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OPTIMISATION OF INVENTORY CONTROL SYSTEMS WITH GENETIC ALGORITHMS

Krisztián BÓNA

Department of Transportation Technology
Faculty of Transportation Engineering
Budapest University of Technology and Economics
H–1111 Budapest, Bertalan Lajos utca 2., Building Z, Room 604, Hungary
Phone: (36 1) 463 19 26; Fax: (36 1) 463 32 69
e-mail: kbona@kku.bme.hu

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Abstract

The inventory control systems are responsible for the optimal operation of the inventory processes of the automotive companies. Generally, the optimisation of the inventory control system manifests itself in a target conflict representing the implementation of the optimal operation in economic and reliability terms. For the process optimisation, the control parameters of the regulation system should be defined. Their actual settings determine the time of placing orders of auto parts and the required quantities for the optimal operation of the processes defined above. This article presents a particular method of exploitation of the opportunities provided by the computer aided simulation and the genetic algorithms for the optimisation of inventory control systems applying classical inventory mechanisms.

Keywords: inventory control, optimization, simulation, genetic algorithms.

1. The Problem

A stock is generally composed of several stock keeping units (SKU). We supposed in our examinations that the optimisation of the inventory processes for each SKU provides the optimum of the entire inventory system as well. This is the principle of the so called SKU-based inventory optimisation (CHIKÁN [2]). Hereafter, this paper analyses the problems of the SKU-based inventory optimisation.

The SKU-based inventory optimisation is a complex, multi-criteria optimisation problem. The main points are the following:

The basic problem is that the expansion of the reliability of the system and the reduction of the operation related costs are conflicting requirements in terms of inventory planning. The continuous operation of the process requires the expansion of the stock levels, while the economic efficiency demands their reduction.

In addition, the parameters of the processes triggering the inventory system change in time, i. e. the dynamic features of the processes can not be disregarded. The varying character of the customers' demand in the inventory control system influences the operation of all other processes in the system shows a simple example for this statement. Consequently, the actual values of the parameters controlling the system should also be set dynamically, i.e. an adaptive inventory control system should be established being able to determine the actual optimum values of the control parameters considering the changes taking place in the inventory system. This is an essential criterion, as – due to the temporal changes in the system – the actual optimum parameter setting may not be optimal in the future.

Accordingly, the optimisation of the inventory processes implies the obtainment of minimum total costs concerning the inventory process for a given period and/or maximum probability of the satisfaction of all demands entering the system within a specified period. Theoretically, the achievement of these objectives is possible by controlling the inventory processes by setting of the various control parameters so that the above mentioned costs and reliability indicators tend to the right direction.

When establishing the inventory control system - among others - two basic questions should be answered (CHIKÁN [2]):

- What control parameters are required for the optimisation of the process according to the criteria indicated above?
- How to determine dynamically the values of the control parameters to achieve the objectives?

The various inventory mechanisms and the inventory models modelling their operation can be applied to give exact answers to these questions. The control parameters are exactly determined by the inventory strategies applied, while for the calculation of the actual parameter values exact mathematical models are available. Up to now, the operations research specialists developed approximately four hundred various inventory models to model the various inventory strategies and describe them mathematically. However, experiences show that there are very few practical applications (as compared to the number of models).

The main reasons of this situation are:

- The application of the models is frequently tied to constraints that can not be met in the real stochastic processes.
- The application of an exact mathematical formalism to define the target function required to the optimisation is usually very difficult or even impossible.
- The target function of the models is able to manage only the cost or only the reliability parameters.
- If the target function is available, the next problem is the exact solution of the extreme value searching problem.

Consequently, the goal is to develop such an inventory modelling method which eliminates the above described problems when calculating the actual values of the control parameters.

2. Adaptive Dynamic Inventory Management

Surveys about practical inventory management show that there is demand for the development of an inventory management system comprising both the above mentioned inventory mechanisms and the methods modelling their operation (TEN HOMPEL and SCHMIDT [5]). A system like this could be used to the backing of the decision preparation process required to the inventory planning and the automation of the adaptive, dynamic inventory management. The main units and unit parts of the system are as follows:

Input unit

- Query database (to store the data created during the operation of the real inventory system);
- Data conversion system (to create the data groups required to the simulation of the processes taking place in the inventory system);
- Input database of simulation (to store the data groups created by the data conversion system);

Core unit:

- The simulator of the inventory mechanisms (to simulate the operation of the inventory system considering the specified inventory mechanism);
- Output database of simulation (to store the data groups created during the simulation);
- Comparative system (to compare the data created by the simulation system model and the real system, i. e. verification of the inventory simulator);

Output unit:

- Optimizer (to specify the actual values of the control parameters of the applied inventory mechanism);
- Database of optimum solutions (to store the output data groups of the optimisation);
- Optimizer adjuster (to set the optimum values of the optimisation parameters).

The above described control system is able to use adaptively and dynamically the data created during the operation of the stockpiling system for the creation of a data structure modelling the real system. A further advantage is that, simulation techniques and special optimisation procedures can be integrated in the system by which the actual value settings of the parameters controlling the system can be calculated.

Nowadays, various enterprise resource planning systems (ERP) are yet able to log continuously the transactions in the inventory system. The data set needed to the estimations are mostly available or they can easily be created from the stored data (query database). A more problematic issue is to evaluate the possible fields of application and the most rewarding ways of application, i. e. how to extract the data groups important for the inventory planning from these logged data (input and output databases of the simulation) and how to determine the actual values of the control parameters applied in the inventory management system in the simplest, most proper and most dynamic way (database of optimum solutions). The simplified process of the operation of an adaptive, dynamic inventory management system is shown in *Fig. 1*.

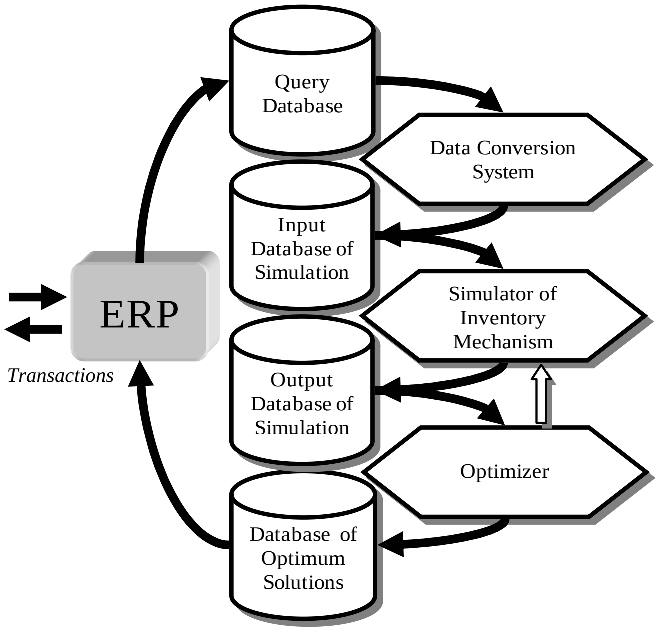

Fig. 1. Process of the dynamic inventory management

The simulator of the inventory mechanisms and optimizer are in tight connection with each other, as one part of the data required to the optimisation is provided by the simulator of the inventory mechanisms through the output database of the simulation.

Hereafter, the presentation of the operation of the simulator of the inventory mechanisms will be explained and a simulator of the inventory mechanisms and genetic optimizer developed by the author using MS Excel and Visual Basic will be presented.

3. Simulation of Inventory Processes

The job of the simulator of the inventory mechanisms is the simulation of the operation of the inventory system by the specified inventory mechanism using the data contained in the input database of simulation. In case of the application of classical inventory mechanisms this can involve the following strategies (CHIKÁN [2]):

[t; q] placing fixed orders (q) in fixed intervals (t);
[t; S] placing orders in fixed intervals (t) and ordering such a quantity, which – when received – completes the stock level to a previously specified maximum level (S);
[s; q] the order should be placed, when the stock level falls below a specified minimum (s) and the quantity to be ordered is fixed (q);
[s; S] the order should be placed, when the stock level falls below a specified minimum (s) and the order specifies a volume, which – when received – completes the stock level to a previously specified maximum level (S).

It is obvious for every classical mechanism, that the optimum values of two control parameters (hereinafter A and B) should be found. In fact, the simulator of the inventory mechanisms is controlled by the optimizer. In every iteration step the optimizer runs the simulator of the inventory mechanisms by setting the actual control parameter. After having run the simulation, the output results should be stored in the output database of the simulation. The optimizer can reach all the required data from this database any time.

The *input database of simulation* should contain the following important data by SKU:

- the statistical parameters describes the elementary processes taking place in the real system (e. g. type of distributions and their parameters describing the demand and supply processes);
- specific cost parameters concerning the operation of the real system (e. g. specific warehouse unit cost, ordering costs by commodities);
- reliability parameters concerning the operation of the real system (required probability levels of the satisfaction of demands);
- parameters characterising the analysed period (e. g. time sections, length of the time sections);
- other auxiliary parameters (e. g. opening stock level, simulation constants).

When initiating the simulation running, the discrete event simulator coded in the simulator of the inventory mechanisms starts to function. The simulator generates the vectors indicated in *Table 1* considering the control parameters of the mechanism applied and the parameters characterising the analysed SKUs for the examination period (T):

Using the elements of the vectors indicated in *Table 1* the following output parameters for every time slot of the examination period (i) can be determined:

- opening stock level – $Q_{ny}^{(i)}$;
- quantities demanded by the customers – $q_{VR}^{(i)}$;
- quantity delivered – $q_{\mathrm{ki}}^{(i)}$;
- unsatisfied demand – $q_{st}^{(i)}$;

Table 1. The simulation vectors

Process element	Interval vector	Quantity vector
Customers' orders	t_{VR}	q_{VR}
Deliveries	t_{ki}	q_{ki}
Warehouse orders	t_{RR}	q_{RR}
Supply processes	t_p	–
Intakes	t_{be}	q_{be}

- quantity ordered by the warehouse – $q_{RR}^{(i)}$;
- received quantity – $q_{\mathrm{be}}^{(i)}$;
- stock in transit – $Q_{ut}^{(i)}$;
- closing stock level – $Q_z^{(i)}$;
- operation related total cost – $K^{(i)}$.

These output parameters should be stored in the *output database of simulation*. The above indicated parameters enable the calculation of all important parameters (e. g. costs, reliability) concerning the simulated operation of the inventory system for the examination period.

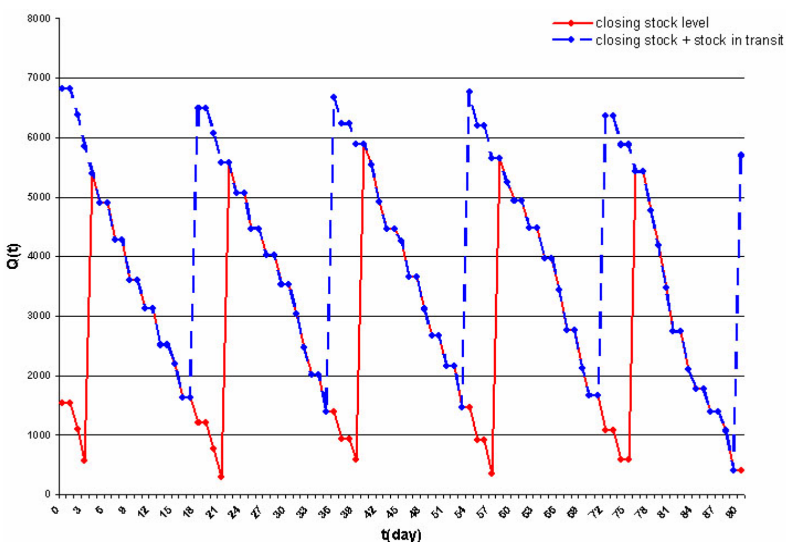

Fig. 2. Operation of the $[t; q]$ mechanism (stock levels)

As show in *Figs.*2 and 3 the simulated operation of a classical $[t; q]$ mechanism. The graphs indicate clearly the quantitative processes taking place in the

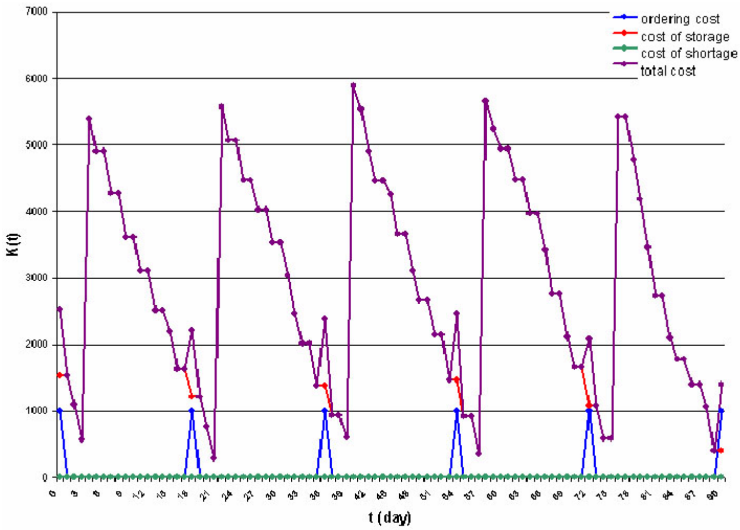

Fig. 3. Operation of the $[t; q]$ mechanism (costs)

inventory system and the time function of the characteristic costs. The most important relations between the above parameters are shown below:

$$Q_{ny}^{(i)} = Q_z^{(i-1)}, \tag{1}$$

$$Q_z^{(i)} = Q_{ny}^{(i)} + q_{be}^{(i)} - q_{ki}^{(i)} - q_{st}^{(i)}, \tag{2}$$

where

$$q_{be}^{(i)} = q_{RR}^{(i-t_P)}. \tag{3}$$

If

$$Q_{ny}^{(i)} + q_{be}^{(i)} \geq q_{VR}^{(i)}, \tag{4}$$

then

$$q_{ki}^{(i)} = q_{VR}^{(i)} \qquad \text{and} \qquad q_{st}^{(i)} = 0.$$

If

$$Q_{ny}^{(i)} + q_{be}^{(i)} < q_{VR}^{(i)} \qquad \text{and} \qquad Q_{ny}^{(i)} + q_{be}^{(i)} > 0 \tag{5}$$

then

$$q_{ki}^{(i)} = Q_{ny}^{(i)} + q_{be}^{(i)} \qquad \text{and} \qquad q_{st}^{(i)} = q_{VR}^{(i)} - q_{ki}^{(i)}.$$

If

$$Q_{ny}^{(i)} + q_{be}^{(i)} < q_{VR}^{(i)} \qquad \text{and} \qquad Q_{ny}^{(i)} + q_{be}^{(i)} < 0 \tag{6}$$

then

$$q_{ki}^{(i)} = 0 \qquad \text{and} \qquad q_{st}^{(i)} = q_{VR}^{(i)}.$$

The size of the travelling stock ($Q_{ut}^{(i)}$) depends always on the date(s) of orders placed before the i^{th} time element, the quantity (quantities) ordered and the expected date of intake(s).

4. Optimisation of Inventory Processes

In case of SKU-based inventory process optimisation the conditions of the optimisation specified earlier should be met, namely:

- the total cost relating to the inventory process for the given examination period should be minimized and/or
- the probability of the satisfaction of the demands entering the system should be maximised.

Of course, the efficiency of the inventory system depends greatly on the mechanism chosen and on the actual values of the control parameters. The basic condition of the process optimisation is the existence of a *target function* by which the above set of (contradicting) criteria can be managed and the optimal settings of the control parameters of the mechanism chosen can be found.

The determination of the inventory related costs is not problematic, as with the application of the specific costs of the process related total cost can be calculated. The two basic specific cost parameters of the model presented are as follows:

- ordering cost (k_t = HUF/order), and
- specific warehousing unit cost (k_r = HUF/pcs*day).

One possible solution for the evaluation of the reliability is a target function being a cost function in which even the reliability of the system is expressed in the form of cost. By means of such a target function both optimisation aspects could be handled on cost basis. A solution for this purpose is the specific deficit cost (k_f = HUF/pcs*day), indicating the losses arising when the system is unable to satisfy the customers' demands. The analyses confirmed unambiguously the points summarised in *Table 2* which can be explained by the learning ability of the system.

Table 2. Relation between the specific deficit cost and the system's reliability

k_f and k_r relation	P(deficit)	Reliability
$k_f \gg k_r$	Small	Big
$k_f \approx k_r$	Medium	Medium
$k_f \ll k_r$	Big	Small

where P(deficit) = the probability of the deficit

The effective reliability of the system can be calculated after the simulation runs and compared with the required reliability. The exact value of the specific costs depends always on the inventory system under review and the commodity. The *target function* (7) can be written in the form:

$$\sum_{i=1}^{T} K^{(i)} = \sum_{i=1}^{T} K_t^{(i)} + \sum_{i=1}^{T} K_r^{(i)} + \sum_{i=1}^{T} K_f^{(i)} \Rightarrow \text{MIN!}, \tag{7}$$

where

$$\sum_{i=1}^{T} K_t^{(i)} = \sum_{i=1}^{T} q_{RR}^{(i)} \cdot k_t ,$$

furthermore

$$\sum_{i=1}^{T} K_r^{(i)} = \sum_{i=1}^{T} Q_z^{(i)} \cdot \frac{\text{sgn}\,(Q_z^{(i)}) + 1}{2} \cdot k_r ,$$

and

$$\sum_{i=1}^{T} K_f^{(i)} = \sum_{i=1}^{T} Q_z^{(i)} \cdot \frac{1 - \text{sgn}\,(Q_z^{(i)})}{2} \cdot k_f .$$

5. Optimisation with Genetic Algorithms

The optimum values of the control parameters of the inventory mechanism chosen will be determined by a binary genetic optimizer. The two parameters (A and B) will be coded by the algorithm in binary form, handled in binary form during the running of the genetic algorithm, then the optimised parameters (A_0, B_0) will be decoded. The operation of the genetic algorithm is shown in *Fig. 4* (GOLDBERG [3]).

The main phases of the preparation (I.) of the optimisation:

- selecting the inventory mechanism to be optimised;
- setting the upper and lower limits of the parameters to be optimised (A_{min}; A_{max}), (B_{min}; B_{max});
- setting the number of the simulation running cycles (N);
- setting the number of the generations (G);
- setting the number of individuals in the generation (E_G);
- setting the number of the offspring in the generation (U_G);
- setting the crossover probability (p_k);
- setting the mutation probability (p_m);
- setting the power of the selection pressure (k).

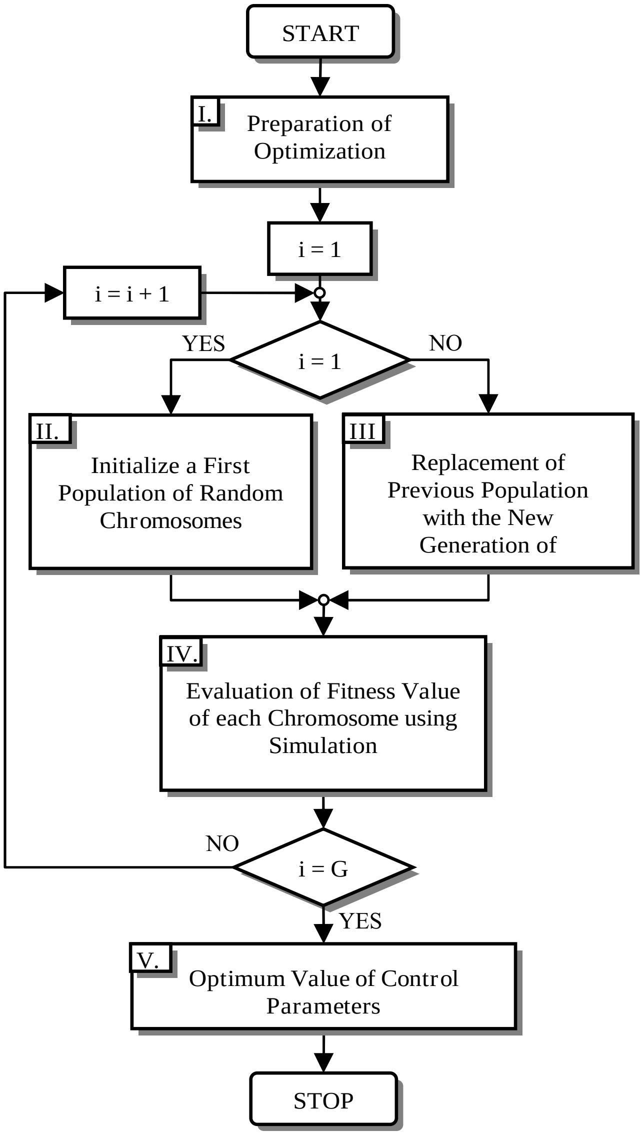

Fig. 4. General flowchart of the operation of the binary genetic optimizer

In case of random initialisation of the chromosomes (II.), a specified (m^{th}) individual of the generation will be created in such a manner that the control parameter A is coded in binary form at the upper A^{bit} bites, while the control parameter 'B' at the lower B^{bit} bits. The coding will be carried out by the following scaling formulae (8) and (9) (MAN, TANG and KWONG [4]):

$$E_m = \underbrace{\overset{A_m^{\text{BIN}}}{[110\ldots101]}}_{A^{\text{bit}}} \; \underbrace{\overset{B_m^{\text{BIN}}}{[001\ldots110]}}_{B^{\text{bit}}}$$

considering that

$$A_{\min} \to A_{\min}^{\text{BIN}} = [000\ldots000]$$

$$A_{\max} \to A_{\max}^{\text{BIN}} = [111\ldots111]$$

$$B_{\min} \to B_{\min}^{\text{BIN}} = [000\ldots000]$$

$$B_{\max} \to B_{\max}^{\text{BIN}} = [111\ldots111]$$

$$\frac{A_m - A_{\min}}{A_m^{\text{BIN}} - A_{\min}^{\text{BIN}}} = \frac{A_{\max} - A_m}{A_{\max}^{\text{BIN}} - A_m^{\text{BIN}}} \tag{8}$$

$$\frac{B_m - B_{\min}}{B_m^{\text{BIN}} - B_{\min}^{\text{BIN}}} = \frac{B_{\max} - B_m}{B_{\max}^{\text{BIN}} - B_m^{\text{BIN}}} \tag{9}$$

The connection between the simulator of the inventory mechanisms and the genetic optimizer will be established at the determination (IV.) of the fitness values belonging to the new individuals of the generation. The fitness values belonging to the actual individual will be calculated by the genetic algorithm pursuant to the mathematical expectation ($M(\sum K)$) of the total cost determined by the simulator of the inventory mechanisms by 'N' simulation runs. Before the simulation runs, the control parameters belonging to the examined individual should be decoded, as the simulator of the inventory mechanisms manages the control parameters in decimal system. After the calculation of the mathematical expectation of the total cost belonging to the m^{th} individual, the genetic optimizer computes the fitness values by the following formula (10) (MAN, TANG and KWONG [4]):

$$F_m = M\left(\sum K\right)_{\min} + \left(M\left(\sum K\right)_{\max} - M\left(\sum K\right)_m\right)^k . \tag{10}$$

The continuous refreshment (III.) of the chromosomes of the generation takes place by the roulette-wheel selection method following the sequencing of the individuals by fitness value, four-point crossover and mutation. Two parents will be selected randomly, and two offsprings are formed, if the relation Random $(0;1) < p_k$ is met. The mutation of the offspring created by the crossover will take place, if Random $(0;1) < p_m$. This process ensures that the iteration results better and better solutions (combinations) and, simultaneously, the diversity of the combinations is maintained (to avoid that the algorithm sticks at a local optimum). After the creation of the appropriate number of generations (G) (V.) – i. e. the execution of the specified number of iteration steps - the algorithm selects the optimum setting values of the control parameters from the last generation by the target function and decodes them by the following formulae (11) and (12) :

$$A_0 = A_{\min} + A_m^{\mathrm{BIN}} \cdot \frac{(A_{\max} - A_{\min})}{2^{A^{\mathrm{bit}}} - 1}, \tag{11}$$

where

A_m^{BIN} is the binary conversion of parameter A.

$$B_0 = B_{\min} + B_m^{\mathrm{BIN}} \cdot \frac{(B_{\max} - B_{\min})}{2^{B^{\mathrm{bit}}} - 1}, \tag{12}$$

where

B_m^{BIN} is the binary conversion of parameter B.

6. Results

To test the $[t; q]$ mechanism, 15 experiments were carried out. The binary genetic optimum searching algorithm was run with varying parameter settings (e.g. number of generations, number of entities or offsprings), but identical input data 100 times. According to the experiences, the search for the optimum parameter setting of the search algorithm is a quite time consuming process, it requires several testing operations and the statistical evaluation of the results. Based on the results of the evaluations, the optimum parameter settings for a given job can be approached by successive approximation. The optimum of the 15 experiments will be shown below.

Parameter settings of the genetic algorithm:

$G = 100$; $E_G = 50$; $U_G = 10$; $N = 20$; $p_k = 85\%$; $p_m = 5\%$;

Results (at a confidence level of 95%):

12.37 days $< t_0 =$ 13.65 days $<$ 14.92 days
3935.28 pieces $< q_0 =$ 4313.12 pieces $<$ 4690.95 pieces
HUF 428340.6 $< K_0 =$ HUF 455023.6 $<$ HUF 481706.5

The second important parameter – the runtime of the optimisation – is depending largely on the capabilities of the computer used. A test carried out on a computer of 1.33 GHz and 128 MB SDRAM, resulted in 69.24 sec $< T_{\mathrm{opt}} =$ 69.99 sec $<$ 70.73 sec. The distribution of the results and the runtime is shown in *Figs.* 5, 6 and 7.

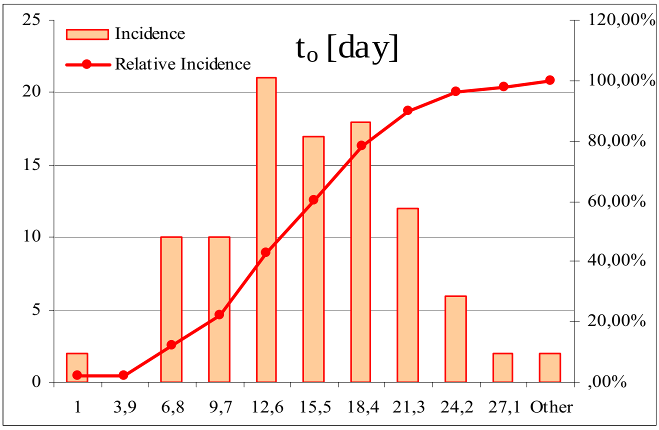

Fig. 5. Distribution of the ordering interval

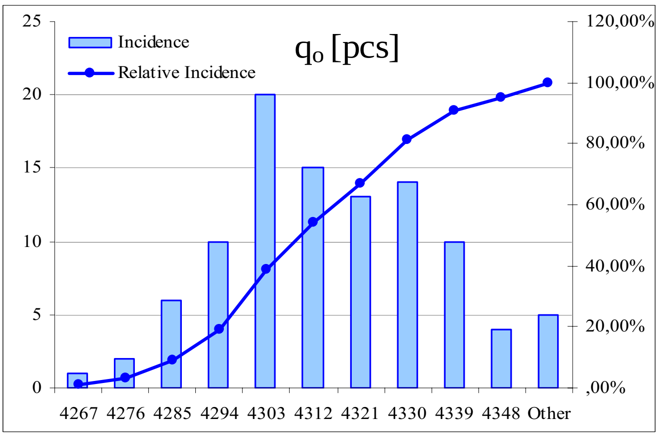

Fig. 6. Distribution of the ordered volume

7. Summary

The simulation inventory model presented in this paper and the binary genetic optimising algorithm determining the control parameters showed beneficial properties in managing of stochastic inventory processes. For the establishment of proper applications, it is worthwhile to examine also the services rendered by the genetic algorithms operating with real number representation, as it is possible that this type of algorithm is able to provide the same results in a faster, more accurate way. Experiences show that the inventory processes in the future may constitute a special application field of the simulation supported optimisation with genetic algorithms.

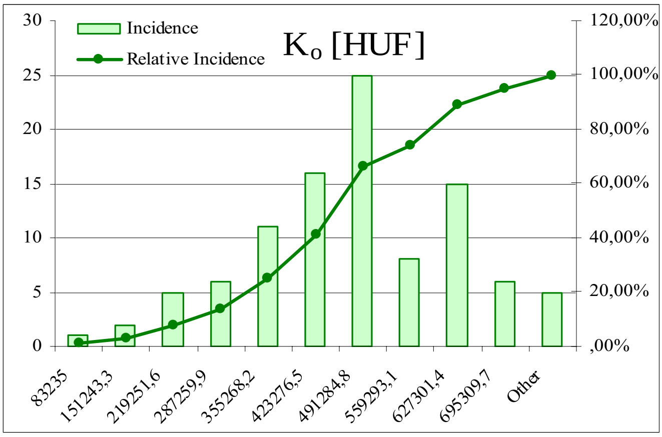

Fig. 7. Distribution of the costs

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