Parameter dependent freeway modelling

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Abstract

The problem of modelling the complex behaviour of freeway flow leads to a nonlinear macroscopic model. Unfortunately, high dimensional non-linear characteristics could not be performed easily. The paper's purpose is to introduce a new, general modelling formalism for freeway traffic flow modelling, respectively control. Linear Parameter Varying (LPV) systems represent a numerically tractable class of complex non-linear systems. The main idea is to derive some arbitrary, time dependent parameters by capturing the nonlinearities in the system. Even if the transformation of the full nonlinear model to affine and quasi Linear Parameter Varying (qLPV) system is not unique, an appropriate qLPV model is presented and a computationally low demanding form is given. More, the paper investigates the problem of selecting the adequate scheduling variables, endogenous parameters and some linear approximations giving a novel way to describe freeway traffic systems. An important aspect of the model selection is the feasibility of the resulted system throughout the controller and observer design. The paper describes the problem of quadratic stabilizability and detectability for LPV flow models. The Linear Matrix Inequality (LMI) conditions are developed to verify these important properties. Finally, a numeric example suggests the application of the LPV structure for a general freeway section with on- and off-ramps. The comparison of the simulation response of the non-linear and the derived nominal LPV model has also been investigated.

Keywords

qLPV · freeway modelling · quadratic stabilizability

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1 Introduction

One of the most progressing research area in traffic modelling is the theory of freeway traffic flow. There is a high demand on creating the most accurate set of models describing the real traffic.

A possible macroscopic technique is based on the analogy between the traffic flow and the streaming fluids or gases. However, the generalized density, speed and traffic volume are commonly used variables, the analogy is not valid for certain specific case. The basic correlation between the traffic variables is described by the well-known fundamental diagram. After Lighthill and Whitham [6, 14] formulated, the theory of kinematic waves were also adapted for freeways. Taking traffic waves into account, the freeway flow model can be extended to a second order macroscopic model. Due to the wave's dynamic, the traffic flow becomes highly nonlinear and complex. The complexity is increased by the segmentation of a freeway therefore a large scale system needs to be considered.

Modern control theory offers the opportunity to handle highway traffic models (and also other road traffic systems [9]) as dynamic systems. Introducing the time dependent freeway model, a more and more complex and liable description is given. Modern, respectively postmodern techniques therefore introduce the states of a freeway dynamical system. The dynamic state equation formulates how the system evolves in time. The state equations describe the variation of the actual states based on the given inputs to the system. Two important questions are arising with respect to the application in freeway traffic. First, the observation of the not measured variables and second, the control of the main flow with variable speed limits and ramp metering. The traffic modelling literature is large enough and contains several solutions for traffic analysis and synthesis [2, 6, 11, 12, 14, 16].

The control objective on freeways could be stated as keeping the main flow volume near to the maximal capacity of the given stretch. Based on the fundamental diagram, this is equivalent to keep the density of the stretch around its critical value. Since the problem is formulated with nonlinear equations, there is a need for the application of nonlinear control techniques. Denote, linear controller can also be used to assure the control per-

formance. Unlike the nonlinear freeway control theory, linear control system design is elaborated [11]. While the nonlinear formalism is used to describe global behaviour, linear systems are applied only to reflect local characteristics around a given operation point. Linearising or simplifying the complete nonlinear plant always leads to loose certain and sometimes important information. Hence, there is a trade-off between the accurate model description and the simplicity of the controller. Since, the final goal is always to achieve an optimal performance level with the appropriate control algorithm, the realization of the closed-loop system needs to be taken into consideration.

As previously it was mentioned, the first part of the freeway model analysis and synthesis is the observation of the real flow. The problem is stated in a linear or in a nonlinear way. The state estimation of non-linear systems is an existing problem. The estimation technique of the Extended Kalman Filter (EKF) is widely applied [4, 12, 16] in the industry. The EKF is based on the linearisation of the nonlinear system around the given operation point depending on the state trajectory. The convergence of the estimation has been investigated and it has been showed that EKF gives a suboptimal solution of the filtering problem. Even if the convergence of the EKF is not guaranteed, it is often used as a "nonlinear" observer. State estimation on freeways could multiple the available set of traffic information, by estimating the non-measured variables.

There is a permanent need to control the motorway traffic in order to avoid traffic jams respectively increase the safety level of a given section. Two main possibilities are applied to directly influence freeways traffic. First, the ramp metering, i.e the freeway on-ramp flow is controlled by signalling. On the other hand, the display of different speed limits throughout Variable Massage Sings (VMS). Traffic control synthesis is based on the results of control engineering [5, 7, 10, 11, 18].

In recent years, a promising approach for nonlinear control theory is certainly the Linear Parameter Varying (LPV) formalism in state space [1, 8, 13, 15, 17]. The LPV class is a specific formulation of the nonlinear systems using measured, computed or estimated parameters. Parameter dependency is given under the time (parameter) dependent variation of the coefficient matrices. The linear represents the casual structure of the dynamic problem in state space where the dynamic and the output equations are the linear combination of the states and the inputs. The LPV description preserves the linear time invariant (LTI) structure, the only difference stays at the computation of the coefficients. The parameter vector is a continuously time-dependent known function. It has been shown that non-linear systems could be cast into an LPV form by several ways. Therefore, the LPV model is not unique. In the particular case when the parameter vector coincides (partially or entirely) with the state vector the system is called quasi Linear Parameter Varying (qLPV) system.

The goal of the paper is the development of a control-oriented LPV model of freeway flow. This model should contain the complex behaviour of traffic flow and should be able to reproduce traffic phenomenons. Moreover the LPV structure will make it possible to apply the LPV design methodology which is an effective way to control and observe non-linear systems.

The paper is divided into 5 sections. After the introductory section, in the problem statement part, the paper describes briefly the freeway traffic model and formulates the problem. The forthcoming part presents the proposed solution for parameter-dependent modelling of the freeway flow. Analytic questions are answered in the next section. Finally simulation results illustrate the accuracy of the qLPV model.

2 Problem statement – freeway models

Recent traffic researches are mainly based on the second order macroscopic traffic flow model [5, 7, 16]. This model uses aggregated traffic variables, such as traffic density, space mean speed and traffic flow to describe freeway flow. Fig. 1 illustrates a freeway stretch.
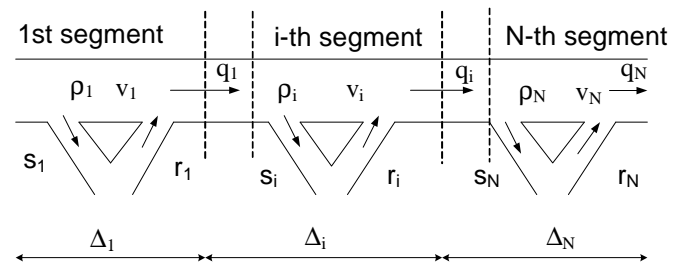

Fig. 1. Freeway division and traffic variables

Due to the complex behaviour, the model is discretized in space; the stretch is subdivided into N segments with length $\Delta_i, i = 1 \ldots N$ and each segment is given by its traffic variables denoted by the subscript as follows:

$\rho_{i(k)}$ denotes the density of the i-th segment at time step k $\left[\frac{veh}{km}\right]$

$v_{i(k)}$ denotes the space-mean speed of the i-th segment at time step k $\left[\frac{km}{h}\right]$

$q_{i(k)}$ denotes the traffic flow leaving the i-th segment at time step k $\left[\frac{veh}{h}\right]$

$s_i(k)$ denotes the off ramp flow of the i-th segment at time step k $\left[\frac{veh}{h}\right]$

$r_i(k)$ denotes the on ramp flow of the i-th segment at time step k $\left[\frac{veh}{h}\right]$

After introducing these variables, the nonlinear difference equations of the second-order macroscopic traffic flow model for a segment i are written by:

$$\rho_i(k+1) = \rho_i(k) + \frac{T}{\Delta_i n} \left[q_{i-1}(k) - q_i(k)\right] + \frac{T}{\Delta_i n} \left[r_i(k) - s_i(k)\right] \quad (1)$$

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$$s_i(k) = \beta_i(k) \cdot q_{i-1}(k) \tag{2}$$

$$v_i(k+1) = v_i(k) + \frac{T}{\tau}[V(\rho_i(k)) - v_i(k)] +$$
$$\frac{T}{\Delta_i} v_i(k)[v_{i-1}(k) - v_i(k)]$$
$$-\frac{\nu}{\tau}\frac{T}{\Delta_i}\frac{\rho_{i+1} - \rho_i(k)}{\rho_i(k) + \kappa} - \frac{\delta T}{\tau \Delta_i}\frac{r_i(k)v_i(k)}{\rho_i(k) + \kappa} + \xi_i^v(k) \tag{3}$$

$$V(\rho) = v_f exp\left[-\frac{1}{a}\left(\frac{\rho}{\rho_{cr}}\right)^a\right] \tag{4}$$

$$q_i(k) = \rho_i(k) \cdot v_i(k) \cdot n + \xi_i^q(k) \tag{5}$$

where T denotes the sample time, n is the number of lanes and a, v_f, ρ_{cr}, κ, τ, δ, ν are additional constant parameters. The macroscopic model was shown to work accurately with segment lengths in the order of 500 meters (or less) [2]. Longer sections could be built up by the interconnection of several segments through the boundary relations (i.e. ρ_{i+1}, v_{i-1}). The second order macroscopic model is used as a basis of different problems regarding the freeway control and surveillance.

The most challenging problem in freeway control engineering is the state (density, speed and volume) observation. Special inductive loop-detectors are installed at distinct locations (usually 4-5 kilometers far from each other) in a freeway's pavement, not in the entire stretch of freeway. These detectors collect traffic data from a single point i.e. no information is available between their installation points. Using the dynamical equations of freeway flow and the theory of state estimation one could design a freeway estimator which filters out the measurement and process noises and gives a suboptimal estimation of the traffic variables between detector stations. This technique multiplies the available set of data, and this additional information could be used for better freeway control and incident detection.

3 Derivation of the qLPV model

The analysis and synthesis of complex systems require complex mathematical techniques. Complex usually covers the nonlinear effect and the large number of state variables. Parameter-dependent models have been motivated by the simplification of the analytic and control design properties.

Let us define the following continuous time, nonlinear and input affine state space model under the form of a Linear Parameter Varying (LPV) system

$$\begin{bmatrix} \dot{x}(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A(p(t)) & B(p(t)) \\ C(p(t)) & D(p(t)) \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} \tag{6}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^{n_u}$ is the control input vector, $y(t) \in \mathbb{R}^{n_y}$ is the measured output vector and $p \in \mathcal{P}$ is the parameter vector over a given compact set $\mathcal{P}$. A, B, C and D are parameter varying coefficient matrices with the appropriate dimensions.

Two alternate classes of the parameters exist. Exogenous and endogenous variables can be defined. With the use of exogenous parameters the dynamic evolution of the nonlinearities occurring in the system can be hidden by replacing them with a variable.

The value of the exogenous parameter needs to be known by measurements, computation or estimation. The entire trajectory of $p(t) \in \mathcal{F}_\mathcal{P}$ is not known, though the value of $p(t)$ must be available at the given time τ, and hence a system might be re-evaluated at A_τ, B_τ, C_τ, D_τ.

Quasi linear parameter-varying (qLPV) systems are applied whenever any of the scheduling parameters becomes a state of the system as well. By definition, the actual value of the parameter is required to the computation of the coefficient of the equation of motion. The selection of quasi LPV model is not unique. Hence, the (quasi) parameter-dependent approach provides a certain modelling flexibility.

The affinity of the parameter dependent description gives a special class of LPV system:

$$A(p(t)) = A_0 + p_1(t)A_1 + p_2(t)A_2 + ... + p_N(t)A_N \tag{7}$$

$$B(p(t)) = B_0 + p_1(t)B_1 + p_2(t)B_2 + ... + p_N(t)B_N \tag{8}$$

$$C(p(t)) = C_0 + p_1(t)C_1 + p_2(t)C_2 + ... + p_N(t)C_N \tag{9}$$

$$D(p(t)) = D_0 + p_1(t)D_1 + p_2(t)D_2 + ... + p_N(t)D_N \tag{10}$$

Eqs. (7)-(10) formulate the linear dependency of the coefficient matrices on the element of the parameter vector p. The resulted parameter-dependent matrix is given by the linear combination of the parameters p_i, $i = 1..n$.

Apart the affine (quasi or not) LPV approach, the polytope method is a general way to describe parameter-dependent systems. The polytope LPV model is a set of linear time invariant plants over a predefined parameter envelope. The linear plants cover a polytope. Exact linear model information is only available at the distinct point of polytope, at the grid points of the parameter envelope. In between the grid points, linear interpolation subjected to the given parameters is applied to compute the model.

Model nonlinearities in the second order macroscopic freeway model arise in several forms. Equations contain exponential relations between states ($V(\rho_i)$), multiplication of states (i.e. $\rho_i \cdot v_i$) and also dividing with states $\left(\frac{v_i}{\rho_i + \kappa}\right)$.

In order to handle these nonlinearities one may first reformulate the basic density-speed relation as:

$$V(\rho) = v_f\left[1 - \frac{\rho}{\rho_{op}}\right] \tag{11}$$

where ρ_{op} is the traffic density value corresponding to the scope of freeway control. The Fig. 2. shows this linear approximation of the fundamental equation. First order curve is fitted on a set of measurement, similar as Greenshields suggested. Data had been collected on a 4.5 km long highway section($M3$) in Hungary. The linear approximation is valid only up to a given density (50 veh/km).

Note that the transformation:

$$f(x) = A(x)x, \quad A(x) = \int_0^1 \frac{\partial f(\lambda x)}{\partial \lambda x}d\lambda \tag{12}$$
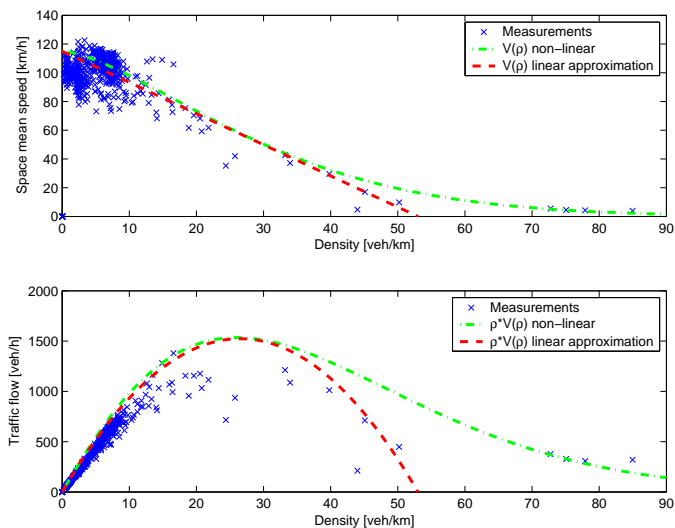
Fig. 2. The nonlinear and the fitted linear fundamental relationship

does not work in this case because $f(0) \neq 0$, so the exact factorization could not be performed.

After this modification the scheduling parameters could be chosen to include the nonlinearities as follows $p_{1i} = v_i$, i.e. the space mean speed of each segment, $p_{2i} = \frac{1}{\rho_i + \kappa}$ and finally $p_{3i} = \frac{v_i}{\rho_i + \kappa}$.

Then a freeway stretch with N segments could be thought as an affine qLPV system with the following state variables:

$$x = \begin{bmatrix} \rho_1 & v_1 & \dots & \rho_i & v_i & \dots & \rho_N & v_N \end{bmatrix}^T \in \mathbb{R}^{2N} \quad (13)$$

The control inputs are the merging flow of segments with on-ramps:

$$u = \begin{bmatrix} r_1 & \dots & r_i & \dots & r_N \end{bmatrix}^T \in \mathbb{R}^m \quad (14)$$

The scheduling parameter vector is partitioned as follows:

$$p = \begin{bmatrix} v_1 \frac{1}{\rho_1+\kappa} \dots v_i \frac{1}{\rho_i+\kappa} \dots v_N \frac{1}{\rho_N+\kappa} \frac{v_1}{\rho_1+\kappa} \dots \\ \frac{v_i}{\rho_i+\kappa} \dots \frac{v_N}{\rho_N+\kappa} \end{bmatrix}^T \in \mathbb{R}^{2N+m} \quad (15)$$

Clearly the first $2N$ co-ordinate of the scheduling vector corresponds to the states, and the remaining m co-ordinates correspond to the control inputs. With this chose, the state matrices are:

$$A(p(k)) = A_0 + \sum_{j=1}^{2N} p_j(k)A_j \in \mathbb{R}^{2N \times 2N},$$

$$A_0 = \begin{bmatrix} 1 & 0 & \dots & \dots & 0 \\ -\frac{v_f}{\rho_{op}} & 1-\frac{T}{\tau} & 0 & \dots & 0 \\ \vdots & -\frac{v_f}{\rho_{op}} & \ddots & & 0 \\ 0 & & & 1 & 0 \\ 0 & & & -\frac{v_f}{\rho_{op}} & 1-\frac{T}{\tau} \end{bmatrix}$$

$$A_{2i-1} = \begin{bmatrix} 0 & \dots & 0 & \dots & & 0 \\ 0 & & \ddots & \dots & & 0 \\ \vdots & \dots & & -\frac{T}{\Delta} & & 0 \\ 0 & \frac{T}{\Delta} & 0 & \frac{T}{\Delta} & & 0 \\ \vdots & & 0 & \frac{T}{\Delta}(1-\beta) & 0 & \ddots & 0 \\ \vdots & & & & 0 & & 0 \\ 0 & \dots & & & & \dots & 0 \end{bmatrix},$$

$$A_{2i} = \begin{bmatrix} 0 & \dots & 0 & \dots & & 0 \\ 0 & & \ddots & \dots & & 0 \\ \vdots & \dots & & \frac{\nu T}{\tau \Delta} & 0 & -\frac{\nu T}{\tau \Delta} & 0 \\ \vdots & & & & \ddots & & 0 \\ 0 & \dots & & & \dots & & 0 \end{bmatrix}$$

$$B(p(k)) = B_0 + \sum_{j=2N+1}^{2N+m} p_j(k)B_j \in \mathbb{R}^{2N \times m},$$

$$B_0 = \begin{bmatrix} \frac{T}{\Delta n} & \dots & 0 \\ 0 & \ddots & 0 \\ & 0 & \frac{T}{\Delta n} \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix}, B_j = \begin{bmatrix} 0 & \dots & & 0 \\ 0 & \ddots & & 0 \\ & 0 & \frac{\delta T}{\Delta \tau} & 0 \\ & & 0 & \ddots \\ \vdots & \vdots & & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The vector of measured disturbance (could be taken into consideration in controller design):

$$W \in \mathbb{R}^{2N \times 1}, W = \begin{bmatrix} q_0 \\ v_0 + v_f \\ 0 \\ v_f \\ 0 \vdots \\ 0 \\ \rho_{N+1} + v_f \end{bmatrix}$$

4 Quadratic stability

This section shows the advantages of the LPV formalism with respect to traffic flow modelling and control.

Before designing a state feedback for control purposes one has to satisfy the stabilizability criteria. Moreover, the dual pre-condition of the state estimator is the detectability that needs to be fulfilled. The stabilizability and detectability check in linear case can be solved by computing and validating Kalman's rank conditions. If the system is non-linear, the equivalent controllability and observability distributions are often hard to compute especially for higher dimensions. One of the main advantage of LPV systems is the simplicity of the above analytic propeties. The notion of quadratic stabilizability and quadratic detectability is known [15, 17]. Just a brief traffic-oriented overview is given in the sequel for highway flow modelling purpose.

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The scheduling parameter $p(k)$ is an $2N + m$ dimensional parameter vector, which takes its value from the *parameter set* $\mathcal{P} \subseteq \mathbb{R}^{2N+m}$.

Each parameter co-ordinate allowed to vary in a convex set: $p_i \in \left[p_i^{min}, p_i^{max} \right]$. Then the $\mathcal{P}$ set is defined by:

$$\mathcal{P} = \left\{ p, p_i \in \left[p_i^{min}, p_i^{max} \right] \text{ for all } i \right\}, \mathcal{P} = conv(\mathcal{P}_0)$$

where $\mathcal{P}_0$ denotes the vertices of $\mathcal{P}$.

The selection of a single Lyapunov candidate might lead to a feasible quadratic stability problem with *parameter independent solution*, P where the system is given by the Eq. (6). The single Lyapunov function and parameter independent gain given by:

$$
\begin{aligned}
V(k) &= x(k)^T P x(k) & (16) \\
u(k) &= K x(k) & (17)
\end{aligned}
$$

The dissipative condition can be written as:

$$V(k+1) - V(k) < 0$$
$$V(k+1) - V(k) = x(k+1)^T P x(k+1) - x(k)^T P x(k) =$$
$$= x(k)^T \left[A(p(k)) + \right.$$
$$B(p(k)) K]^T P$$
$$[A(p(k)) + B(p(k)) K]$$
$$x(k) - x(k)^T P x(k) =$$
$$= x(k)^T \left[[A(p(k)) + B(p(k)) K]^T P \right.$$
$$[A(p(k)) + B(p(k)) K] - P] x(k) < 0$$
$$(18)$$

which is equivalent with:

$$[A(p(k)) + B(p(k)) K]^T P [A(p(k)) + B(p(k)) K] - P \prec 0$$
$$P - [A(p(k)) + B(p(k)) K]^T P [A(p(k)) + B(p(k)) K] \succ 0$$

Using the Schur decomposition the following matrix inequality is given

$$\begin{bmatrix} P & [A(p(k)) + B(p(k)) K]^T \\ [A(p(k)) + B(p(k)) K] & P^{-1} \end{bmatrix} \succ 0$$

With a simple pre-multiplication [3] the inequality is transformed to

$$\begin{bmatrix} G^T P G & G^T A(p(k))^T + G^T K^T B(p(k))^T \\ A(p(k)) G + B(p(k)) K G & P^{-1} \end{bmatrix} \succ 0$$
$$(19)$$

Let us denote Q as the inverse of P and apply $Y = KG$ with a lower bound approach for the remaining Q^{-1} term.

$$\begin{bmatrix} G^T Q^{-1} G & G^T A(p(k))^T + Y^T B(p(k))^T \\ A(p(k)) G + B(p(k)) Y & Q \end{bmatrix} \succ 0$$

Finally, the following LMI condition for the quadratic stability of the discrete time system can be given by

$$\begin{bmatrix} G^T + G - Q & G^T A(p(k))^T + Y^T B(p(k))^T \\ A(p(k)) G + B(p(k)) Y & Q \end{bmatrix} \succ$$
$$0 \text{ for all } p \in \mathcal{P} \quad (20)$$

Since $\mathcal{P}$ is the convex hull of $\mathcal{P}_0$, it is sufficient to verify quadratic stability for all $p \in \mathcal{P}_0$. This implies a finite number of LMIs at the 2^{2N+m} vertices.

Parameter dependent quadratic *gain* can be formulated with a single Lyapunov function. Let us assume the affine parameter dependent gain:

$$K(p(k)) = K_0 + p_1 K_1 + p_2 K_2 + \ldots p_{2N+m} K_{2N+m} \quad (21)$$

Recall the closed loop system to be quadratically stable. The definite condition derived above takes the following form:

$$P - [A(p(k)) + B(p(k)) K(p(k))]^T$$
$$P [A(p(k)) + B(p(k)) K(p(k))] \succ 0$$

The problem is in the latter inequality, which in general not affine in p. As a consequence, for fixed $x \in \mathbb{R}^n$ the function $f_x : \mathcal{P} \to \mathbb{R}$ defined by:
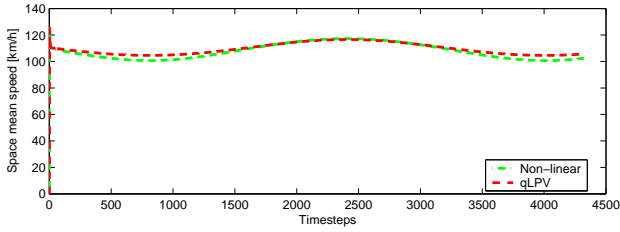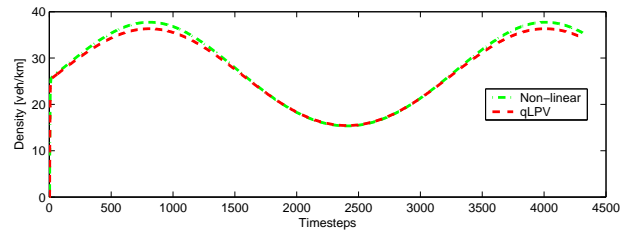
$$f_x(p) = x^T \left[P - [A(p(k)) + B(p(k)) K(p(k))]^T \right.$$
$$P [A(p(k)) + B(p(k)) K(p(k))]] x$$
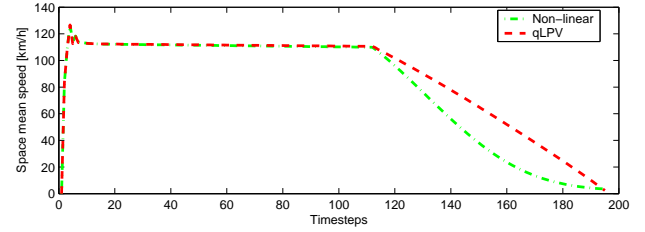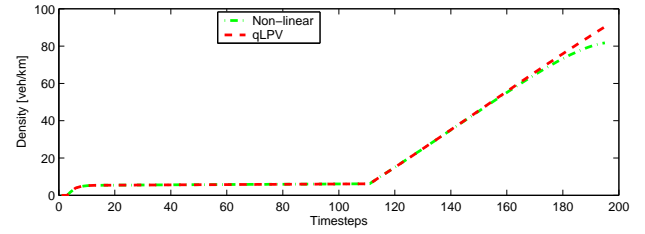
will not be convex so that the implication

$$\{ f_x(p) < 0 \text{ for all } p \in \mathcal{P}_0 \} \Rightarrow \{ f_x(p) < 0 \text{ for all } p \in \mathcal{P} \}$$

used in the previous section will not hold [15]. The solution is that the $f_x(p)$ should be partially convex in each of its arguments p_i. Since f_x is a twice differentiable function, the partial convexity implies:

$$f_x^{''} = \frac{\partial^2 f_x}{\partial p_j^2} \geq 0$$

Fig. 3. The accident free and the accidental flow simulation

So the solution for parameter dependent gain will be calculated by the following non-strict LMIs:

$$\begin{bmatrix} G^T + G - Q & G^T A (p (k))^T + Y (p (k))^T B (p (k))^T \\ A (p (k)) G + B (p (k)) Y (p (k)) Q \end{bmatrix} \succ 0 \qquad (22)$$

$$\text{for all } p \in \mathcal{P}_0 \qquad (23)$$

$$P = P' \succ 0 \qquad (24)$$

$$\begin{bmatrix} 0 & Y_2^T B_2^T \\ B_2 Y_2 & 0 \end{bmatrix} \succ 0 \qquad (25)$$

A generic solution of quadratic stability problem can be derived from parameter dependent Lyapunov function ($P(p)$) with parameter dependent gain ($K(p)$). Let us suppose the inverse of the $P(p)$ is an affine function of p by

$$Q (p (k)) = Q_0 + p_1 (k) Q_1 +$$
$$p_2 (k) Q_2 + \ldots p_{2N+m} (k) Q_{2N+m}$$

Taking the parameter variation in time into consideration, the change of the inverse can be computed under the form:

$$p_i (k + 1) = p_i (k) \pm \lambda_i \qquad (26)$$
$$Q (p (k + 1)) = Q (p (k)) \pm \lambda_1 (k) Q_1 \pm \lambda_2 (k) Q_2 +$$
$$\ldots \pm \lambda_{2N+m} (k) Q_{2N+m} \qquad (27)$$

Based on the dissipative energy condition the parameter dependent Lyapunov criteria might be driven back to the LMI condition

$$\begin{bmatrix} G^T + G - Q (p (k + 1)) & G^T A (p (k))^T + Y (p (k))^T B (p (k))^T \\ A (p (k)) G + B (p (k)) Y (p (k)) & Q (p (k)) \end{bmatrix} \succ 0 \qquad (28)$$

$$\text{for all } p \in \mathcal{P}_0 \qquad (29)$$

$$Q = Q' \succ 0 \qquad (30)$$

The ensure quadratic stabilizability of the qLPV freeway model the upper and lower bounds of the scheduling parameters (and their variations) were determined from real traffic measurements. Finally the derived LMI conditions were found to be feasible at the vertices of the parameter set defined by the parameter bounds.

5 Numerical example

This section gives an example to compare the fully nonlinear and the qLPV traffic models.

Using the same constant parameter values determined by identification for all segments, the comparison of the non-linear model and the derived qLPV model are carried out. To perform simulation, a simple freeway stretch was built in MAT-LAB/Simulink. The stretch consists of three 500 meters long segments, each with two lanes. There is an on-ramp in the middle segment. First the two model were compared under slowly varying traffic flow, the typical characteristics of the morning and evening rush hours are represented through changing flow and speed. Simulation response of the models for the case of normal flow and interrupted (accident) flow are shown on Fig. 3.

In the second case an accident was simulated in the third segment, by suddenly decreasing the outflow. The responses are given on Fig. 3.

As it could be seen on Fig. 3, the nominal qLPV model can accurately simulate the dynamics of freeway flow. Clearly the response of the qLPV model is more like linear under fast variation, due to the linear approximation of the fundamental diagram. Also a small difference between the models appears when the density rises over the critical values, denoted by ρ_{op}. On the other hand these effects could be taken into consideration through robust qLPV framework, which will be in focus of our further research.

6 Conclusion and further research

The paper presents a generic model formalism, the Linear Parameter Varying (LPV), in order to handle nonlinearities in a complex highway flow model.

The paper clearly implies the advantages of the modelling technique and derives feasible stability conditions. Quadratic stabilizability and detectability questions are answered using Linear Matrix Inequalities (LMI). Single Lyapunov function is assumed to make the closed loop system feasible.

In the near future, the advantage of formulating a parameter dependent Lyapunov function, or parameter dependent gain ($K(p(t))$ or $L(p(t))$) will be given. On the other hand, further works will be carrying on the control of highway flow with the help of LPV systems.

Acknowledgement

References

1 **Bokor J, Balas G**, *Detection filter design for lpv systems - a geometric approach*, Automatica **40** (2004), 511-518.

2 **Cramer M, Papageorgiou M**, *Parameter identification for a traffic flow model*, Automatica **17** (1981), 837-843.

3 **de Oliveira MC, Bernussou J, Geromel JC**, *A new discrete-time robust stability condition*, Systems and Control Letters **37** (1999), 261-265.

4 **Hegyi A, Girimonte D, De Schutter B, Babuska R**, *A comparison of filter configurations for freeway state estimation*, 2006.

5 **Hegyi A, De Schutter B, Hellendoorn H**, *Model predictive control for optimal coordination of ramp metering and variable speed limits*, Transportation Research Part C **13** (2005), 185-209.

6 **Lighthill MJ, Whitham GB**, *On kinematic waves. part I: Flood movement in long rivers. part II. a theory of traffic flow on long crowded roads.*, Proceedings Royal Society **A229** (1955), 281-345.

7 **Kotsialos A, Papageorgiou M, Mangeas M, Haj-Salem H**, *Coordinated and integrated control of motorway networks via non-linear optimal control*, Transportation Research Part C **10** (2002), 65-84.

8 **Kulcsár B, Bokor J**, *Robust LPV detection filter design using geometric approach*, 2004.

9 **Kulcsár B, Varga I**, *Simulation of turning rates in traffic systems.*, 2004.

10 **Luspay T, Kulcsár B, Varga I, Bokor J**, *Modelling freeway traffic flow: an LPV approach*, 2007.

11 **Blosseville JM, Papageorgiou M, Haj-Salem H**, *Modelling and real-time control of traffic flow on the southern part of Boulevard Peripherique in Paris: Part Imodelling. part II: Coordinated on ramp metering.*, Transportation Research Part A **24A** (1990), 345-370.

12 **Mihaylova L, Boel R, Hegyi A**, *Freeway traffic estimation within particle filtering framework*, Automatica **43** (2007), 290-300.

13 **Peni T, Bokor J**, *Robust model predictive control for a class of linear parameter varying systems*, 2007.

14 **Richards PI**, *Schock waves on the highway*, Opns Res **4** (1956), 42-51.

15 **Scherer C, Weiland S**, *Linear Matrix Inequalities in Control*, Delft Center for Systems and Control, Delft, The Netherlands, 2004.

16 **Wang Y, Papageorgiou M**, *Real-time freeway traffic state estimation based on extended Kalman filter: a general approach*, Transportation Research Part B **39** (2005), 141-167.

17 **Wu C**, *Control of Linear Parameter Varying Systems*, University of California at Berkeley, 1995.

18 **Zhang HM, Jayakrishnan R, Richie SG**, *Coordinated traffic-responsive ramp control via nonlinear state feedback*, Transportation Research Part C **9** (2001), 337-352.