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RESEARCH ARTICLE

# LQ Servo control design with Kalman filter for a quadrotor UAV

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***Abstract***

*This paper deals with reference signal tracking control of a quadrotor UAV (Unmanned Aerial Vehicle).*

*Before controller design, a nonlinear simulation model is needed, to be the base of design and the first testbed for the resulted controllers. The next step should be the linearization of the nonlinear model in hovering mode, and the reduction of the resulted linear model. The reduced linear model is controllable and observable.*

*The control goal was to track a spatial trajectory with the helicopter center of gravity. For this purpose, an LQ Servo controller (with double integrator) was designed, augmented with a Kalman filter state observer.*

*The resultant controller provided good tracking performance for a slowly varying reference signal, also on the nonlinear model! After the transient response, the tracking error was below 1 cm which provides safe handling even in indoor applications. The time of transients was approximately 4 seconds which is acceptable.*

***Keywords***

*LQ Servo control · Kalman filter · Nonlinear simulation · Quadrotor UAV*

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## 1 Introduction

Nowadays the autonomous control of UAVs is an actual research topic, because they are spread all over the World. They will be used in cooperative tasks in a few years, what needs their precise spatial control.

Last year a joint research project was initiated with the participation of HAS Computer and Automation Research Institute, BME Department of Control and Transport Automation and BME Department of Control Engineering and Information Technology. The project goal is to build a quadrotor helicopter model and use it in indoor applications [9, 10]. This type of application also needs very precise spatial trajectory following, mainly because of obstacles.

This paper summarizes a part of the work done by BME Department of Control and Transport Automation. This covers two topics, the building of detailed nonlinear simulation model and the design of a spatial trajectory following controller.

Nonlinear equations of motion for a quadrotor can be found in the literature in more or less detailed forms [3–5, 7, 8]. One can use either one of these models or create a new (similar) model. Building a new model has the benefits of having the opportunity to decide about the details to consider in it, and knowing everything about it (even every parameter which are usually omitted in articles). After the construction of the simulation model in Matlab Simulink, simulations have to be performed to test it. Tests can be done for the basic motions of the quadrotor helicopter (hovering, ascending, descending, yawing and straight horizontal flight).

After model generation, the controller design task has to be solved. The literature has several examples for this, which mainly solve only the stabilization of the helicopter, or pilot augmentation [3–5, 7, 8]. Spatial trajectory tracking controllers are rarely designed, and they are sophisticated nonlinear, adaptive ones: [6]. However, motion simulations showed that ([14, 15]) with slowly varying reference signal the quadrotor does not leave the linear region considered around hovering mode. So it is worth designing simple linear controllers and test them on the nonlinear system.

A linear model can be obtained from the nonlinear with linearization. If the resulted linear model is not controllable or observable, model reduction has to be applied on it.

Then the linear tracking controller can be designed using LQ (Linear Quadratic) Servo technique and state estimation can be solved using a Kalman filter.

After controller design, tests with the linear and nonlinear systems have to be done. During the tests the transient time and the tracking error have to be estimated.

The paper is divided into the following sections: 2. the construction and implementation of the nonlinear simulation model, 3. linearization and model reduction, 4. controller design and test results, 5. conclusions.

## 2 The construction and implementation of a nonlinear simulation model of the quadrotor helicopter

The modelling of multibody systems has to be started with the definition of inertial frame and moving coordinate systems (hereafter abbreviated as coord. sys.) [1]. In case of an indoor quadrotor, the earth can be considered as an inertial frame of reference and so the *earth coord. sys.* can be used. The helicopter body moves with respect to this, so a moving frame of reference called as *body coord. sys.* has to be joined to it (it moves and rotates together with the body). Another type of coordinate system has to be created, in order to consider the gyroscopic effects of the rotors. This coord. sys. has to be fixed to the rotor blades axis (we considered two bladed propellers) and called as *rotor coord. sys.* Of course it rotates together with the rotor and every rotor has its own *rotor coord. sys.* . The defined coord. systems are shown in Fig. 1.

The equations of motion were built using the principles of linear (translations) and angular (rotations) momentum (see [14], [15]). The rotor inertial effects and the effects of rotor oblique inflow were all considered in these equations.

The solution of these servo equations requires the knowledge of the model's parameters. The mass and inertia values were calculated with a CAD program in which the 3D solid body model of the quadrotor was built. The rotor thrust and torque are quadratic functions of rotor angular velocity. The thrust and torque coefficients were selected to achieve the proper thrust on a given rotor angular velocity value. Later the measured characteristics (with the apparatus developed in [9] and [10]) will be integrated into the model.

The simulation was built in Matlab Simulink using S-function blocks which provide the flexibility of modelling (for details see [14] and [15]).

Tests were done for the basic motions of the quadrotor helicopter which are hovering, ascending, descending, yawing and straight horizontal flight. All of them were started from a pre–calculated trim point and they worked well. The most complicated is the straight horizontal flight which needs to create a small pitch angle on which the helicopter is in equilibrium. In this way the rotor thrusts have horizontal components and the quadrotor starts to fly horizontally. Because of very small air drag, this needs a pitch angle around $1^\circ$ (and smaller) which is in the linear region around hovering state. For this case only an approximate trim point was calculated, but the simulation showed good results. After a short transient, the quadrotor preserved its equilibrium state (see Fig. 2).

## 3 Linearization and model reduction

Before controller design, the model was completed with an input transformation block. This provides the opportunity to fly the quadrotor as a conventional helicopter, so the pilot can give commands for pitch, roll, yaw and ascend/descend separately. These commands are transformed to electric motor angular velocities in the proper combination to achieve the commanded motion (for details see [15]).

The nonlinear model was linearized in hovering mode using Matlab linmod command. The inputs were the pilot commands (the autopilot can also give them), the outputs were the measurable states of the system. The outputs can be measured with gyroscopic sensor and a vision-based system (or thermal anemometry in case of velocities).

The resulted model is described with the well-known state dynamic and measurement equations,

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{1}$$

and has 16 states which are the following:

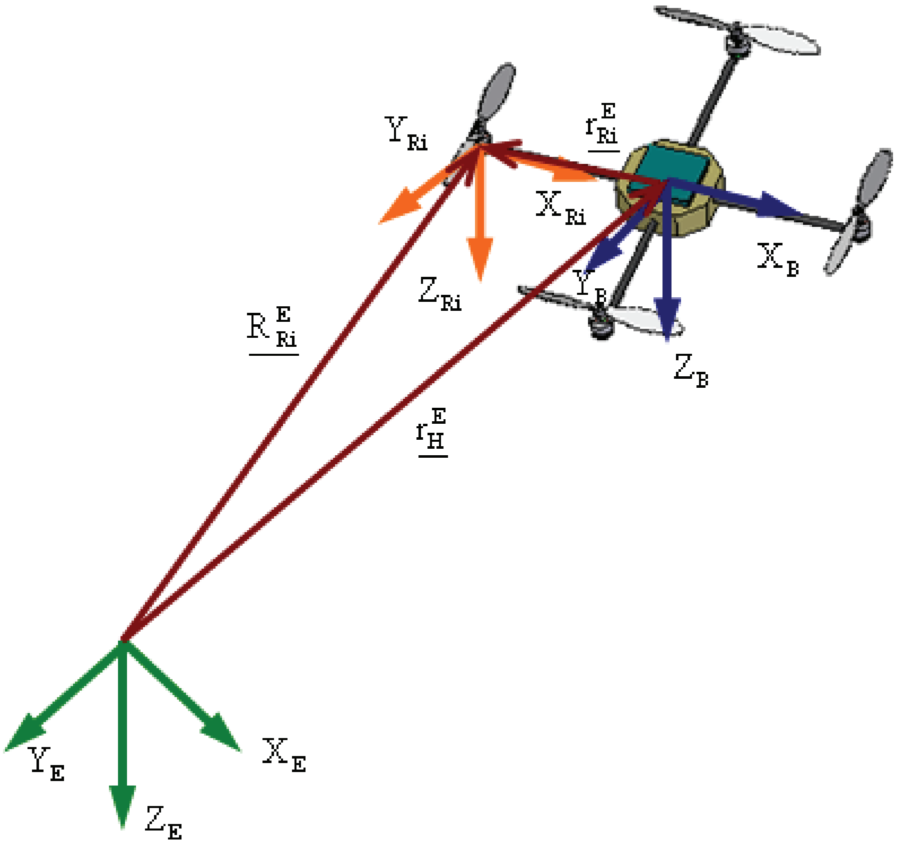
x=[position in *earth coord. sys.* (X Y Z), angular velocities of the four rotors ($\eta_1 \;\; \eta_2 \;\; \eta_3 \;\; \eta_4$), quadrotor velocity components in *body coord. sys.* (u v w), quadrotor angular velocity components in *body coord. sys.* (P Q R), Euler angels ($\varphi \;\; \theta \;\; \psi$)]

The selected (measured) outputs are: y=[position in *earth coord. sys.* (X Y Z), quadrotor velocity components in *body coord. sys.* ( u v w), quadrotor angular velocity components in *body coord. sys.* (P Q R)].

However, this linear model is not controllable and observable so it has to be reduced. The reduction was done considering the structure of the model and deleting the states which do not have effects on the other states. At first, the angular velocities of the rotors were deleted. The reduced model was controllable, but was not observable. To make it observable, the third Euler angle (yaw angle $\psi$) was deleted, because it did not have effect on the remaining states. In this way a controllable and observable model resulted. However, if $\psi$ is deleted from the model, the orientation of the helicopter can not be given. Later, this has to be solved with introducing other measurable outputs to improve the observability. Now only the control of center of gravity spatial position was the goal, so the orientation of the quadrotor could be neglected.

Among the poles of the linear model, there were values with positive real part so the goal of control was also the stabilization of the system. During model reduction, only zero poles were

**Fig. 1.** The defined coordinate systems and defined vectors between their origins




deleted, so the reduction was correct (unstable poles are not allowed to be deleted). The final state space model has a state vector with 11 states.

### 4 Controller design and test results

After linearization and model reduction, a spatial trajectory tracking LQ Servo controller was designed, completed with a Kalman filter state observer (because not all of the states required to state feedback are measured and the quadrotor has system and measurement noises).

The spatial trajectory can be given with p(t)=[X(t) Y(t) Z(t)] position vector in *earth coord. sys.*, so the tracking of the first three states has to be solved. Slowly varying ramp type signals were considered as reference inputs, so an LQ Servo controller with double integrator was needed according to the internal model principle (see [2] page 310). The augmented system for LQ Servo double integral controller design was the following (it was derived using [2] page 308):

$$\begin{bmatrix} \dot{x} \\ \dot{e}_I \\ \dot{e}_{II} \end{bmatrix} = \begin{bmatrix} A & 0 & 0 \\ -I_r & 0 & 0 \\ 0 & I & 0 \end{bmatrix} \begin{bmatrix} x \\ e_I \\ e_{II} \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix} r \qquad (2)$$
$$y = \begin{bmatrix} C & 0 & 0 \end{bmatrix} \bar{x}$$

In (4) $\bar{x}$ is the state vector augmented with the error signals $e_I$ and $e_{II}$ and $I_r$ is the matrix which selects the states from $x$ which have to track the reference. The derivative of $e_I$ is the vector of differences between reference signals and system states. In our case:

$$\dot{e}_I = \begin{bmatrix} X_{ref} - X \\ Y_{ref} - Y \\ Z_{ref} - Z \end{bmatrix}$$

$e_{II}$ is the integral of $e_I$, so:

$$\frac{de_{II}}{dt} = e_I$$

The augmented system is also controllable. The quadratic functional to minimize was the following:

$$J(\bar{x}, u) = \frac{1}{2} \int_0^\infty \left( \bar{x}^T Q_w \bar{x} + u^T R_w u \right) dt \qquad (3)$$

The state weighting matrix is noted with $Q_w$ the input weighting matrix with $R_w$.

Most of the elements of the $Q_w$ matrix were selected by using the suggestions in [2] pages 194-195 and the method of inverse squares. Of course, the control goals and the physical limits of the system were all considered. The limits were the following:

1 for $u$ $v$ $w$ the selected limit was 0.5m/s because of the assumption of slowly varying reference signal and the hard limits of indoor application

2 for $P$ $Q$ $R$ the value $2^\circ/s$ was selected to maintain slow motion and avoid sudden leaving of the linear region

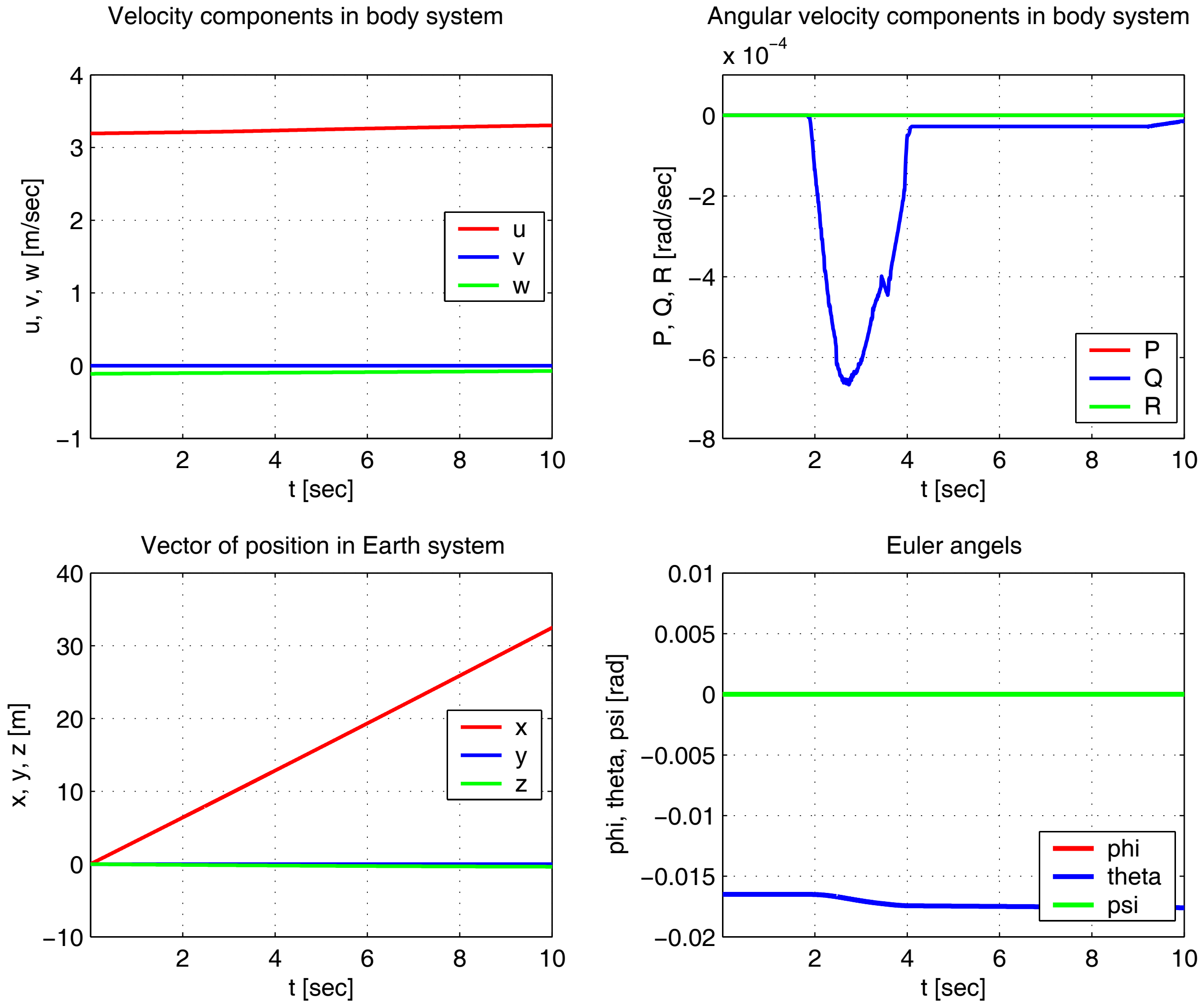



**Fig. 2.** Straight horizontal flight simulations

3 for $\varphi \quad \theta$ the value $5^\circ$ was selected which is the bound of the linear region

The other elements of $Q_w$ were selected with trial and error to minimize the tracking error (the weights on $e_{II}$ are the largest, $e_I$ has weights which are two order smaller and (X Y Z) have small weights to be free to track the reference).

The elements of $R_w$ refer to the inputs which have equal effects on the system, so the same weighting was selected for all of them, using the method of inverse squares. The considered limit was $\pm 1000$ change in electric motor RPM.

After the selection of weighting matrices the controller was designed. The poles of the closed loop system resulted all stable.

In the next step, a Kalman filter state observer was designed for the original (11 state) system. $e_I$ and $e_{II}$ are implemented in the controller, so they do not need to be observed.

The modified (noise augmented) state equations for Kalman filter design are the following:

$$\begin{aligned} \dot{x} &= Ax + Bu + Gx_n \\ y &= Cx + y_n \end{aligned} \tag{4}$$

Here $x_n$ is the vector of system (state) noise and $y_n$ is the vector of measurement noise. All of the noises were considered as stationary white noise processes (see [2] page 66). The elements of the spectral density matrices ($W$ for $x_n$ and $V$ for $y_n$) were constructed considering the maximum possible errors for the states and measurements. The errors were determined considering physical aspects and sensor data sheet.

The considered measurement errors are the following:

1 for $X \quad Y \quad Z$ the precision of a vision-based position measurement system is $\pm 1cm$

2 for $u \quad v \quad w$ the precision of thermal anemometry (or other velocity measurement procedure) can be $\pm 0.1 m/s$

3 for $P \quad Q \quad R$ the data sheet for the gyros publish $0.05^\circ/s$ precision (see [11])

State noises were considered only for velocities ($u \quad v \quad w$) and angular velocities ($P \quad Q \quad R$), because they come from windgusts (from draught in indoor situation) and this effects the velocities first (this means that G is an 11 x 6 rectangular matrix (see (4)). The maximum values are the following:

1 for $u \quad v$ it is $0.1 m/s$

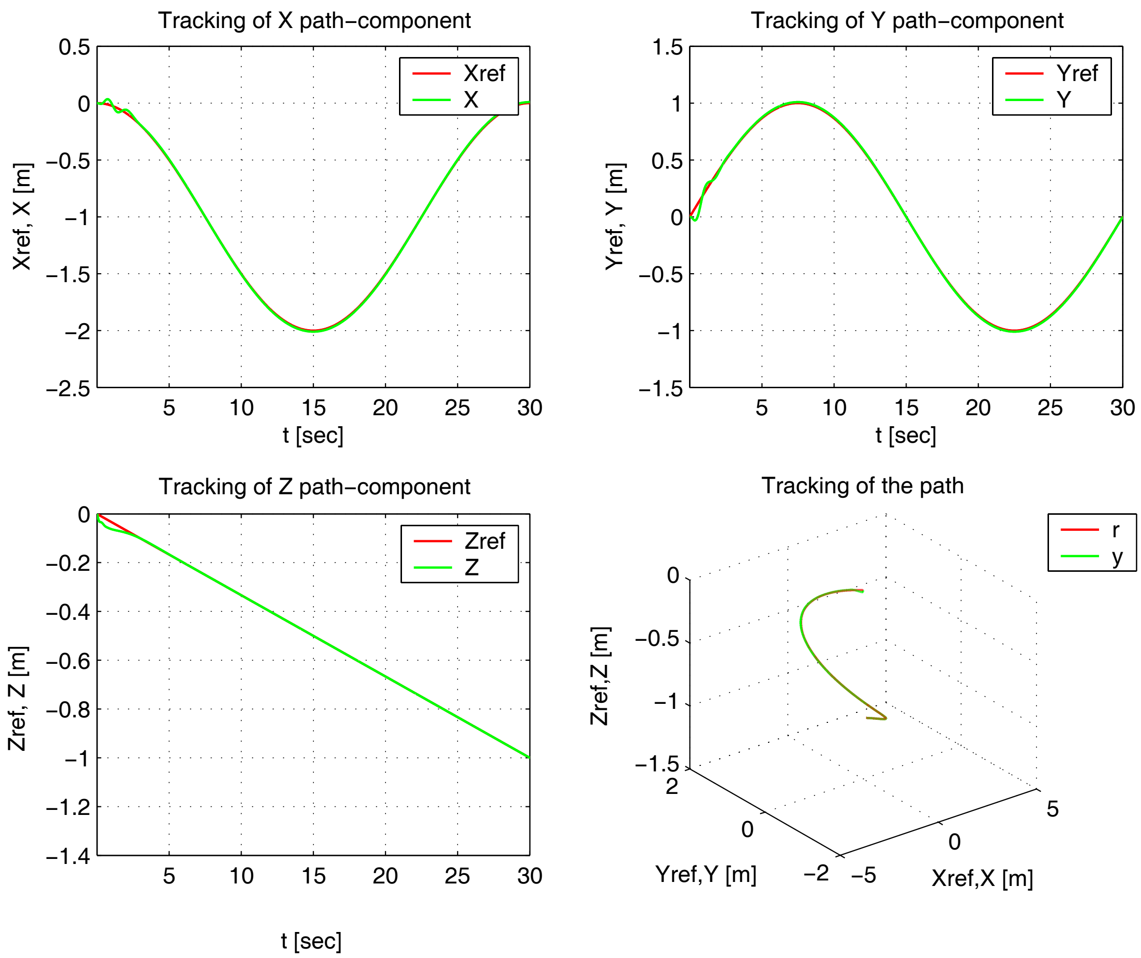



**Fig. 3.** Trajectory tracking with double integrator and Kalman filter (nonlinear model)

2 for $w$ vertical velocity component $0.02m/s$ because of the stabilization effect of rotor in- and outflow

3 for $P\quad Q$ it is $5^{\circ}/s$ because of large drag surface in Z (vertical) direction

4 for $R$ it is $2^{\circ}/s$ because of more smaller drag surfaces in X and Y directions

The designed Kalman filter was stable and gave satisfactory results during simulations.

Simulations were done with the linear model and also with the whole nonlinear system using both the Servo controller and the Kalman filter. The model was started from hovering mode, the servo controller was started from zero error states, meanwhile the Kalman filter started from an initial state estimation error: $x_e$=[1 cm for X Y Z, 0.1m/s for u v w, $1^{\circ}/s$ for P Q R and $1^{\circ}$ for $\varphi\quad\theta$]

Noises were not applied during the simulations. The results are very good, the design goals were achieved and the constraints were satisfied.

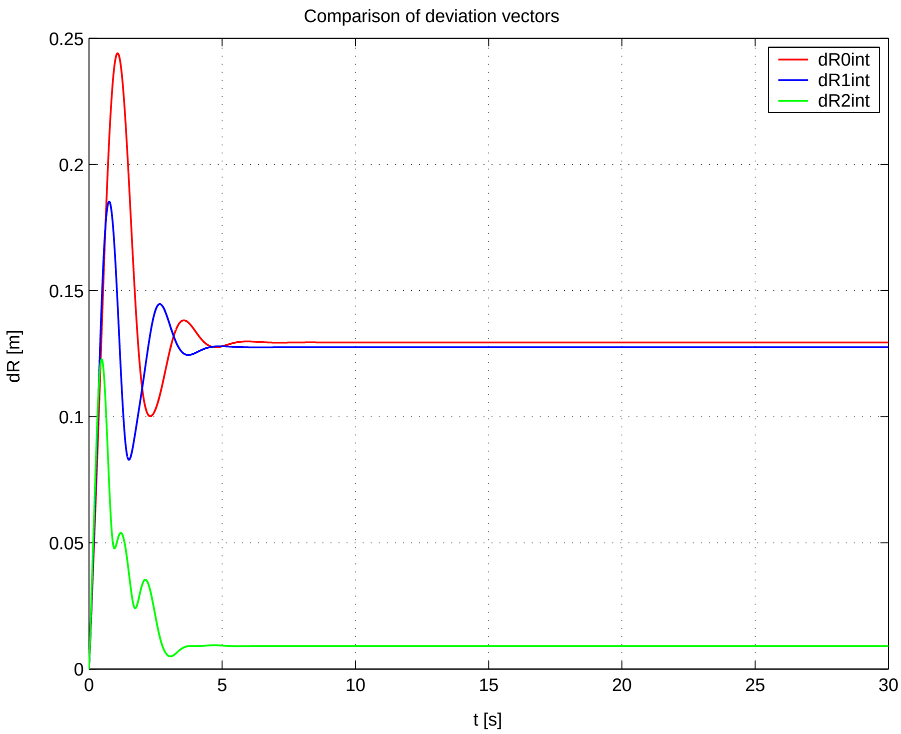
In Fig. 3 the tracking of a spatial descending spiral trajectory with the whole nonlinear system can be seen. After the transients the tracking is very good. This proves our statement about the controllability of the nonlinear system with linear controller, if the reference signal slowly varies. This can be also seen in Fig. 4 where dR2int refers to the tracking error with double integrator. After the transients, this error is below 1cm which is acceptable even in indoor situations. The transient time is approximately 4 seconds. In this figure, tracking error results also with state transformed LQ controller (dR0int) and LQ Servo controller with one integrator (dR1int) can be seen. These controllers do not satisfy the internal model principle (for the given reference signal) so they are not capable to track the trajectory as can be seen in the figure (errors around 13cm).

## 5 Conclusions

This paper presents a nonlinear model of a quadrotor UAV, and introduces a possible spatial trajectory tracking controller design technique.

The nonlinear simulation model was implemented in Matlab, and controller was designed for a linear model obtained from it in hovering mode.

The controller consists of an LQ Servo part with double integrator and a Kalman filter state observer. Weighting matrices were selected considering indoor constraints, data sheets and physical aspects. The resulted controller can track well a spatial position reference signal, meanwhile it is not capable to control the orientation of the model (because the orientation informa-




![ht]

**Fig. 4.** Tracking errors

tion was removed during model reduction). So the control of orientation needs to complete the system with other sensors to solve the observability of orientation.

However, the linear controller provided good reference signal tracking even on the nonlinear system model (see Fig. 3). This is, because with slowly varying reference signal, the system can stay inside the linear region around hovering. This means that possibly the highly nonlinear quadrotor can be controlled with a relatively simple linear controller.

In the future, other linear control design techniques which consider uncertainties in a more straightforward manner have to be applied, such as $\mathcal{H}_\infty$ design. These future designs have to control also the orientation of the quadrotor.

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