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ESTIMATION OF DYNAMIC ORIGIN DESTINATION MATRIX OF TRAFFIC SYSTEMS

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Abstract

The paper suggests the necessity of the estimation of unmeasured variables of traffic systems. The presented method proposes constrained state estimation of unmeasured traffic variable such as turning rates. The weighted constrained state approach uses moving horizon along the state trajectory which permits to handle equality and inequality constraints belonging to the nature of the traffic model. A numerical example illustrates the importance of constrained estimation.

Keywords: traffic simulation, turning rate, moving horizon.

1. Introduction

Automotive technologies grain ground in modern traffic control systems, since there is a perpetual growing need of traffic automation.

In many cases, control-related variables are almost inaccessible for design unless applying estimation techniques. In such a situation the approximation, computer-based estimation of these variables could be useful. The applications of traffic simulation can be classified in several parts. Some basic classifications are the division between microscopic, mesoscopic and macroscopic, and between continuous and discrete time approaches. The methodology of static and dynamic analysis of traffic systems is known. Several state variables, derived from the description of the dynamic, can be used for operational and planning aspects.

A newly emerged area is the demand estimation through microscopic traffic modelling. The dynamic aspect of traffic simulation in a traffic system needs the previously measured or estimated volumes of vehicles. Since the measurement of certain variables in the dynamic description is rather costly, one tries to estimate them. The observation of permanently varying turning rates in a simple intersection is rather costly, however, the amount of the turning vehicle could be applied for traffic light harmonization, generally speaking for traffic light control.

Many estimation techniques exist for giving reliable estimation on dynamic OD(Origin Destination) matrix, their results, however, could be different. The short review on OD estimation begins with the Least Squares (constrained or not), statically based methods such as Likelihood methods [16] and Kalman filtering [5, 3, 4], or Bayesian estimator [26]. Sometimes, combined estimators, using constraints or apriori knowledge about the intersection can be applied.

The greatest drawback of most estimation techniques is the lack of constraints. Usually, to formulate constraints in the estimation either equality or inequality is rather difficult.

Constraints must be taken into account in course of a dynamic process, mainly in OD estimation. A class of optimal state estimation methods is called Moving Horizon Estimation (MHE) methods [7, 18, 22]. On the way of getting a closer look to Moving Horizon Estimation (MHE) processes one can enumerate the contribution of several researchers. After formulating the estimation in a recursive form, Tyler and Morari showed the property of stability of the linear filter. Findeisen summarized and featured the advantage of MHE against the existing widespread methods. Rao has elaborated the filter stability even for nonlinear, constrained highly complicated dynamic systems.

The MHE can be concerned as the dual of the Model Predictive Control, though some special assumptions must be given for filter stability. Another advantage of the Moving Horizon Estimation can be the fact that constraints assumption can be considered in the estimation process. In the following space the Moving Horizon state estimation method is applied in intersection model.

The paper offers the contribution in 5 chapters. After a short introduction, the intersection as a basic element of the traffic system is detailed in the first section. The second section summarizes briefly the estimation techniques for split rate approximation and shows how to apply them for a basic traffic system. The third part gives a numerical example. The conclusion contains further research problems.

2. Traffic Modelling

One of the basic elements in traffic network systems is the intersection. One divides the intersection into three parts such as entry, exit and internal flows. The measurement of both the entry and the exit flows might be assumed. Traffic density cannot be measured without error, so the idealized flow plays role only in theoretical aspects. A model setup of entry-exit travel demands regarding an intersection allows estimation methods to determine the internal link flows. The key of the model buildup is the split parameter ratios. The split rate determines the turning percentage of the vehicles entering a traffic system. If one assumes that these turning rates are slowly varying split probabilities, the methods to determine probabilities are called split ratio methods [5, 15]. The split rates define a turning proportion.

It is supposed that the proportions of entry-flow split, according to the destinations, are variant. At this intersection there are no traffic lights and the right of way is not regularized, since from point of view of the estimation one only takes into account the time varying input and output volumes. However, traffic regulation can be applied in model description. In this case the mathematical model for the dynamic process of exit volume is rather elementary.

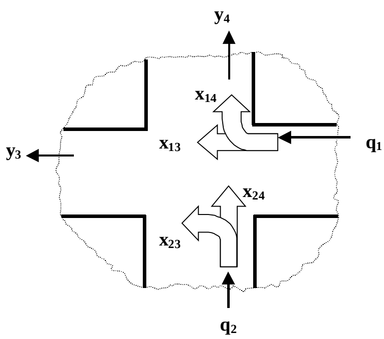

Fig. 1. A simple intersection with two inputs and two outputs

To show the problem the following variables are defined:

- $q_i(k)$ the traffic volume (the number of vehicles over a time period) entering the intersection from entrance i, during time interval $k = 1, 2, \dots, N$
- $y_j(k)$ the traffic volume (the number of vehicles over a time period) leaving the intersection from exit j, during time interval $k = 1, 2, \dots, N$
- $x_{ij}(k)$ the percentage of $q_i(k)$ (split rate) that is destinated to exit j, $k = 1, 2, \dots, N$.

Let us consider the following intersection model

$$y_j(k) = \sum_{i=1}^{m} q_i(k)x_{ij}(k) + v_j(k), \tag{1}$$

where $i = 1, ..n$ and $j = 1, ..m$. $v_j(k)$ is a zero mean noise term. The input measurement is a noisy term, since $q_i(k) = \tilde{q}_i(k) + \zeta_i(k)$, with the same assumption for the noise $\zeta_i(k)$ as above.

Split variables are independent trials. The model and its constraints are given by

$$x_{ij}(k+1) = x_{ij}(k) + w_{ij}(k) \tag{2}$$

$$0 \le x_{ij} \le 1 \tag{3}$$

$$\sum_{j=1}^{m} x_{ij}(k) = 1. \tag{4}$$

The random variation in split parameter is small, and the $w_{ij}(k)$ is a zero mean random component. All random components ζ, v, w are mutually independent terms. The scheme of the MH observer is given in *Fig.2*. For the sake of simplicity,

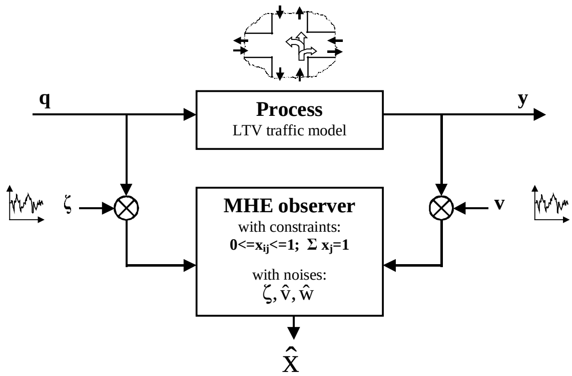

Fig. 2. The MHE observer

let us arrange all elements of the OD matrix in a single vector and use the following notations:

$$
\begin{aligned}
x_k &= [x_{ij}(k)]^T \\
w_k &= [w_{ij}(k)]^T \\
v_k &= [v_j(k)]^T
\end{aligned}
$$

The problem is to observe the x_k states under certain conditions. The latest estimation of the split parameters can be treated as a filtering problem. The difficulty of the task is that constraints have to be taken into consideration. In the presented case, two types of constraints are applied (inequality and equality), but further constraints may be implemented. When using state estimation, constraints can only be put on the observer with difficulty. In the following section one tries to emphasize the effectiveness of the constraint Moving Horizon Estimation (MHE) method as a reliable state observer of split ratios of the intersection layout shown in the *Fig.1*.

3. Receding Horizon State Estimation

The following section describes, in an inductive manner, the general receding horizon approach. Starting form the presentation of the simple one step back estimation process throughout the N stepped one, the final conclusion is the general infinite stepped estimator (Batch Estimator) subjected to constraints. The proof of the filter stability is not outlined in this article.

Let us consider the following discrete linear time-variant system

$$x_{k+1} = Ax_k + Gw_k \tag{5}$$
$$y_k = C_k x_k + +v_k \tag{6}$$

with x_0 given. One can denote that for the first view, without assuming anything about the noise components, w_k is the state error and v_k is the measurement noise vector, the description remains totally deterministic. $x(t)$ is the state vector. A, G are constant parameter matrices, C_k is time-variant output map of the dynamic system.

In our case one can neglect the control input (see previous section to understand the nature of the intersection model), since the split rate are random trials, with $G = A = I_n$ where n is the number of states (i.e. the number of turning rates). In that case the random components can be filled with real content up. These parameters are unknown, because in most of the cases only input and output detectors are installed in intersections.

If one chooses the horizon equal to one ($N = 1$), the one stepped moving estimation process uses always the 1 back stepped measurement and the actual one. Let as denote the actual step by k, and the one stepped estimator is given by

$$\min_{(\bar{x}_0, \hat{w}_{k-2|k}, \hat{w}_{k-1|k})} \Psi_k \tag{7}$$

$$\Psi_k = \hat{w}_{k-2|k}^T Q_0^{-1} \hat{w}_{k-2|k} + \hat{w}_{k-1|k}^T Q^{-1} \hat{w}_{k-1|k} + \hat{v}_{k-1|k}^T R^{-1} \hat{v}_{k-1|k} + \hat{v}_{k|k}^T R^{-1} \hat{v}_{k|k} + \Psi_0 \tag{8}$$

or in a more compact form

$$\begin{aligned} \min_{(\bar{x}_0, \hat{w}_{k-2|k}, \hat{w}_{k-1|k})} & \begin{bmatrix} \hat{w}_{k-2|k} & \hat{w}_{k-1|k} \end{bmatrix} \begin{bmatrix} Q_0^{-1} & 0 \\ 0 & Q^{-1} \end{bmatrix} \begin{bmatrix} \hat{w}_{k-2|k} \\ \hat{w}_{k-1|k} \end{bmatrix} \\ & + \begin{bmatrix} \hat{v}_{k-1|k} & \hat{v}_{k|k} \end{bmatrix} \begin{bmatrix} R^{-1} & 0 \\ 0 & R^{-1} \end{bmatrix} \begin{bmatrix} \hat{v}_{k-1|k} \\ \hat{v}_{k|k} \end{bmatrix} + \Psi_0 \end{aligned} \tag{9}$$

subjected to the following dynamic equality constraint

$$\hat{x}_{k-1|k} = \bar{x}_{k-1} + G\hat{w}_{k-2|k} \tag{10}$$
$$\hat{x}_{k|k} = A\hat{x}_{k-1|k} + G\hat{w}_{k-1|k} \tag{11}$$

and with the following measurements:

$$y_{k-1} = C_{k-1}\hat{x}_{k-1} + \hat{v}_{k-1} \tag{12}$$
$$y_k = C_k\hat{x}_k + \hat{v}_k. \tag{13}$$

One needs to note that the C output map is a time-dependent one, since the elements of C are the input measurements. Henceforth one defines the supplementary equality and inequality constraint coming from the geometry of the intersection (see 3,4).

Geometric (see *Fig. 1*) equality constraints are given by

$$\hat{x}_{13} + \hat{x}_{14} = 1 \tag{14}$$

$$\hat{x}_{23} + \hat{x}_{24} = 1. \tag{15}$$

One can augment the estimation process with some other constraints subjected to noise as well.

Q and R are weighting matrices. If the expected output is small, R^{-1} has to be chosen large compared to Q^{-1}, and the resulting sensor noise vector becomes small, compared to $\hat{w}_{j|k}$. On the other hand, if the measurements are not reliable, Q^{-1} should be chosen large, compared to R^{-1}.

Ψ_0 is the so-called arrival cost to the analogue of the *cost to go* in MPC technique. The arrival cost summarizes all knowledge about the best estimation before the N-th step. For the unconstrained linear case, the arrival cost can be expressed explicitly. If state or noise inequality constraints, or nonlinearities are present, we do not have an analytic expression to generate the arrival cost. Though an analytic approach is unavailable, an *approximate* cost may be given. When inequality constraints are inactive, the approximation is exact. Therefore, the poor choice of the arrival cost leads to the filter's instability.

The Moving Horizon Estimation scheme can be seen in *Fig. 3*.

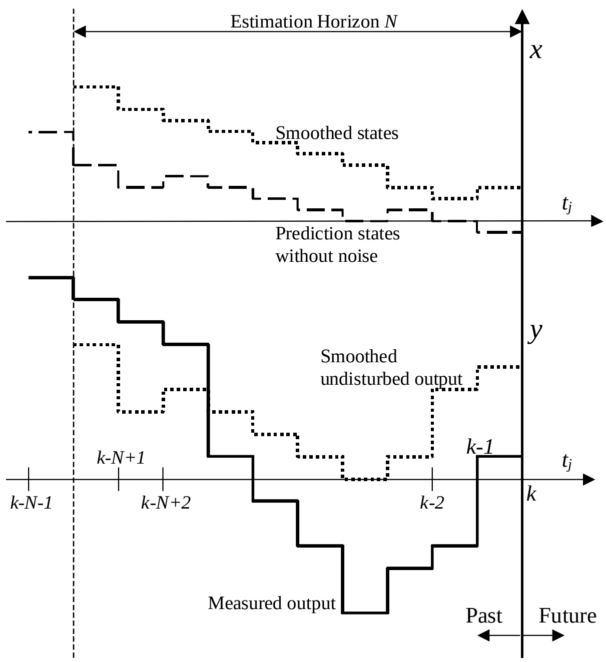

Fig. 3. General Moving Horizon Estimation process

Let the generalized MHE optimization criteria with a horizon N be defined by the following functional

$$\min_{(\bar{x}_{k-N-1},\hat{w}_{k-N-1|k},\ldots,\hat{w}_{k-1|k})} \Psi_k \tag{16}$$

$$\Psi_k = \hat{w}^T_{k-N-1|k} Q_0^{-1} \hat{w}_{k-N-1|k} + \sum_{j=k-N}^{k-1} \hat{w}^T_{j|k} Q^{-1} \hat{w}_{j|k} + \sum_{j=k-N}^{k} \hat{v}^T_{j|k} R^{-1} \hat{v}_{j|k} + \Psi_0 \tag{17}$$

subject to:

$$\hat{x}_{k-N|k} = \bar{x}_{k-N} + \hat{w}_{k-N-1|k} \tag{18}$$

$$\hat{x}_{j+1|k} = A\hat{x}_{j|k} + G\hat{w}_{j|k} \qquad j = k-N-1,\ldots,k-1 \tag{19}$$

$$y_j = C\hat{x}_{j|k} + \hat{v}_{j|k} \qquad j = k-N-1, \tag{20}$$

$$0 \le x_k \le 1 \tag{21}$$

$$\sum_{j=1}^{m} x_{jk} = 1,\ldots,k \tag{22}$$

with R^{-1}, Q^{-1} which are symmetric positive semi-definite noise weighting matrices. While $Q_{-N|k}$ penalizes the $\bar{x}_{k-N}$ initial state, R^{-1} weights the output prediction error and Q^{-1} penalizes all estimated state noise.

To find the initial condition of general MHE, we used a batch estimation for the first $N-1$ step estimates. The stability of the MHE filter is effected by the choice of the initial condition and the weighting matrices, as well. The batch estimator is an infinite horizon state estimator. When applying batch estimation, the entire past behavior of the system is known.

$$\min_{(\bar{x}_0,\hat{w}_{-1|k},\ldots,\hat{w}_{k-1|k})} \Psi_k \tag{23}$$

$$\Psi_k = \hat{w}^T_{-1|k} Q_0^{-1} \hat{w}_{-1|k} + \sum_{j=0}^{k-1} \hat{w}^T_{j|k} Q^{-1} \hat{w}_{j|k} + \sum_{j=0}^{k} \hat{v}^T_{j|k} R^{-1} \hat{v}_{j|k}, \tag{24}$$

subject to:

$$\hat{x}_{0|k} = \bar{x}_0 + \hat{w}_{-1|k} \tag{25}$$

$$\hat{x}_{j+1|k} = A\hat{x}_{j|k} + G\hat{w}_{j|k} \tag{26}$$

$$y_j = C\hat{x}_{j|k} + \hat{v}_{j|k} \tag{27}$$

$$0 \le x_k \le 1 \tag{28}$$

$$\sum_{j=1}^{m} x_{jk} = 1. \tag{29}$$

However, Batch Estimator, even for a small state space is intractable from the point of view of the numerical computation, as the Batch Estimator window is infinite.

Sliding alies estimation windows and contains an initial condition to the method. There is more than one method to assure the transit between to windows, the most plausible approximation is to use the estimated, filtered state. However, there is no direct probabilistic relation for the use of the one step before calculated smoothed state estimation, cycling behaviour of estimation can be avoided. To slide between windows the filtered estimate update is preferred.

4. Simulation Results

In the following space the constrained general MHE is solved for a simple traffic system. Let us assume that the q_1 and q_2 input volumes, entering the intersection and y_3, y_4 volumes leave the intersection. The simulation of such a traffic system creates 4 split rate.

A software for traffic simulation was elaborated and treated as the reference real environment of intersection, furthermore the data provided by it are the 'real' data of the split rate estimation.

This software is a modular simulation environment, what means that – except for some fundamental issues – the simulation kernel can be easily changed. This feature makes the environment able to provide the proper data structure for our scope and the Matlab implementation. The fundamental structure possesses the 'TSimulation' and the 'TNet' classes which are responsible for the kernel and the topological functionality of the microscopic model. For the complete topological modelling, multiple basic classes have been implemented, such as 'TLine', 'TLane', 'TCell' and 'TNode'. Using these classes together, any basic road network can be easily built. For the modelling of the moving entities (such as cars) in the environment, the 'TCar' class has been introduced. Any moving object can be derived from this class. Yet, this environment had not got any interface, signal or control properties, so the definition of a generic class was necessary. The 'TObj' base class is the solution of implementing any other structure into the kernel. All other classes are inherited from this interface class, and this is the feature that makes this environment modular, and able to communicate with other processes. There are multiple inherited classes of the TObj responsible for the more precise modelling, such as 'TCarGenerator', 'TLamp', 'TSimEvent', 'TMeasure', 'TNodeMeasure' etc... When we are talking about microscopic traffic modelling, it means that we are studying the individual vehicle behaviour. The model examines the current state of the vehicle (i.e. speed, position, acceleration etc), its environment (i.e. speed limit, priorities, signals, nodes), its desired travel parameters (i.e. speed, routing), and the vehicles interacting with it (cars before, after, in the neighboring lanes, in the opposite lane etc). By using all known parameters, the vehicle itself decides its way of behaviour for the current moment, [10]. The main aspect of microscopic models is the 'car-following model', where the vehicle adjusts its acceleration (or speed) using the states of the interacting vehicles. Other base decisions are the lane changing, overtaking, and object related interactions. The inbuilt car behaviour model uses Fuzzy-Sugeno type [13] procedures for acceleration, deceleration, lane changing, and for other object-dependent decisions. The input fuzzy sets of the car following are the distance, speed difference and the acceleration difference between the vehicle and the vehicle in front. The lane changing model examines the potential of free flow in the current lane, and the potential for the neighbouring lanes. The car generation routine generates the following times as a Gaussian distribution about the reciprocal mean traffic flow (frequency). The deviation is half of the frequency. Under this scope, the vehicle entities do not have a preferred routing, they decide their directions of turning randomly at each node entrance.

The measurement of the incoming cars is done by the 'TMeasure' class. The entities of this class can simulate a 'measurement at a point' process, which means that they can detect the vehicles passing by them. The measurement records the time of passing, and the speed of the vehicle. The proceeding of the traffic flow uses a moving time frame at each measurement point, with a frame length of ten minutes. The output of this process is the sum of the cars passing by the measurement point in the last ten minutes. (Traffic flow in [veh/10 min]) The data required for the project is the in- and outgoing traffic flow of the node, so measurement points are placed at the end of each incoming lane, at the beginning of each outgoing lane, and at each virtual turning lane in the node for validation purposes.

In the following one tries to gain the turning rates using MHE approach. After describing the discrete time model of the system one can assume to have

$$\begin{aligned} x_{k+1} &= I_n x_k + w_k \\ y_k &= C x_k + v_k, \end{aligned}$$

where the time-dependent C contains the elements of $q_{1,2}$. The structure of C depends upon the layout of the intersection.

Let us suppose we have 5 samples in every second. For solving MHE numerically, one may use a quadratic programme-solver. Let the horizon comprise 1–5 samples, and let us apply the diagonal scaling for R, Q the following results are gained. The simulation covered 300 samples, which took 1 minute. Simulating the split parameter for intersection with 1, 3 or 5 samples a long horizon can be seen in *Figs.* 4, and 5.

In each Figure the 'real' split rates are denoted by dotted line.

Not only the horizon length has an important role on state estimation but also the weighting factors Q and R. The performance of the estimation contrasted can be seen in *Figs.* 6 and 7.

The MHE simulation time is relevant. Even for a horizon of 5 samples and having constraints the elapsed time between starting and stoping was more than five minutes in a Pentium 4, 2.4 GHz PC machine.

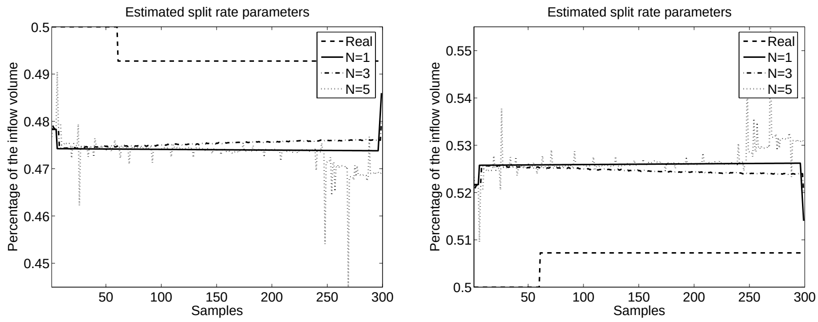

Fig. 4. Real and estimated split rates turning from input 1 to 3, 4

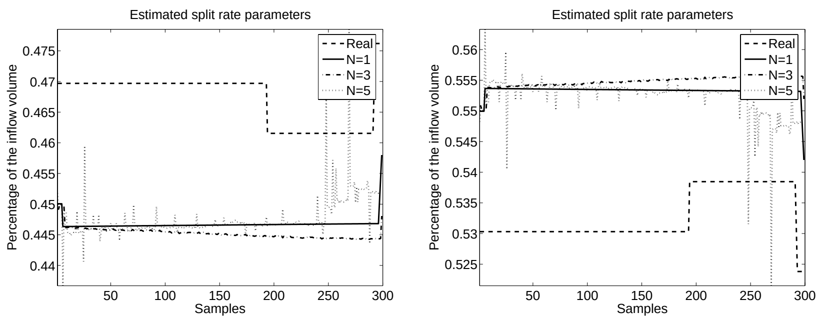

Fig. 5. Real and estimated split rates turning from input 2 to 3, 4

5. Conclusion

The article summarizes the MHE approach for a simple traffic system, for an intersection. In traffic engineering the estimation of split variables is important and could create the base of further control problems. A numerical example has been shown to demonstrate how to apply the Moving Horizon technique for split rate observation.

The MHE optimal estimation method shows a possible way for including constraints into the design procedure. One could possibly extend the state estimation, based on MHE algorithm with some additional constraints in inequality form on states, noise or other variables.

The general MHE technique could be applied to nonlinear processes which will be in the focus of our traffic system estimation research.

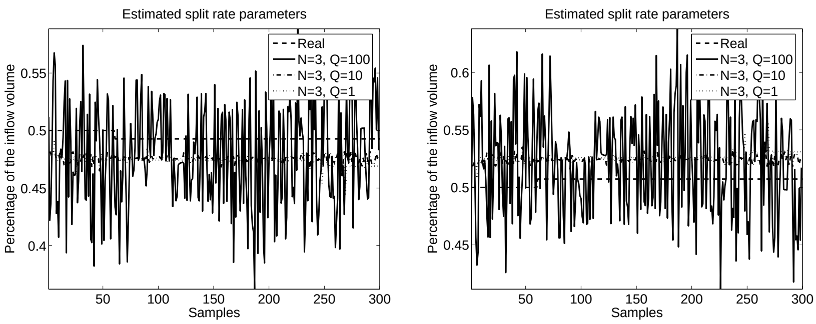

Fig. 6. Influence of the Q weighting on estimated split rates turning from input 1 to 3, 4

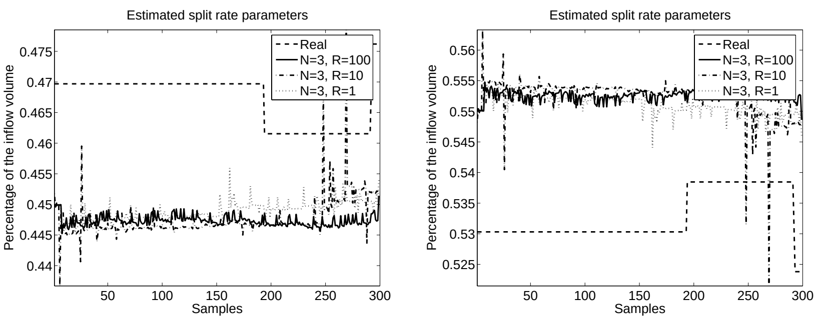

Fig. 7. Influence of the R weighting on estimated split rates turning from input 2 to 3, 4

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