PERIODICA POLYTECHNICA SER. TRANSP. ENG. VOL. 51, NO. 1, PP. 35–44 (2007)

THE INFLUENCE OF ROUNDED EDGES ON THE SHAFT-HUB CONTACT

Ferenc KOLONITS

Department of Railway Vehicles
Budapest University of Technology and Economics
H–1521 Budapest, Hungary
Phone: (+36-1) 463-1939, Fax: (+36-1) 463-4382
e-mail: kolonf@iif.hu

Received: Sept. 14, 2006

Abstract

The interfacial pressure in press fits displays local peaks at the edges. Theoretically, if there were perfect square corners, they would be infinite as singularities. However, studies of plane contacts showed that even a small rounding yields finite peaks. In the present work, the appearance of this feature in cylindrical geometry is studied, with its possible consequences on the corrosion-assisted fatigue, the fretting. A substitute model of rigid hub vs. elastic shaft leads to an integral equation, which is solved by a numeric integration.

Keywords: press fit, contact edge, hub and shaft, fretting, rounding.

List of Symbols

E	Modulus of elasticity, MPa
G	Shear modulus, MPa
h	Nominal interference, mm or related to shaft radius
I_n	Modified Bessel function of first kind, n-th order
I	Integral calculated by trapezoid rule
L	Half length of hub, or related to shaft radius
p	Pressure on the mating surfaces
p_L	Pressure according to Lamé
R	Nominal fillet radius characterizing also the level-off of interference
r, φ, z	Radial, tangential and axial co-ordinate, respectively
r_1	Radius of the shaft, unit of relative length scale
u	Radial displacement of surface
$\mathbf{Z}$	Flexibility matrix or a minor of that
α	Length of the level-off of interference, related to L
ν	Poisson number
ω	Influence function of pressure on an elemental ring surface onto displacements
ζ	Axial co-ordinate in relative units

1. Introduction: The Fretting in Shaft-hub Contact

The press fit or shrink fit is a widely applied solution for joining machine elements in order to transmit torque (while withstand to bending). An important example for that is the mounting of wheels onto railway axles. The rotating shafts of rail vehicles had been of paramount importance in identifying fatigue at all and building up its theory. Under the "classical" concept of fatigue (i.e. crack initiation by repeated load cycles with peaks in space and time), there is emerging a more complex effect of growing significance: the fretting.

If two bodies are pressed together and slip on each other, there is a certain amount of friction work generated. That is originated by breaking and deforming of surface roughness peaks and finally dissipated as heat. There will be abrasive products appearing and a corrosion process starts. The process also generates local stresses, which in case of repetition mean a specific kind of surface fatigue, assisted by corrosion. At end's end, it results incipient cracks. For fretting, there are interface pressure, friction and repeated relative motion as originating cause necessary. What they can result, that depends on the stress state of the sub-contact area concerned. The widely accepted criterion of Ruiz and Chen (in its modified form [3]) considers the product of surface shear stress, slip amplitude and highest principal stress, respectively, as an index for fretting danger. Therefore, a survey of local stresses and relative, even extremely small relative motions is necessary.

2. Overview of Press Fit Characteristics

The design of press fits in the general practice is performed with respect to the torque to be transmitted. The basic formula used is that of Lamé for thick wall tube, from that and with a friction coefficient one might calculate the shaft-hub interference needed. There may be some semi-empirical factors in order to assess strength and life expectancy. It is well known that the contact pressure grows high toward the edges of the hub and there is a good deal of experimental work presented on that along with the moment sustainability of the joint. The details, however, are approached mainly by FEA, which is although a powerful tool, needs a judicious application to "go down enough" into the realm of highly localized effects. Here, even rough models might be of value, if they showed some main characteristics of an effect. They can also give some guidance, how to model them in FEA if necessary.

The theoretical approach of press fits seems to be rather abandoned. Following the classical papers of OKUBO [6], LUR'E [5] and LIVSHIC [4] back in the '50s, there are yet some publications [7] somewhat later which try to handle the problem on a theoretically developed base, however essentially with numerical means. The computational possibilities of those times set narrow limits to these efforts.

3. Developments Concerning Two-dimensional Flat Contacts

An interesting revival of theoretical methods happened recently to the related problem of two-dimensional contacts. The practical interest had been awakened by the dovetail roots of gas turbine blades, turbo-generator coil wedges etc. These problems are planar ones and a half-space approach can be used to. Here, the contact of two elastic bodies might be substituted by a rigid punch intruding into a half-space, elastic parameters having suitably adjusted of [2]. Were the punch square-edged, the theory identifies infinite singular pressures at the corners. These are, however, rather imaginary, because being limited by plasticity and extremely sensitive to even slight round-offs as well. The effect of the latter, analysed in the book of SHTAYERMAN [8] about sixty years ago, had been re-discovered by CIAVARELLA [1] and made starting point for further important investigations. The Shtayerman analysis yielded a rather intricate but closed-form solution. According to that, rounding the edges eliminates the singularity. Instead of an abrupt non-smooth jump of tangents of mating surfaces while they going to separate, there will be a "friendly divorce" taking place with a common tangent at the separation point (see *Fig. 1*). There will be no infinite peak, the Boussinesq criterion holds true i.e. the interfacial pressure decreases to zero at the contact limit. The greater the rounding radius, the lower is the existing finite peak. As the radii on the two sides of the punch merge into one, the contact shall be a Hertzian and the two finite peaks at the sides melt into a half-elliptic pressure distribution according to that.

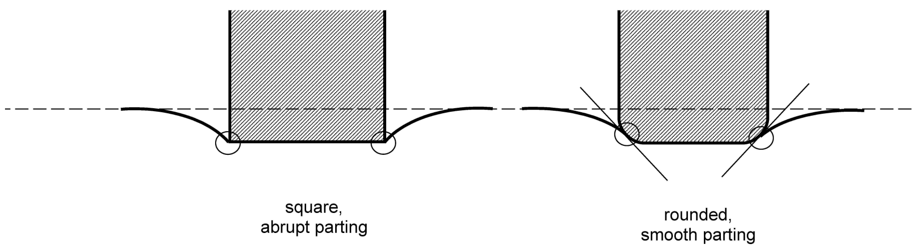

Fig. 1.

4. An Approach to Cylindrical Contact

4.1. Preliminaries

Efforts aimed at utilizing Shtayerman's result as an approach to cylindrical contacts remained fruitless. However, they more than justified a search for something similar for those sort of problems. For this purpose, the analysis of Livshic's mentioned paper was reconsidered, taking into account the computing power greatly enhanced since then. He had developed solutions in infinite series separately for hub and shaft, respectively, and coupled them by a kind of collocation. Relatively few, 6-10 members were used, at even less locations. In the following, the results for an elastic shaft mated with a rigid and symmetric hub will be discussed – this is a parallel to Shtayerman's model.

Livshic studies the elastic shaft loaded with an arbitrary distributed pressure by Papkowitch's method. There, the displacements and stresses are produced by generally three stress functions, two of them being enough for the problem at hand. The displacements are received in form of complex integrals, the evaluation follows by the contour method of Lur'e. Finally, the radial displacement on the surface of the shaft is

$$u(\zeta) = -\frac{r_1(1-\nu)}{2G(1+\nu)} \int_{-L}^{+L} p(t) \left[-\frac{1-\nu}{2} \omega\left(|\zeta - t|\right) \right] dt. \tag{1}$$

(Livshic denominates the full content in brackets as ω. The lengths inside the integral are related to r_1). The influence factor of a pressure load on an infinitesimal band at ζ=0 is

$$\omega = \sum_{s=1}^{\infty} (a_s \cos \gamma_s \zeta + b_s \sin \gamma_s \zeta) e^{-\delta_s \zeta}. \tag{2}$$

Here γ_s, δ_s are defined by the γ_s+iδ_s $= \beta_s$ solutions of the equation

$$\psi(\beta) = \beta^2 \; I_0^2(\beta) - I_1^2(\beta) \; - 2(1-\nu) I_1^2(\beta) = 0, \tag{3}$$

a_s, b_s are defined by the equation

$$b_s + i a_s = 8 I_1^2(\beta_s)/\psi'(\beta_s). \tag{4}$$

The Bessel functions in the above equations may be approximated from sixth root on by asymptotic formulae. In the present analysis, they are used as starting approaches and the defining equation is solved repeatedly by Newton's method. This worked well up to the 279^{th} root, then, the MATLAB routines showed "out of application range". A check was made for this set of solutions applying a formula of Livshic for infinite summation:

$$-(1+\nu)\Sigma(a\delta + b\gamma)/(\delta^2 + \gamma^2) = 1. \tag{5}$$

With the 279 solutions, a sum 0.99811 was obtained.

4.2. Simplifying the Contact Equation

In order to perform the analysis in a possibly general form, let u also be measured in r_1 units and the pressures related to p_L, the pressure pertaining to a constant h

interference (according Lamé):

$$p_L = Eh/(1-\nu). \tag{6}$$

Applying these substitutions,

$$u/h = \frac{1+\nu}{2}\int_{-L}^{+L} p(t)\omega\,(|\zeta - t|)dt. \tag{7}$$

As a further check, the above equation had been investigated for an infinite hub, $L \to \infty$, with a constant interference. Here, the pressure must be constant and $u/h = -1$, respectively. The integral becomes an improper one. The ωt-series will be absolutely convergent because of the negative exponential and may be integrated term by term. Their improper integrals are tabulated ones, finally they yield a series of terms corresponding the *Eq.* 5, the sum being easily obtainable from. At end's end, from the *Eq.* 7 one obtains $p = 1$, the Lamé pressure, as might be expected. The asymptotic behaviour of *Eq.* 7 is verified. The function $\omega(\zeta)$ is shown on *Fig.*

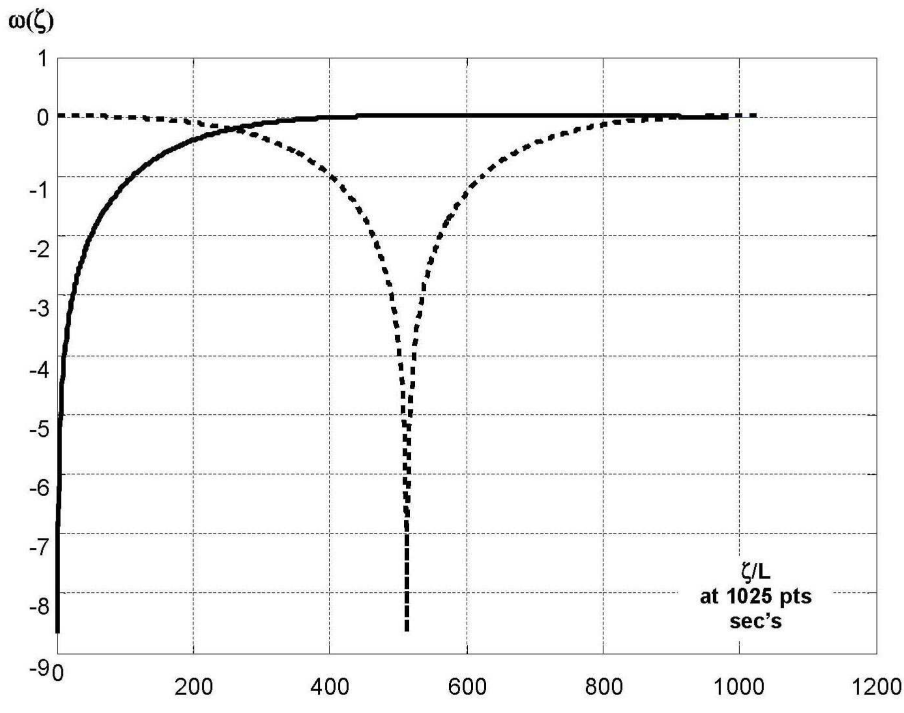

Fig. 2.

2 for $L = 1, 2$, along 1025 points by equal steps in $-L \ldots + L$ interval, both in central and links shifted position. The minute fluctuations along the curves are due the MATLAB point-wise plotting. In the course of calculating the integral, a section between $-2L$ and $2L$ will be shifted in a $-L \ldots L$ window, multiplied by the actual pressure at the cusp and then summed up for each ζg location. For a symmetric hub, the pressure will be also a symmetric one. A load on every infinitesimal band has its counterpart on the opposite side of the hub – being associated of course with a different ω-value. However, if the "wing" of the negative side would be revolved onto the positive half-plane and summed with the $\omega g - g$ values existing there, the full integral could be calculated with multiplying the p-s on the positive side by the summarized, "compound" ω-s. Let us consider n equally distributed points along the half of the hub and be this compound from an original being shifted its cusp to the k-th location denoted with ω_k. The constituent parts of ω_{1000}, i.e. the influence function of a band load at the 1000-th point, the cusp being shifted there, are shown on *Fig. 3*. On the upper part, the rotated curve from the negative side takes place (dotted line), the compound ω is marked with a thick line.

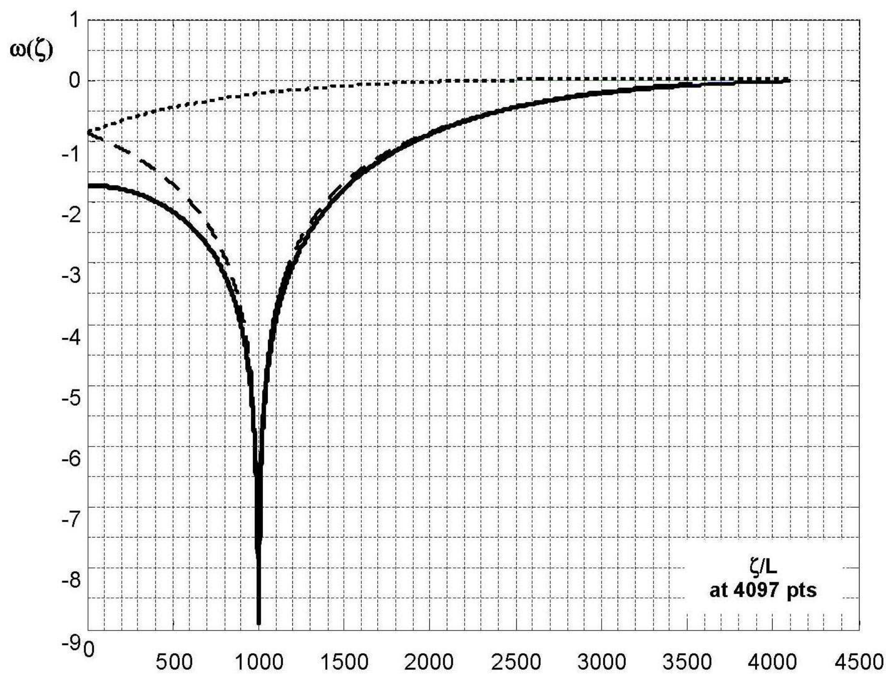

Fig. 3.

4.3. Numerical Scheme for the Contact Equation

Now, taking up $p_1, p_2, \ldots, p_{n-1}, p_n$ (yet unknown) pressures, the integral pertaining to the k-th point might be expressed by the trapezoidal rule:

$$I_k = p_1\omega_{k,1}/2 + p_2\omega_{k,2} + \ldots + p_{n-1}\omega_{k,n-1} + p_n\omega_{k,n}/2. \tag{8}$$

With that, an n-point collocation scheme can be established with a given u/h, k varying from 1,…,n. The rounding of the hub profile is approximated by a parabola with a corresponding curvature, like in Shtayerman's analysis. For the profile,

$$\begin{aligned} u_h &= 0, && \text{if} \quad \zeta \le \zeta_{plain}, \\ u_h &= \zeta - \zeta_{plain}{}^2/(2R) && \text{if} \quad \zeta > \zeta_{plain}. \end{aligned} \tag{9}$$

The total radial displacement of the shaft surface is

$$u = u_R/R - 1, \tag{10}$$

$$u_R = \zeta - \zeta_{plain}{}^2/2. \tag{11}$$

Eq. (7) is an integral equation first kind of Fredholm type. This sort of equations is notoriously ill-conditioned and prone to numerical difficulties. The boundary conditions:

$$p(L) = 0, dp/d\zeta|_{L=0} = 0. \tag{12}$$

The second one is granted by the symmetry. There is a welcome checking possibility with respect to the central portion of the pressure:

$$p(\text{around}\, 0) \approx 1. \tag{13}$$

The u indentation function consists of two parts, i.e. the profile and the general interference, respectively. The equation may be solved independently and the solutions superposed:

$$p = p_R/R + p_{-1}. \tag{14}$$

The calculation starts with a certain L indentation length and plane profile part, respectively. From *Eq.* (14) the R rounding radius will be obtained, at which the boundary condition, i.e. Boussinesq principle becomes satisfied. A drawback of this approach is that, in the course of calculations, a difference of great numbers must be set to zero by an appropriate R. This may me circumvented by taking the indenting profile height at the end at L as variable and iterating that for a zero pressure.

The solution curves exhibit satisfactorily the condition (13). At large, they show similar characteristics as the Shtayerman's ones: the smaller of the radius of rounding, the higher and narrower the peak at the edge of the hub. *Fig. 4* shows a series of pressure distributions, for varying relative rounding indentations:

$$\alpha = 1 - \zeta_{plain}/L = 0,1\ldots0,005. \tag{15}$$

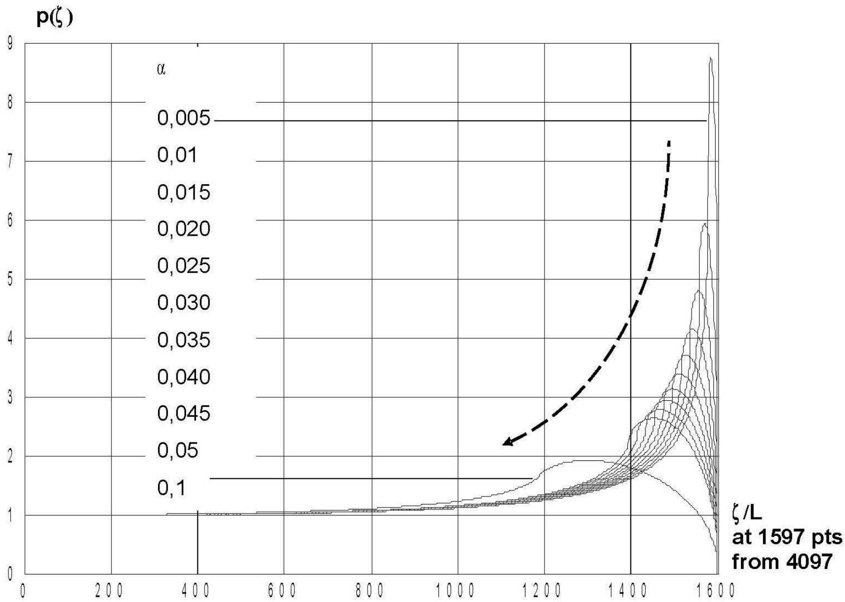

Fig. 4.

There are results at 4097 points. The curves show the parts between 2500-4097 (i.e. 1…1597), the lower section being practically 1. The R goes down to about 30 mm. As α and the rounding radius diminish, the curves turn less and les smooth, primarily at the more susceptible steep changes in pressure. This might be counteracted by variable steps along the hub – accepting in turn, however, a lot of calculations for ω_k, instead of plain shifting. A further correction is possible by approaching pω with a parabola in the last interval instead of the line of trapezoid.

4.4. Local Character of the Edge Effect

A further verification of the solution can be obtained by analysing whether the pressure distribution due to rounded edges preserved the local character as it should be. For this end, a series of calculations had been made, for $L = 0.8\ldots8$, over 4097 points (*Fig. 5*). The peaks are of same height and become ever narrower. If they transform onto the same length, the curves cover each other near to line thickness (*Fig. 6*, the last 1997 points shown).

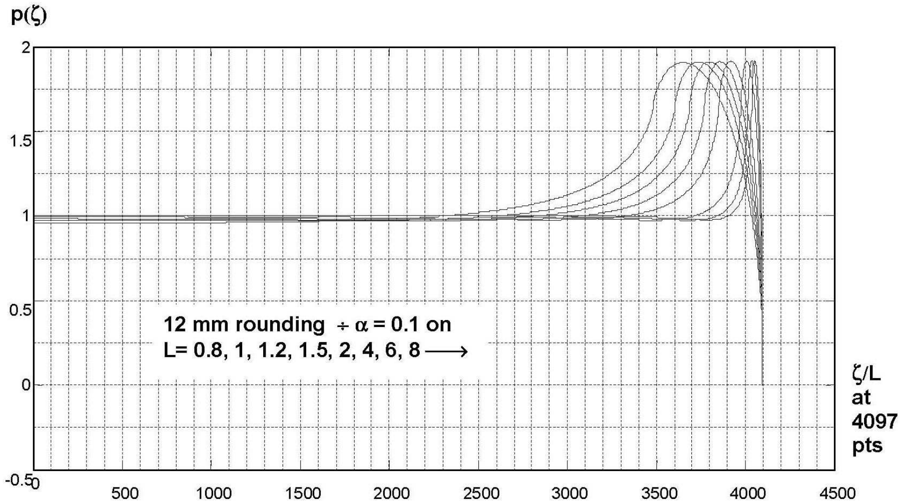
p(ζ)
2
1.5
1
0.5
0
-0.5
0
500
1000
1500
2000
2500
3000
3500
4000
4500
12 mm rounding ÷ α = 0.1 on
L= 0.8, 1, 1.2, 1.5, 2, 4, 6, 8 ⟶
ζ/L
at
4097
pts

Fig. 5.

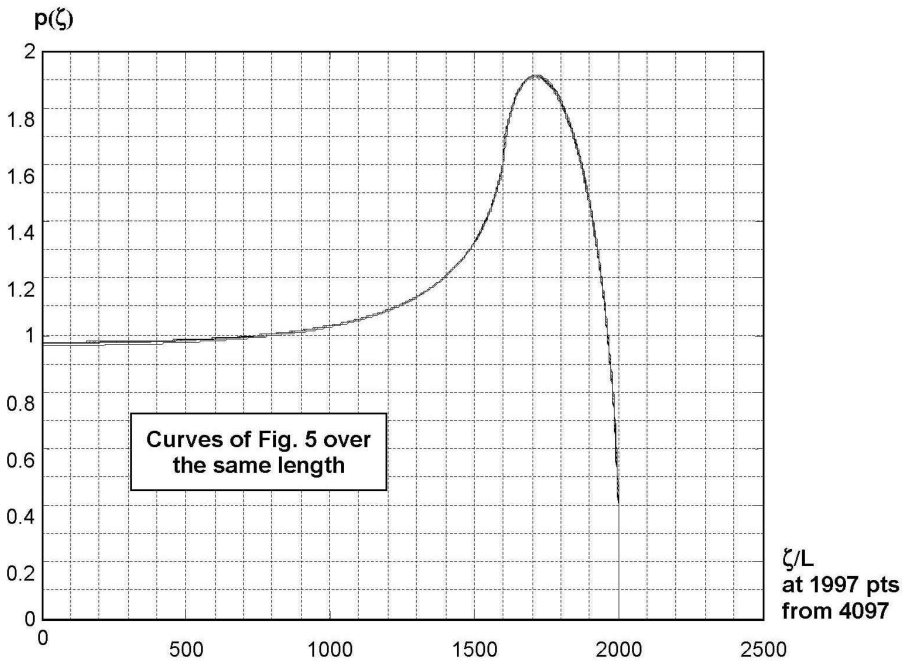
p(ζ)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
500
1000
1500
2000
2500
Curves of Fig. 5 over
the same length
ζ/L
at 1997 pts
from 4097

Fig. 6.

5. Conclusions

As for practical consequences with respect to fretting, let us consider the result got for $\alpha = 0.1$. Here, the peak pressure is about two-fold of Lamé's one and the R would be about 2000 mm. This is not a rounding but rather a level-off of the interference towards the edge of the hub. It might be intentional for assisting the press-in at mounting, or casual, made by repeated laceration. Toward the smaller, technically more real rounding, the peak will be more and more high what must result local plastic deformation. That will limit a less and less, ever-narrowing strip, where the interfacial pressure sinks to zero. On the other hand, the level-off geometry results a more wide, less loaded strip, where an additional bending moment shall have wider sections with greater amplitude to slip on each other. This is in keeping with experience on railway wheels, where fretting had been observed mainly at internal side of the wheels. Here, the hub is prepared with an introductory section for mounting and this is the part of the hub being lacerated by the full length of the shaft while pressing in.

References

[1] CIAVARELLA, M. – HILLS, D. A.– MONNO, G., The Influence of Rounded Edges on Indentation by a Flat Punch, *Proc. Instn. Mech. Engrs* **212** (1998) Part C, pp. 319–328.
[2] HILLS, D. A. – NOWELL, D. – SACKFIELD, A., Mechanics of Elastic Contacts, Butterworth-Heinemann, Oxford, 1993.
[3] LEIDICH, E., Einfluss des Schwingungsverschleißes auf die Tragfähigkeit von Welle-Nabe-Verbindungen, in: Welle-Nabe Verbindungen, *VDI-Berichte* **1790** (2003),VDI-Verl. Düsseldorf, pp. 1-29.
[4] LIVSHIC, I. Z., O raspredelenjii naprjazhenyiy po kontaktnoj poverhnostyi…, *Izv. Akd.Nauk SSSR, Otd. Tehn. Nauk,* (1955) No.4, pp. 22–42.
[5] LUR'E, A. I., Napryazhennoe sostoyanie v uprugom cylindre…, *Inzh. Sbornik, AN SSSR,* **17** (1953) pp. 43–58.
[6] OKUBO, H., The Stress Distribution in a Shaft Press-fitted with a Collar, *ZAMM* **32** (1952) 6., pp. 178–186.
[7] SAVVUSHKIN, E. S., Raspredelenie kontaknovo davlenija v stupice kolesa…, *Vestnik Vses. Nauch-issl. Inst-a Zh-d. Transporta,* (1958) 1, pp. 49–53.
[8] SHTAYERMAN, I. JA., Kontaktnaya zadacha teorii uprugosti, Gosztekhizdat, Moskva, 1949.