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ROBUST CONTROL DESIGN BASED ON HIGH-COMPLEXITY MODELS

Péter GÁSPÁR* and István KUTI**

*Computer and Automation Research Institute
Hungarian Academy of Sciences
H–1111 Budapest, Kende u. 13–17, Hungary
Fax: +36 14667503
e-mail: gaspar@sztaki.hu
**Department of Chassis and Lightweight Structures
Budapest University of Technology and Economics
H–1111 Budapest, Műegyetem rkp. 3, Hungary
Fax: +36 14635574
e-mail: kuti@kme.bme.hu

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Abstract

In this paper a high-complexity finite element structure of the truck model is proposed in the control design of active suspension systems. A significant step of the method is the selection of a reduced-order model by using the relevant frequencies up to 30 Hz from the modal representation of the model. The model uncertainty can also be approximated with the difference between the high-order and the reduced-order models. Then the model is augmented with the specifications of the performance demands and the multiplicative uncertainty structure. The control design itself is based on the H_∞/μ method, which guarantees that the controlled system achieves robust stability and meets performance demands.

Keywords: finite element model, H_∞/μ synthesis, unmodelled dynamics, uncertainties, performances, vehicle dynamics, active suspension.

1. Introduction and Motivation

The control design methods of the active suspension system usually use simple model structures with masses, dampings and springs. These low-complexity models contain the basic properties of the actual plant to be controlled [3, 7]. Assuming a rigid body structure for the plant, significant frequency values may be ignored, e.g. information about the tire dynamics, screw movement between the front and the rear parts, movement from the bending dynamics, or super-harmonic components.

For numerical stability reasons of the control design algorithms, the model of the actual plant should be relatively low-complexity. The control reduction methods used in the control field usually lead to large modelling errors and unmodelled dynamics. The message of the paper is the selection of a reduced-order model by selecting the significant frequencies from the high-order model. The frequencies which are important in the controlled system sense are selected and the other frequencies are ignored. At the same time the difference between the reduced-order model and the high-complexity model can be calculated, which is an estimation of the error bound.

In this paper the model, which is the basis of the control design, is constructed by a finite element method. This method usually results in a high-complexity model, which approximates the actual plant more accurately. In this paper, the control design itself is based on the H_∞/μ method, which is applied to the reduced-order model. This model guarantees robust stability and meets performance demands for the system designed, see [2].

The structure of the paper is the following. In Section 2, a reduced-order model is constructed by selecting the significant modal frequencies from finite element structure of a flexible truck model. In Section 3, the model is augmented with the performance specifications and the multiplicative uncertainties. In Section 4, the design process is demonstrated in an illustrative example.

2. Constructing a Reduced-Order Model

In this paper a high complexity model of a truck model is applied in the design of an active suspension system. The finite element model is 2148 degrees-of-freedom (DOF). The construction of the finite element model of the truck has been summarized in previous research works [4, 5]. The structure of the truck model is illustrated in *Fig. 1*.

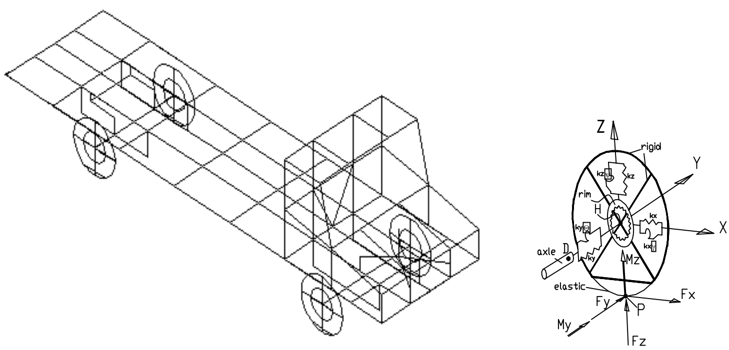

Fig. 1. Skeleton structure of the truck and the tire

The differential equation of motion in the space of the modal displacements can be written as:

$$M\ddot{y} + K\dot{y} + Sy = u + K_w w, \tag{1}$$

where M is the lumped mass matrix, K is the damping matrix, S is the stiffness matrix, y is the vector of vertical displacements, u is the actuator forces, w is the excitation of road roughness, K_w is the road matrix.

After the eigenproblem for *Eq*. (1) the eigenvectors have been solved that can be arranged into the modal matrix Φ with respect to the ascending order of the related natural frequencies. Define the inverse modal transformation by $y = \Phi q$ in such a way that the coordinates q in the new space will be the modal displacements. Applying the transformation, the following differential equation can be formalized:

$$\ddot{q} + K_v \dot{q} + \Lambda q = F_u u + K_d w, \tag{2}$$

where q is the vector of modal displacements. $\ddot{q}$ and $\dot{q}$ denote the modal velocity and acceleration vectors, respectively. Λ is the diagonal matrix of the squares of natural frequencies relating to the applied natural modes.

Then a reduced vector q_r with m components is defined in order to create a reduced-order model of the truck. Select $m \ll n$ and consider the reduced matrix Φ_r, the q_r contains the first m eigenvectors. Let y_r denote the approximation of generalized displacements and q_r the approximation of the m modal displacements. The inverse modal transformation of the approximation displacements is defined as $y_r = \Phi_r q_r$. Thus, the following differential equation is formalized for the approximation:

$$\ddot{q}_r + K_r \dot{q}_r + \Lambda_r q_r = F_r u + K_{dr} w. \tag{3}$$

Here q_r is selected in such a way that the reduced-order equation consists of the significant natural modes up to 30 Hz, since this is the relevant frequency domain in the active suspension design sense. For the design of active suspensions 12 natural modes have been selected, seven of them relate to the suspension deflections (vertical, rotational and pitch motions of the body, and the vertical and rotational motions of axles) and the remaining five belong to the elastic deformations of the body.

3. Modelling for Control Design

In this section the model to be controlled is augmented with the uncertainty models and performance objectives. The control design is based on the H_∞/μ method. Thus, the motion differential equation transformed into the state space representation form:

$$\dot{x} = Ax + B_1 w + B_2 u, \tag{4}$$

where $x = \begin{bmatrix} \dot{q}_r & q_r \end{bmatrix}^T$ is the state vector, w is the disturbance, u is the control input and the state matrices are

$$A = \begin{bmatrix} -K_r & -\Lambda_r \\ I & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} K_{dr} \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} F_r \\ 0 \end{bmatrix}.$$

The performance outputs are the sprung mass accelerations $\ddot{z}_b$, the suspension deflections z_{sd}, the wheel travels z_t, and the control inputs z_u at the left hand and right hand sides of the front and of the rear. The measured outputs are the suspension deflections in the left hand and right hand sides of the front and of the rear. The performance and the measured outputs can be expressed in the state vectors:

$$z = C_1 x + D_{11} w + D_{12} u \tag{5}$$
$$y = C_2 x, \tag{6}$$

where the forms of the matrices are the following:

$$C_1 = \begin{bmatrix} -\Phi_{body} K_v & -\Phi_{body} \Lambda \\ 0 & \Phi_{rel} \\ 0 & \Phi_t \end{bmatrix}, \quad D_{11} = \begin{bmatrix} \Phi_{body} K_g \\ 0 \\ 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} \Phi_{body} F_u \\ 0 \\ 0 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 0 & \Phi_{rel} \end{bmatrix}.$$

The weighting function W_p represents the performance signals of the active suspension system. The weighting function of the sprung mass acceleration is selected in the $W_{p_1} = 0.0014 \frac{S+350}{S+1}$ form, and the weighting function of the suspension deflection is $W_{p_2} = 2.857 \frac{S+350}{S+1}$. It is assumed that in the low frequency domain disturbances at the accelerations of the sprung mass should be rejected by a factor of 0.5 and at the suspension deflection by a factor of 3. The control force is limited by a constant value defined by $W_{p_3} = 0.0018$. We assume that the maximum road disturbance is 0.10 m and hence we choose $W_w = 0.1$. We set $W_n = 0.001$, thus essentially it is assumed that the sensor noise is 0.001 m in the whole frequency domain. The frequency weighting function of the unmodelled dynamics is selected in such a way that in the low frequency domain, the uncertainties are about 15% and in the upper frequency domain they are up to 100%: $W_r = 1.05 \frac{S+30}{S+200}$.

The control design applied in the paper is based on the H_∞/μ synthesis. The goal of this method is to minimize overall stabilizing controllers K, the lower LFT form of the interconnection structure:

$$\min_K \|\mathcal{F}_l(P, K)\|_\mu. \tag{7}$$

The optimization problem can be solved in an iterative way by using D and K. The procedure is called $D - K$ iteration. The formula in the frequency domain:

$$\sup_\omega \inf_{D\omega \in \mathcal{D}} \bar{\sigma}[D_\omega \mathcal{F}_l(P, K)(j\omega) D_\omega^{-1}] \tag{8}$$

which can be solved pointwise in the frequency domain. The stable and minimum phase scaling matrix D is selected by an interpolation technique or a graphical matching. A detailed description of this method can be found in [1, 9].

4. Numerical Experiments

In this section the effectiveness of the designed active suspensions is studied by considering the different driving conditions, such as travelling along a straight road at a constant speed. The shape of the applied road defect (bump) is given in the upper part of *Fig. 2*, and its position, which is at the beginning of a smooth road, is shown in the bottom part of *Fig. 2*. Its height is 0.10 m, and stretches under the wheels on both sides resulting in symmetric kinematic road excitations in vertical direction. The velocity of the applied truck model during cornering and in straight motion is 20 m/s, while during braking its initial velocity is 20 m/s and its final velocity is 4 m/s. For the sake of simplicity, the actuator that is designed for an elastic chassis is called elastic actuator, and the other actuator that is designed for a rigid chassis is called rigid actuator. For better comparability the vertical vibrations of the vehicle chassis are studied for each case at the right-hand-side front wheel.

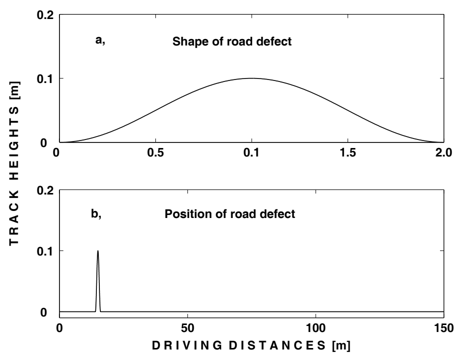

Fig. 2. The applied road defect

In the first analysis the effectiveness of the actuators is studied during the straight motion of the truck when it passes over the road defect. *Fig.3.a* and *3.b* show clearly that both of the elastic and rigid actuators decrease the vertical vibrations effectively, when they are applied to the elastic and rigid chassis, respectively. An important difference is that the controlled vertical motion, produced by the rigid actuator reaches to its equilibrium position very slowly (*Fig.3.b*). By contrast, in case of the elastic actuator the equilibrium position is reached very quickly (*Fig.3.a*). This result shows the advantage of the elastic actuator over the rigid one. Since the elastic chassis compared to the rigid chassis approximates the structure of the actual vehicle better, it is interesting to study the behaviour of the rigid actuator if it is applied to the elastic chassis (*Fig.3.c*). It is also interesting to apply the elastic actuator to the vehicle with rigid chassis (*Fig.3.d*). Comparing these figures it can be seen that the controlled motion from the equilibrium position produced by the rigid actuator in an elastic chassis in *Fig.3.c* is larger than in *Fig. 3.d*, where it is applied to the rigid chassis. It shows that there is a small decrease in the effectiveness of the rigid actuator when it is applied to the elastic chassis. It is noted, that very similar results can be achieved if only the vertical oscillations are simulated without the horizontal motion. It means that the excitation does not have an important role in the simulation of the horizontal motion. Summing up the elastic actuator is more suitable to be used in the simulations.

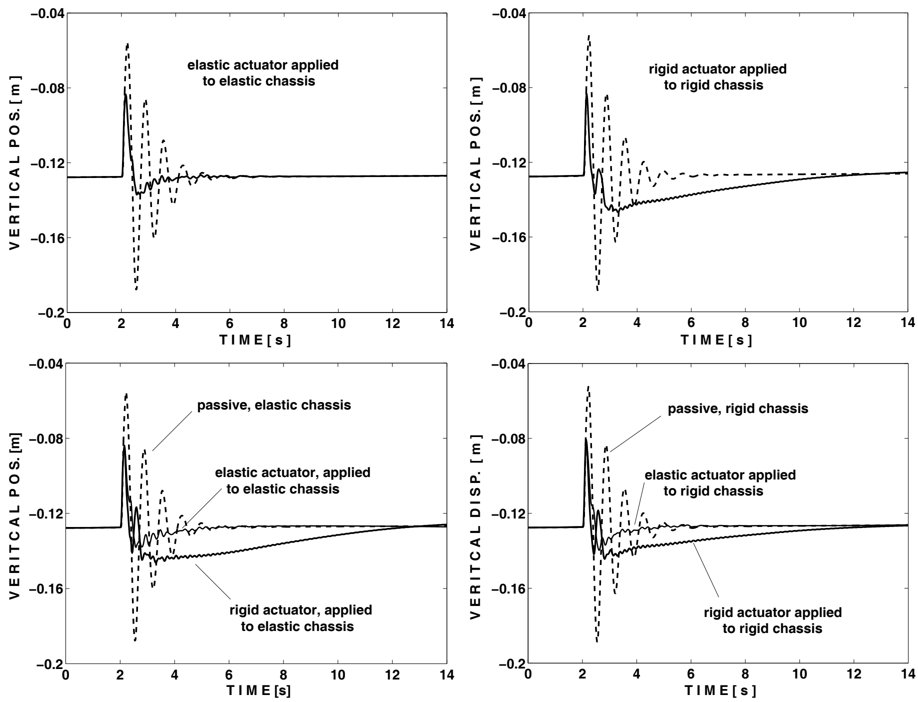

Fig. 3. Straight motion. The vibration of the chassis at the right-hand-side front suspension

In the second analysis the actuators are studied during braking. *Fig.4.a* and *Fig. 4.b* show clearly that both of the elastic and the rigid actuators decrease the vertical vibrations effectively when they are applied to the elastic and rigid chassis, respectively. Similarly to the straight motion the controlled motion produced by the rigid actuator reaches its equilibrium position very slowly (*Fig.4.a*). In these figures a surprising disadvantageous effect of both actuators can be seen. That is the actuators work in such a way that they increase the pitch motion of the vehicle chassis during the complete braking process. In order to illustrate this negative effect the corresponding passive vertical motions of the chassis for smooth road are shown in *4.a* and *Fig. 4.b*. A natural requirement for these actuators is not to increase the pitch motions of the chassis compared motions produced by the passive suspension. This unexpected behaviour of the actuators can be interpreted such a way, that the vehicle models used in the design of active suspensions do not contain any information about the braking process. In *Fig. 4.c* and *Fig. 4.d* the applications of the rigid and elastic actuators to the elastic and rigid chassis, respectively, are demonstrated. It is shown clearly in *Fig. 4.d* that the elastic actuator works well on the rigid chassis provided that the pitch motion of the chassis is ignored. However, the rigid actuator applied to the elastic chassis produces abnormal controlled motion (*Fig. 4.c*).

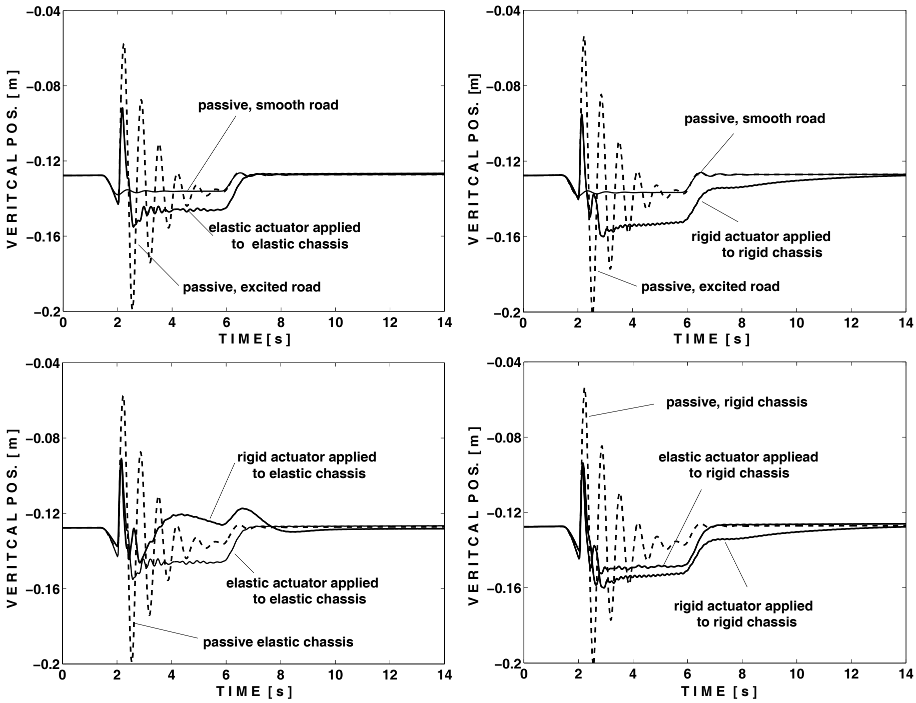

Fig. 4. Braking. The vibration of the chassis at the right-hand-side front suspension

5. Conclusions

In this paper a method which combines the finite element method and the robust control design method has been presented. The model which is the basis of the control design is low-order complexity. This model is created by using the relevant frequencies of the high-complexity model generated by the finite element method. The control design itself is based on the H_∞/μ method. With this method both the performance demands and the model uncertainties can be taken into consideration. The diagrams presented in the last section for various road conditions prove conclusively the effectiveness and robustness of the active suspensions designed.

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