A NEW APPROACH IN URBAN TRAFFIC CONTROL SYSTEMS

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Abstract

The paper proposes some new methods for traffic control systems design. The goal was to find solutions that make possible to give constraints in course of the estimation process of the road traffic parameters and the traffic controller system design. The proposed solutions are built on the constrained Moving Horizon Estimation and the Fault Detection Filter technique.

Keywords: traffic control, parameters estimation, traffic light.

1. Introduction

Traffic engineering makes the dynamic or static analysis and synthesis of automotive vehicle technologies possible. The main goal engineering is to design and manage of traffic systems. Quite a long time passed since traffic issues have been first taken into consideration; in fact, traffic lights were reported to exist already more than a century ago. At their introduction, to ensure safety of participants in the traffic has been the only reason for utilization, but as time went by and traffic increased, due to safety reasons control issues have been added. Nowadays, traffic lights are used in order to:

- disable concurrent traffic flows to enter at the same time in the same intersection,
- decrease congestion and overall delay in a certain intersection,
- harmonize transport modes.

The majority of intersections all over the world base the signals optimization on the stage and cycle duration concept [14]. The program in operation at intersections where the concept of stage is applied has a predefined duration during which the colour of each traffic light remains unchanged. Modern urban traffic control
systems do not follow the concept of cycle, and instead, introduce a high density of sensors for real-time traffic measurement. The explosive growth of population and the continuously augmenting level of development experienced in the past centuries induced numerous changes also in traffic and transportation. Modern world is characterized by a massive increase in the number of vehicles, which often use a transportation infrastructure with limited capacity at the same time. When this occurs, traffic congestion is very frequent, queuing and delays are experienced. Furthermore, traffic congestion leads to the degradation of the available infrastructure, reduced safety, and increased environmental pollution.

The paper is organized in 5 sections. After the introductory part, the new control structure is detailed in the second section. The third section briefly summarizes the estimation techniques for the split rate approximation and shows how to apply them to a basic traffic system. The fourth section describes a design method for congestion detection filter. The next section presents a traffic light control, based on the length of vehicle queue. The fifth part gives a numerical example regarding the estimation and control. The conclusion contains further research issues.

2. The Control Structure

The factors above presented the necessity of the implementation of control techniques in traffic and the introduction of innovations in the transportation area. In order to reach a safer, more efficient and less polluting transportation, optimization in the utilization of the available infrastructure is required. This can be attained by applying suitable traffic control methods. It is to be noticed that the efficiency of traffic control directly depends on the efficiency and relevance of the employed control methods. The advanced traffic control strategies are applied in: Urban road networks, freeway networks, and route guidance and information systems. The approaches in the paper deal only with urban road network control. On the other areas results may be achieved with systems built in a similar way.

The behaviour of the traffic flow is influenced by two main factors: the control inputs, respectively, the disturbances incurred. The control inputs are directly related to corresponding control devices such as traffic lights, variable message signs, etc. The manipulation of disturbance values is not possible, but there are cases when their measurement is possible (e.g. demand), detectable (e.g. incident) or predictable over a certain time horizon.

The performance of the network is measured through suitable parameters such as the total time spent by all the vehicles in the network over a time horizon. The information required by the control strategy applied and the human operators, and provided by measurement devices can be enhanced through surveillance. The base of the control loop is control strategy, the task of which is to specify the control inputs in real time, based on available measurements and estimations.

The main control measure in urban road networks is the traffic lights at intersections. Traffic lights, besides ensuring the safety of road crossings, may also
help in the minimization of the total time spent by all the vehicles in the network, provided that an optimal control strategy is applied [15].

### 2.1. Traffic Systems as Positive Systems

Positive features have special significance in several areas. State variables and parameters of road traffic systems also have positive features, the values of the state variable - based on their original physical meaning (*naturally positive*) - are always positive (e.g. traffic flow, number of vehicles, traffic density). Moreover the control inputs are also positive. In this view one should apply the theory and results of positive systems to road traffic process. Such systems have strict conditions (terms) of controllability, so it is reasonable (practical) to treat naturally positive systems as general linear systems and control them in the positive orthans with controllers.

### 3. Measuring and Estimation

In order to examine the dynamic aspects of the traffic control, it is necessary to have information on certain variables such as the number of vehicles. Measuring variables in a dynamic system can be very costly, therefore, their prediction is a commonly adapted solution. The prediction of the variables is addressed in one of our earlier papers, as well [8].
The permanently varying demand on road networks, regarding mainly intersections or motorways, needs to be estimated. Firstly, because traffic volumes can only be measured at the input or at the output point of the section, not in the intersection, itself. Secondly, the most important task of traffic planning is the co-ordinate control of more than one intersection, which is based on estimated variables. The need could be defined by the dynamic OD (Origin Destination) matrix, which shows the links among the entries and exits in a certain intersection.

One of the basic elements in traffic network systems is the intersection. One divides the intersection into three parts such as entry, exit and internal flows. The measurement of both the entry and the exit flows might be assumed. Traffic density cannot be measured without error, so the idealized flow plays role in theoretical aspects only. A model setup of entry-exit travel demands regarding an intersection allows to use estimation methods to determine the internal link flows. The key of the model buildup is the split parameter ratios. The split rate determines the turning percentage of the vehicles entering a traffic system. If one assumes that these turning rates are slowly varying split probabilities, the methods to determine probabilities are called split ratio methods [2, 11]. The split rates define a turning proportion.

The state variables are the proportions of entry-flow split, according to the destinations. At an intersection like the above there are no traffic lights and the right of way is not regularized, since, from estimation point of view, one takes only input and output volumes varying in time into account. However, traffic regulation can be applied in model description. In this case the mathematical model for the dynamic process of exit volume is rather elementary.

Let us consider the following intersection model

\[
y_j(k) = \sum_{i=1}^{m} q_i(k) x_{ij}(k) + v_j(k),
\]

where \( q_i(k) \) is the traffic volume entering the intersection from entrance \( i \) at time \( k \), \( y_j(k) \) is the traffic volume leaving the \( j \)th way of intersection, \( x_{ij}(k) \) is the percentage of \( q_i(k) \) (split rate) that is destined to exit \( j \), \( v_j(k) \) is a zero mean noise term. The input measurement is a noisy term, since \( q_i(k) = \tilde{q}_i(k) + \xi_i(k) \), with the same assumption for the noise \( \xi_i(k) \) as above \( (i = 1,..n \text{ and } j = 1,..m) \).

Split variables are independent trials. The model and its constraints are given by

\[
x_{ij}(k+1) = x_{ij}(k) + w_{ij}(k)
\]

\[
0 \leq x_{ij}(k) \leq 1
\]

\[
\sum_{j=1}^{m} x_{ij}(k) = 1.
\]

The random variation in the split parameter is small, and the \( w_{ij}(k) \) is a zero mean random component. All random components \( \xi, v, w \) are mutually independent terms.
The problem is to observe the \( x_{ij}(k) \) states under certain conditions. The difficulty of the task is that constraints have to be taken into consideration. In the presented case, two types of constraints are applied (inequality and equality), but further constraints may be implemented. When using state estimation, constraints are difficult to be inserted into the observer as this would lead to a constraints state estimation problem.

A possible way to estimate states is to apply a finite back stepped state observer. A class of optimal state estimation methodologies is called Moving Horizon Estimation (MHE)/Receding Horizon Estimation (RHE) method. First the initial state is applied to the estimation of nonlinear systems. Findeisen [6] summarized the advantages of MHE and featured the different update formulae. Rao [16] presented questions related to filter stability even regarding nonlinear, constrained stochastic systems. [17] combined the estimation with hybrid systems. [3] examined the probabilistic aspects of the MHE technique. [10] used a stable MHE subjected to inequalities.

If one chooses the horizon equal to one, the one stepped moving estimation process always uses the 1 back stepped measurement and the actual one. Let as denote the actual step by \( k \), and the \((N = 1)\) estimator is given by

\[
\Psi_k = \min_{\hat{x}_0, \hat{w}_{k-2|k}, \hat{w}_{k-1|k}} \psi_k
\]

subject to the following dynamic equality constraint:

\[
\begin{align*}
\hat{x}_{k-1|k} &= \hat{x}_{k-1} + G \hat{w}_{k-2|k} \\
\hat{x}_{k|k} &= A \hat{x}_{k-1|k} + G \hat{w}_{k-1|k}
\end{align*}
\]

and the following measurements:

\[
\begin{align*}
y_{k-1} &= C_{k-1} \hat{x}_{k-1} + \hat{v}_{k-1} \\
y_k &= C_k \hat{x}_k + \hat{v}_k.
\end{align*}
\]

One needs to note that output map \( C \) is time-dependent, since the elements of \( C \) are the input measurements. Henceforth one defines the supplementary equality and inequality constraint coming from the geometry of the example intersection.

\( Q \) and \( R \) are weighting matrices. If the expected output is small, \( R^{-1} \) has to be chosen large, compared to \( Q^{-1} \), and the resulting sensor noise vector becomes small, compared to \( \hat{w}_{j|k} \). On the other hand, if the measurements are not reliable, \( Q^{-1} \) should be chosen large, compared to \( R^{-1} \).

\( \psi_0 \) is the so-called arrival cost to the analogue of the cost to go in MPC technique. The arrival cost summarizes all knowledge about the best estimation before the \( N \)-th step. Regarding the unconstrained linear case, the arrival cost can be expressed explicitly. If state or noise inequality constraints or nonlinearities are
present, we do not have an analytic expression to generate the arrival cost. Though an analytic approach is unavailable, an approximate cost may be given. When inequality constraints are inactive, the approximation is exact. Therefore, the poor choice of the arrival cost leads to the filter’s instability. The results have been published in more details in our earlier papers [8, 7].

4. Traffic Light Control

4.1. Local Control of Intersection

Theoretically, the optimal control problem may be readily solved for any road network. What is, in fact, difficult is to realize the problem in a control loop and obtain the solution in real-time. This is so, due to real-world issues such as:

- Many unpredictable and hardly measurable disturbances (incidents, illegal parking, pedestrian crossings, intersection blocking, etc.) that may perturb the traffic flow,
- the red-green switching of traffic lights requires the introduction of discrete variables, which renders the optimization problem combinatorial,
- the problem is that size of a whole network is very large,
- traffic conditions are measured mostly locally (via inductive loop detectors) and is highly noisy,
- tight real-time constraints.

The above factors make the solution to a detailed optimal control problem for more than one intersection infeasible. In order to cope with this issue, researchers and practitioners have introduced different kinds of simplification in the road traffic control strategies they propose. The introduction of these simplifications, although effective in certain configurations, renders the corresponding control strategies less suitable to address traffic saturation phenomena.

In the following the control of a single intersection (of 4 ways) will be designed. It is supposed that the vehicles inflow and exit from the queue when the traffic light is green, according to the following model:

\[ x_{opt}(k) = Q_m u(k) \]  \hspace{1cm} (11)

In reality, the number of vehicles that exit the intersection may differ from this:

\[ x_{opt}(k) = x_{real}(k) + x_f(k) \]  \hspace{1cm} (12)

The \( x \) state represents the number of vehicles standing in a certain branch of the intersection.

\[ x(k) = x_1(k), x_2(k), x_3(k), x_4(k) \]  \hspace{1cm} (13)
The queue evolves, according to the formula below:

\[ x(k + 1) = x(k) + x_{in}(k) - x_{out}(k) \quad (14) \]

The state equation:

\[ x(k + 1) = Ax(k) + Bu(k) + x_{in}(k) + v_q(k) + x_f(k) \quad (15) \]

The measuring equation:

\[ y(k) = Cx(k) + v_y(k) \quad (16) \]

The control problem will be solved by the LQ control approach proposed by M. Papageorgiu [12]. The cost function is:

\[ J = \frac{1}{2} \sum_{k=0}^{K} (x^T(k) Q x(k) + u^T(k) R u(k)) \quad (17) \]

In the extension we proposed it was taken into consideration that it was not always possible that \( Bu \) vehicles exit the intersection.

\[ \text{Fig. 2. Local intersection controller (LQ) with Congestion Detection Filter} \]

The Congestion Detection Filter (CDF) predicts the \( x_f \) value that will be used in the controller. The CDF is a kind of fault detection filter. There are various options
to detect an unknown input in a dynamic system. The most promising approaches are the design of an unknown input observer (UIO) [1], a system inversion regarding the unknown input [5], detection filter design [9] for a complete isolation of the unknown input and the noise effect in the filter output error space [4] or the parity space approach.

4.1.1. Simulation Results

As an illustrative example, let us consider the following simple traffic intersection of four ways. The simulation results have shown that the reconfigurable controller (LQ with Congestion Detection Filter) is significantly reducing the number of vehicles in a given ramification of the analysed intersection (See Fig. 3, Fig. 4, Fig. 5).

Fig. 3. The number of vehicles in queue with fix-time control

4.2. Model Predictive Control in Traffic Network

Road traffic networks with more junctions could be controlled based on the model described in 4.1. The state-equation can be written in the following form:

\[ x(k + 1) = Ax(k) + Bu(k) + x_{in}(k) + w(k) \]  (18)
Fig. 4. The number of vehicles in queue with reconfigurable controller (LQ with Congestion Detection Filter)

where \( x \) denotes the number of vehicles standing in queue, \( u \) denotes the input signal, \( x_{in} \) denotes the number of vehicles entering in the network and \( w \) denotes the sum of the not measurable failures. These not measurable failures include the process- and the sensor (measurement) noise. The measurement equation:

\[
y(k) = Cx(k) + v(k)
\]

where \( v \) denotes the zero-mean, Gaussian white sensor-noise. The above system is controllable with Model Predictive Control also if the following assumptions are valid:

\[
u_i \geq t_{MIN} \quad u_i \leq t_{MAX} \quad \forall i \quad \sum_{i=1}^{O_j} u_i \leq t_{j MAX}^{MAX} \quad j = 1...J
\]

where \( u_i \) denotes the effective green times, \( O_j \) denotes the number of traffic lights in the \( j \) intersection. While the state variables are the queue-lengths the system is naturally positive so the control in the positive orthans should be done with controller. To access the whole positive orthans one could need several steps to be considered when choosing the horizon of MPC. The prediction horizon should be chosen for such a number of steps that the entire positive orthans is accessible in those steps.
Fig. 5. Green time in case of reconfigurable controller (LQ with Congestion Detection Filter)

5. Conclusion

In the paper it was shown that in the control of traffic systems there exist techniques that can properly deal with the constraints in these systems. In the estimation process of the road traffic parameters and in the design of traffic controller systems results from the control and systems theory can be utilized effectively. The utilization of some of the techniques presented in the paper is particularly important in traffic control. The techniques and their utilization can be summarized as follows:

- MHE technique: utilizing the constraint MHE technique also nonlinear traffic system processes may be observed.

- Fault Detection Filter technique: with the help of the Fault Detection Filter technique congestions in local intersections may be detected.

- MPC technique: the MPC technique can be used (giving the state variables’ positive characteristics as constraints) for controlling the linear time invariant system in such a way that the system stays in the positive orthans.
References