important features in railway systems include high speed, relatively cheap operation and relatively low maintenance cost as well as a safe and environmentally friendly service. Therefore, railways play an essential transport role in the 21st century. The cost efficiency in railway operations can be explored from different points of view such as running speed, ride comfort, safety, rail/wheel contact wear, maintenance cost and so forth. The bogie system of high speed trains contains primary and secondary suspension components which can significantly affect the overall dynamic behavior of railway vehicles in different operational scenarios (Bideleh and Berbyuk, 2016a; Bideleh et al., 2016b). The kinematic and dynamic characteristics of railway wheelsets are now well understood. Although passive suspension systems might provide satisfactory running behavior at low to medium speeds, application of such systems at high speeds might lead to poor ride comfort and steering problems such as instability and reduced performance when taking curves (Goodall and Li, 2000).

In order to overcome the drawback of the passive system, special attention has been paid to the bogie suspension system with the active control design. Active elements are often used in substitution, or in combination with passive components to improve the train's dynamics. The mainstream of the active control design in railway applications is considered for the secondary suspension system. However, the increasing interest in the idea of actively-controlled wheelsets gives a new perspective to the possibilities for

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achieving better stability (Wickens, 1991), and it is therefore valuable to re-evaluate the dynamic equations in a manner which will facilitate a control engineering approach.

Several active systems have been developed during the past few decades in order to meet various design requirements and improve railway vehicles' performance from different perspectives such as ride comfort, safety, and wheel/rail contact wear (Bideleh and Berbyuk, 2016a; Bideleh et al., 2016b; Matamoros-Sanchez and Goodall, 2015; Zong et al., 2013). This paper presents a new LPV control design for an active wheelset system in order to improve wheelset stability.

1.2 Related works

From the available literature some related works are listed as below:

- In the works of Bideleh and Berbyuk (2016a) and Bideleh et al. (2016b), a robust controller is designed for active steering of a high speed train bogie with solid axle wheelsets to reduce track irregularity effects on the train's dynamics and to improve stability and cornering performance. A half-car railway model with seven degrees of freedom equipped with practical accelerometers and angular velocity sensors is considered for the H_∞ control design. The results showed that for the case of nonlinear wheel and rail profiles, significant improvements in the active control performance can be achieved using the proposed compensation technique.
- In the work of Goodall and Li (2000), the authors present the unconstrained wheelset equations in a block diagram form, illustrating the feedback action created by a combination of creep and conicity. It then identifies a re-structuring and simplification from which the basic kinematic oscillation can readily be predicted, this also helps to expose the issues relating to stabilization through passive means. The analysis is extended using a similar approach to a wheelset with independently rotating wheels, including the effect of longitudinal creep upon the relative speed of the two wheels.
- In Pérez et al.'s work (2000), an improvement of the curving behavior of conventional railway vehicles with mounting bogies and solid wheelsets through active control was investigated. Various possible control goals are considered and implemented using optimal control techniques. Then suitable sensor types and locations are selected for each control strategy and results are obtained taking into account stochastic disturbances.
- In the work of Li and Goodall (1998), an assessment of a railway wheelset in control engineering terms was presented. It addresses a linearized dynamic model and some simulation results for straight and curved track; it also gives an interpretation of the fundamental stability problem based upon the control system stability analysis, and suggests theoretical possibilities for active control laws.

1.3 Paper contributions

Based on the idea in the works of Li and Goodall (1998); Goodall and Li (2000) and Pérez et al. (2000), here the authors present preliminary research results on the H_∞/LPV active wheelset control system with the aim of improving the wheelset stability. Hence the following contributions are made:

- We propose an LPV wheelset system by considering the forward velocity, the longitudinal and lateral creep coefficients as the three varying parameters. The two exogenous disturbances used include the curvature and the cant angle. The control input includes the lateral force and yaw torque which are controlled by the active controller.
- We use the grid-based LPV approach to synthesize the H_∞/LPV controller which is self-scheduled by the forward velocity, and the longitudinal and lateral creep coefficients. The aim of the controller is to reduce the lateral displacement and yaw angle of the wheelset. Simulation results show that the proposed controller ensures the above targets in the considered frequency domain up to 100 rad/s.

The paper is organized as follows: Section 2 presents a single active wheelset LPV model. Section 3 develops the H_∞/LPV control synthesis for an active wheelset system to improve stability. Section 4 presents some simulation results in the frequency domain. Finally, some conclusions are drawn in Section 5.

2 Wheelset modeling

A single wheelset model shown in Fig. 1 is considered for the robust LPV control design. It has a solid axle with linear wheel profiles, therefore the model has 2 degrees of freedom (DOF) (Li and Goodall, 1998; Pérez et al., 2000). There are

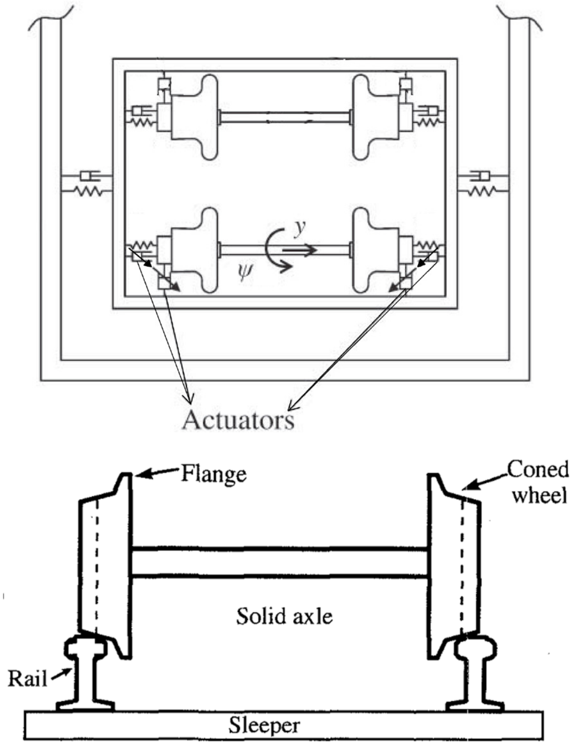

Fig. 1 A single wheelset model (Bideleh and Berbyuk, 2016a; Bideleh et al., 2016b; Li and Goodall, 1998)

four actuators which can be used for this model, with two actuators to create the lateral force (F_y) and two actuators for the yaw torque (T_ψ). The parameters and variables of the wheelset model are detailed in Table 1. The motion differential equations are formalized as in Eq. (1):

$$\begin{cases} \ddot{y} = -\dfrac{2f_{22}}{mv}\dot{y} + \dfrac{2f_{22}}{m}\psi + \dfrac{F_y}{m} + \dfrac{v^2}{R_0} - g\theta \\ \ddot{\psi} = -\dfrac{2f_{11}l\lambda}{Ir_0}y - \dfrac{2f_{11}l^2}{Iv}\dot{\psi} + \dfrac{T_\psi}{I} + \dfrac{2f_{11}l^2}{IR_0} \end{cases}. \quad (1)$$

The motion differential Eq. (1) can be rewritten in the LPV state-space representation with the three varying parameters $\rho = [\rho_1, \rho_2, \rho_3]$ ($\rho_1 = v, \rho_2 = f_{11}, \rho_3 = f_{22}$) as follows:

$$\dot{x} = A(\rho)\times x + B_1(\rho)\times w + B_2(\rho)\times u \quad (2)$$

with the state vector x, the exogenous disturbance includes the curvature and the cant angle w, the control input includes the lateral force and the yaw torque u.

$$x = [\dot{y}, y, \dot{\psi}, \psi]^T$$

$$w = \left[\frac{1}{R_0}, \theta\right]^T$$

$$u = [F_y, T_\psi]^T$$

Table 1 Variables and parameters of single wheelset model (Goodall and Li, 2000; Li and Goodall, 1998)

Symbols	Description	Value	Unit
m	Wheelset mass	1250	kg
v	Forward velocity	-	km/h
l	Half gauge	0.75	m
I	Wheelset yaw inertia	700	kg m^2
r_0	Nominal wheel radius	0.5	m
R_0	Curve radius	-	m
θ	Track cant angle	-	rad
λ	Conicity	0.15	-
f_{11}	Longitudinal creep coefficient	10^7	N
f_{22}	Lateral creep coefficient	10^7	N
y	Lateral displacement	-	m
ψ	Yaw angle	-	rad

3 H_∞/LPV controller design for active wheelset system

3.1 LPV control for the active wheelset system

One of the key factors in railway operations (especially at high speeds) is running stability which is particularly important for safety and ride comfort. Therefore, the controller should first of all stabilize the wheelset motion. In order to satisfy the above mentioned design requirements, it is not necessary to control all the states of the system. A global sensitivity analysis on the wheel/rail contact properties with respect to the wheelset dynamics proved that the contact properties such as creeps, contact forces, and contact patch dimensions are mostly sensitive with respect to the wheelset lateral and yaw motions. The lateral wheelset motion should be below some limit (8 mm in most of the cases) to avoid a flange contact. On the other hand, the wheelset yaw motion can significantly affect the contact forces. In order to describe the control objective, the model Eq. (2) has a partitioned representation in the following way:

$$\begin{bmatrix} \dot{x}(t) \\ z(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A(\rho) & B_1(\rho) & B_2(\rho) \\ C_1(\rho) & D_{11}(\rho) & D_{12}(\rho) \\ C_2(\rho) & D_{21}(\rho) & D_{22}(\rho) \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \\ u(t) \end{bmatrix} \quad (3)$$

where $z(t) = [y, \psi, T_\psi, F_y]^T$ is the performance output vector and $y(t) = [\ddot{y}, \ddot{\psi}]^T$ is the measured output vector.

The bounds $(\bar{\upsilon}, \underline{\upsilon})$, of the varying parameters are taken into account. The control goal is to minimize the induced L_2 norm of the closed-loop LPV system $\Sigma_{CL(\rho)} = \mathrm{LFT}(G(\rho), K(\rho))$, with zero initial conditions, which is given by

$$\left\|\Sigma_{CL(\rho)}\right\|_{2\to2} = \sup_{\substack{\rho\in P \\ \bar{\upsilon}\le\dot{\rho}\le\underline{\upsilon}}} \sup_{\substack{w\in L_2 \\ \|w\|_2\neq0}} \frac{\|z\|_2}{\|w\|_2}. \quad (4)$$

If $\Sigma_{CL(\rho)}$ is quadratically stable, this quantity is finite. The quadratic stability can be extended to the parameter dependent stability, which is the generalization of the quadratic stability concept (Gaspar et al., 2005; Wu et al., 1996).

3.2 H$_\infty$/LPV control synthesis

In this section, the forward velocity, the longitudinal and lateral creep coefficients are considered as the three varying parameters ($\rho = [\rho_1, \rho_2, \rho_3], \rho_1 = v, \rho_2 = f_{11}, \rho_3 = f_{22}$). The forward velocity can be measured directly by sensors, whereas the two creep coefficients vary a lot as the train moves.

3.2.1 H$_\infty$/LPV control design

Fig. 2 shows the control scheme for the H$_\infty$/LPV control design. It includes the feedback structure of the nominal model $G(\rho)$, the controller $K(\rho)$, the weighting functions and the performance objectives. In this diagram, u is the control input, y is the measured output, n is the noise measurement, z is the performance output and w is the disturbance signal.

The main objective of the active system is to reduce the lateral displacement and yaw angle. Furthermore, the lateral force and yaw torque should be kept as small as possible in order to avoid the saturation of the actuators. Therefore the weighting functions are chosen as given in Table 2.

The LPV controller $K(\rho)$ in Fig. 2 is defined as

$$\begin{bmatrix} \dot{x}_c(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} A_c(\rho) & B_c(\rho) \\ C_c(\rho) & D_c(\rho) \end{bmatrix} \begin{bmatrix} x_c(t) \\ y(t) \end{bmatrix}. \quad (5)$$

The closed-loop system $\Sigma_{CL(\rho)} = \mathrm{LFT}(G(\rho), K(\rho))$ can be derived from the generalized plant $G(\rho)$ Eq. (3) and the controller $K(\rho)$ Eq. (5) as follows:

$$\begin{bmatrix} \dot{\xi}(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}(\rho) & \boldsymbol{B}(\rho) \\ \boldsymbol{C}(\rho) & \boldsymbol{D}(\rho) \end{bmatrix} \begin{bmatrix} \xi(t) \\ w(t) \end{bmatrix}, \quad (6)$$

where

$$\begin{cases} \boldsymbol{A} = \begin{bmatrix} A(\rho) + B_2(\rho) D_c(\rho) C_2(\rho) & B_2(\rho) C_c(\rho) \\ B_c(\rho) C_2(\rho) & A_c(\rho) \end{bmatrix} \\ \boldsymbol{B} = \begin{bmatrix} B_1(\rho) + B_2(\rho) D_c(\rho) D_{21}(\rho) \\ B_c(\rho) D_{21}(\rho) \end{bmatrix} \\ \boldsymbol{C} = \left[C_1(\rho) + D_{12}(\rho) D_c(\rho) C_2(\rho) D_{12}(\rho) C_c(\rho) \right] \\ \boldsymbol{D} = D_{11}(\rho) + D_{12}(\rho) D_c(\rho) D_{21}(\rho) \end{cases} \quad (7)$$

with $\xi(t) = \left[x^T(t), x_c^T(t)\right]^T$.

The quadratic LPV γ-performance problem is to compute the parameter-varying control matrices $A_c(\rho)$, $B_c(\rho)$, $C_c(\rho)$, $D_c(\rho)$ in such a way that the resulting closed-loop system is quadratically stable and the induced L_2 norm from $w(t)$ to $z(t)$ is less than γ. The existence of a controller that solves the quadratic LPV γ-performance problem can be expressed as the feasibility of a set of Linear Matrix Inequalities (LMIs), which can be solved numerically (Gaspar et al., 2005; Wu, 2001; Wu et al., 1996).

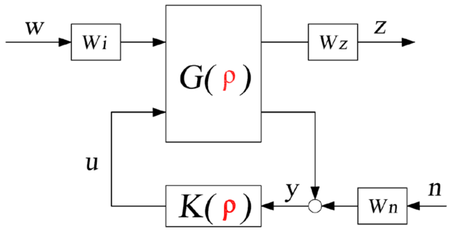

Fig. 2 Closed-loop interconnection structure with LPV active controller

Table 2 The weighting functions of the closed-loop structure (Bideleh and Berbyuk, 2016a; Bideleh et al., 2016b)

Weighting function	Description	Value
W_z	Weighting functions for the performance output	$diag\left[W_{zy}, W_{zpsi}, W_{zFy}, W_{zTpsi}\right]$
W_{zy}	Weighting function for the lateral displacement	$100\dfrac{40s^2 + 60s + 100}{1s^2 + 200s + 100}$
W_{zpsi}	Weighting function for the yaw angle	$100\dfrac{10s^2 + 20s + 1}{1s^2 + 200s + 100}$
W_{zFy}	Weighting function for the lateral force	5e-8
W_{zTpsi}	Weighting function for the yaw torque	5e-8
W_i	Weighting functions for the disturbance signals	$diag\left[W_{i1}, W_{i2}\right]$
W_{i1}	Weighting function for the curvature	0.5
W_{i2}	Weighting function for the cant angle	0.5
W_n	Weighting functions for the noise measurement	$diag[0.01, 0.01]$

3.2.2 Solution for the H$_\infty$/LPV control

Several approaches can be used to design an LPV controller, based on the LPV model Eq. (3): Linear Fractional Transformations (LFT) (Apkarian and Gahinet, 1995; Packard, 1994), Polytopic solution (Gahinet et al., 1996; Scherer et al., 1997), Linearizations on a gridded domain (grid-based LPV) (Wu, 1995). The grid-based LPV approach is interesting since it does not require any special dependence on the parameter vector. This method is used in this paper together with the LPVTools™ (Hjartarson et al., 2015) to synthesize the H$_\infty$/LPV controller.

For the interconnection structure shown in Fig. 2, the H_∞ controllers are synthesized for 10 values of the forward velocity in a range of $\rho_1 = v$ = [50–130] km/h, 50 values of the longitudinal creep coefficient in a range of $\rho_2 = f_{11}$ = [5–10] MN, 50 values of the lateral creep coefficient in a range of $\rho_3 = f_{22}$ = [5–10] MN. The spacing of the grid points is based upon how well the H_∞ point designs perform for the plant around each design point. The grid points and the LPV controller synthesis using LPVTools™ are expressed by the following commands:

$$v = pgrid('\text{rho1}', linspace(50/3.6, 300/3.6, 10));$$
$$f_{11} = pgrid('\text{rho2}', linspace(5e6, 10e6, 50));$$
$$f_{22} = pgrid('\text{rho3}', linspace(5e6, 10e6, 50));$$
$$[Klpv, normlpv] = lpvsyn(\text{H}, nmeas, ncont).$$

At all of the grid points, the proposed weighting functions are applied to the entire grid parameter space and the effect of the scheduling parameter is ignored. In the H_∞ control design, the γ iteration results in an optimal γ value and an optimal controller. However, if the weighting functions were changed, another optimal γ and another optimal controller would be obtained.

4 Simulation results analysis

The parameters of the wheelset model are detailed in Table 1. In this section we will evaluate the effectiveness of the H_∞/LPV active wheelset control system in the frequency domain for the three cases:

- **First case:** the forward velocity ($\rho_1 = v$) varies from 50 km/h to 300 km/h with 10 grid points. The longitudinal and lateral creep coefficients ($\rho_2 = f_{11}$, $\rho_3 = f_{22}$) are kept at the nominal value of 10 MN.
- **Second case:** the longitudinal creep coefficient ($\rho_2 = f_{11}$) varies from 5 MN to 10 MN with 50 grid points. The lateral creep coefficient ($\rho_3 = f_{22}$) is kept at the nominal value of 10 MN and the forward velocity ($\rho_1 = v$) is considered at 20 m/s.
- **Third case:** the lateral creep coefficient ($\rho_3 = f_{22}$) varies from 5 MN to 10 MN with 50 grid points. The longitudinal creep coefficient ($\rho_2 = f_{11}$) is kept at the nominal value of 10 MN and the forward velocity ($\rho_1 = v$) is considered at 20 m/s.

4.1 First case: $\rho_1 = v$ = [50–300] km/h, $\rho_2 = \rho_3 = f_{11} = f_{22}$ = 10 MN

In this subsection, the authors consider the varying parameter of the forward velocity $\rho_1 = v$ from 50 km/h to 300 km/h with 10 grid points.

Figs. 3 and 4 show that in the case of the "no control" system, when the forward velocity changes, the transfer function of the variables changes a lot, so the application of LPV control with the forward velocity as a varying parameter is very necessary.

We can also see that the H_∞/LPV active wheelset control system reduces significantly the magnitude of the above variables in most frequency ranges. It shows that the active system can generate a roll stability, when compared with the "no control" system.

4.2 Second case: $\rho_2 = f_{11}$ = [5–10] MN, $\rho_3 = f_{22}$ = 10 MN, $\rho_1 = v$ = 20 m/s

In this section we consider that the longitudinal creep coefficient ($\rho_2 = f_{11}$) varies from 5 MN to 10 MN.

Figs. 5 and 6 show that, although the longitudinal creep coefficient varies from 5 to 10 MN, the transfer function magnitude of the variables varies negligibly. The results show that, when this coefficient varies over a wider range (e.g., 0.1–100 MN), the transfer function magnitude of the variables vary greatly. In the process of moving the train, due to the different characteristics between the rail and the wheel, as well as the weather conditions and the material, it is difficult to accurately determine the value of this coefficient. Therefore, considering this coefficient as a varying parameter is still meaningful in practice.

From Figs. 5 and 6 we see that the H_∞/LPV active wheelset control system achieves the goal to reduce the lateral displacement, yaw angle, as well as their acceleration in most of the frequency ranges of interest.

4.3 Third case: $\rho_3 = f_{22}$ = [5–10] MN, $\rho_2 = f_{11}$ = 10 MN, $\rho_1 = v$ = 20 m/s

In this section we consider that the lateral creep coefficient ($\rho_3 = f_{22}$) varies from 5 MN to 10 MN.

Figs. 7 and 8 show that the transfer function magnitude of the lateral displacement and its acceleration do not change too much when the coefficient changes. However, the yaw angle and its acceleration are sensitive to the variations of this coefficient. So it is a good idea to consider this coefficient as a varying parameter.

As with the two cases considered above, the H_∞/LPV active wheelset control system can reduce the transfer function magnitude of the variables, thereby enhancing the stability of the wheelset.

5 Conclusions

In this paper, we propose an LPV wheelset system by considering the three varying parameters: the forward velocity,

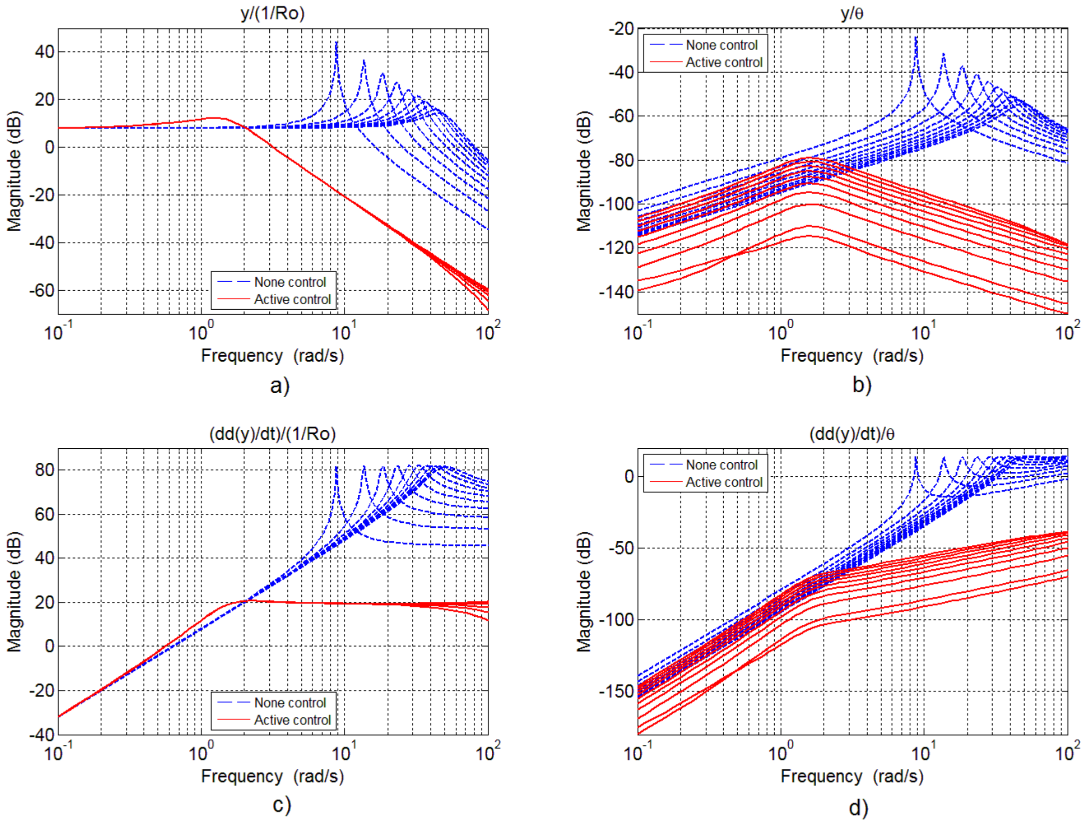

Fig. 3 First case: transfer function magnitude of a) $y/(1/R_0)$, b) y/θ, c) $\ddot{y}/(1/R_0)$, d) $\ddot{y}/\theta$

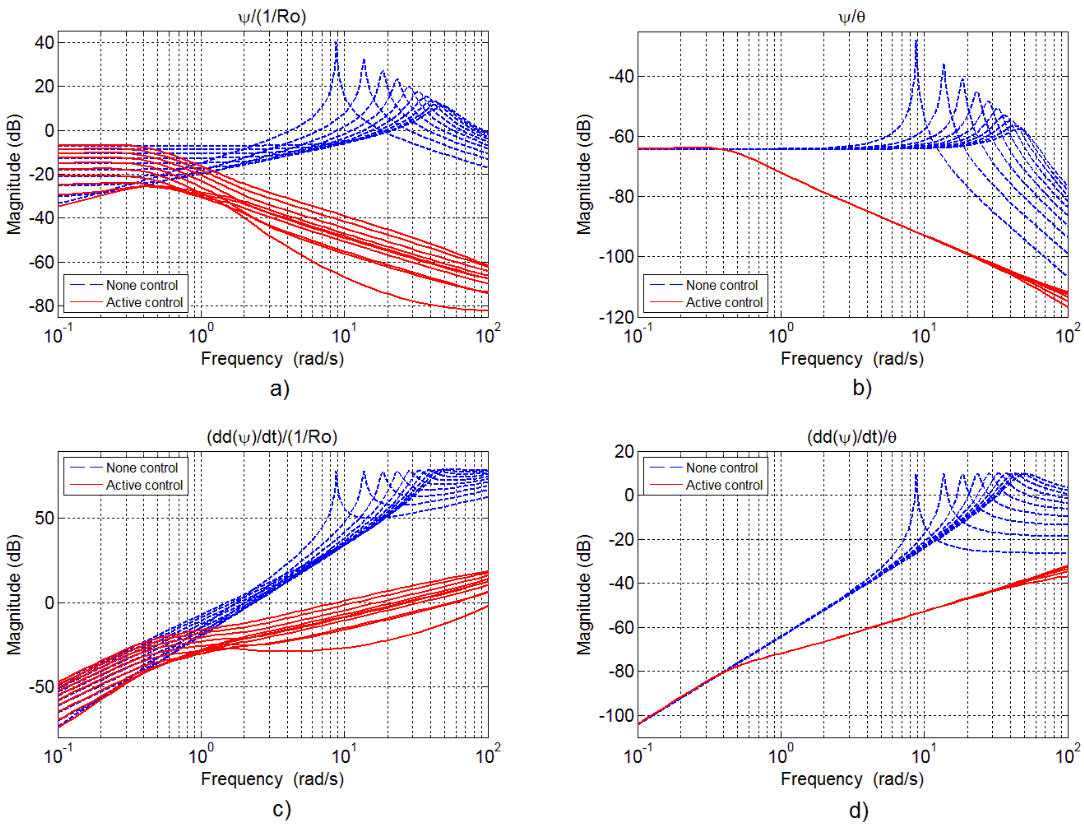

Fig. 4 First case: transfer function magnitude of a) $\psi/(1/R_0)$, b) ψ/θ, c) $\ddot{\psi}/(1/R_0)$, d) $\ddot{\psi}/\theta$

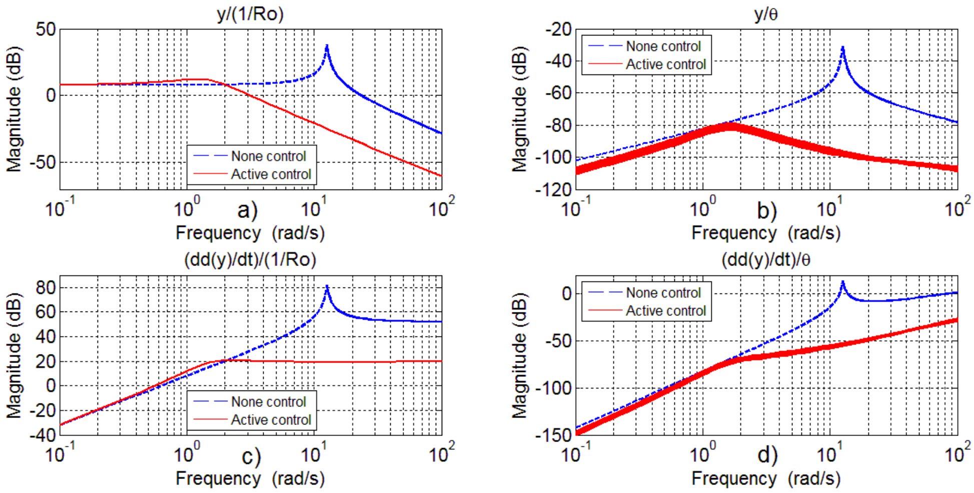

Fig. 5 Second case: transfer function magnitude of a) $y/(1/R_0)$, b) y/θ, c) $\ddot{y}/(1/R_0)$, d) $\ddot{y}/\theta$

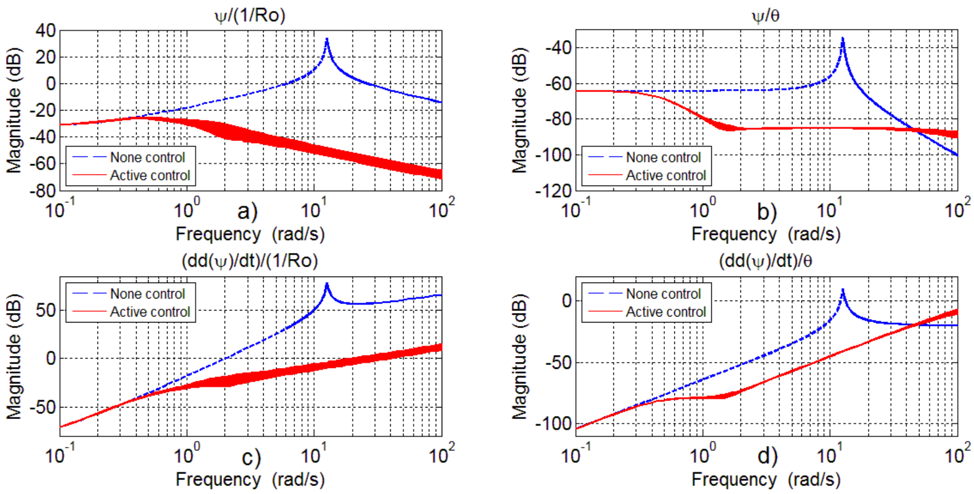

Fig. 6 Second case: transfer function magnitude of a) $\psi/(1/R_0)$, b) ψ/θ, c) $\ddot{\psi}/(1/R_0)$, d) $\ddot{\psi}/\theta$

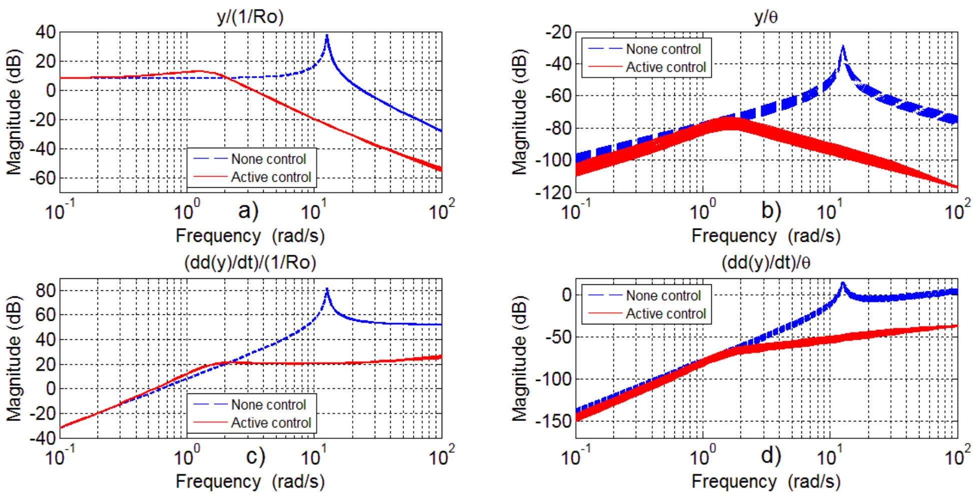

Fig. 7 Third case: transfer function magnitude of a) $y/(1/R_0)$, b) y/θ, c) $\ddot{y}/(1/R_0)$, d) $\ddot{y}/\theta$

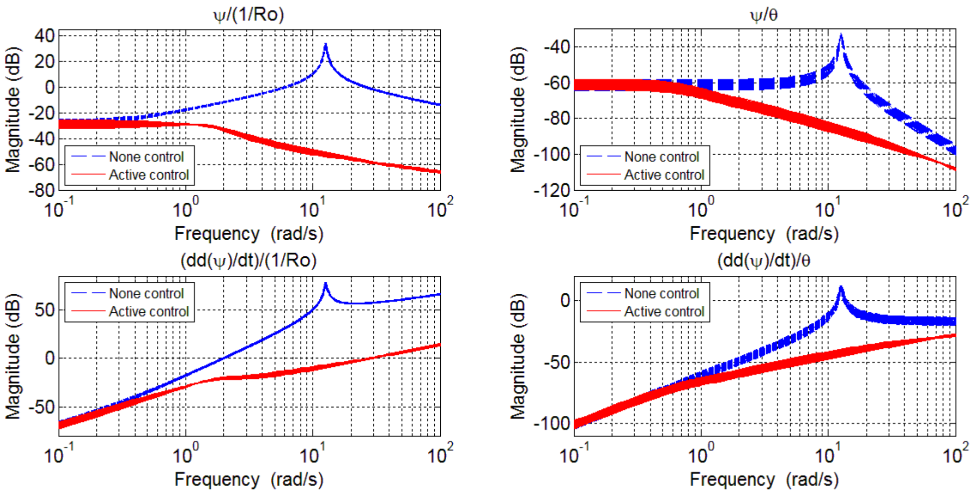

Fig. 8 Third case: transfer function magnitude of a) $\psi/(1/R_0)$, b) ψ/θ, c) $\ddot{\psi}/(1/R_0)$, d) $\ddot{\psi}/\theta$

the longitudinal and lateral creep coefficients. The grid-based LPV approach is used to synthesize the H_∞/LPV controller which is self-scheduled by the above three varying parameters. The aim of the controller is to reduce the lateral displacement and yaw angle of the wheelset. Simulation results in the frequency domain show that the controller achieves the improvement of the wheelset stability in the desired frequency range. In the future, the application of the H_∞/LPV control to a more complete train model is essential. Furthermore, combining the dynamic model of the actuators will ensure that the research is more realistic and practical.

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