DESIGN OF LPV CONTROLLERS FOR ACTIVE VEHICLE SUSPENSIONS

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Abstract

In the paper the linear parameter varying (LPV) method is applied to active suspensions. The suspension structure contains nonlinear components, i.e. the dynamics of the dampings and the springs. The model is augmented with weighting functions specified by the performance demands and the uncertainty assumption. By selecting scheduling parameters an LPV model is generated in which the model structure is nonlinear in the parameters but linear in the states. The design of the active suspension is illustrated in the demonstration example.

Keywords: automotive control, vehicle dynamics, model for control, uncertainty, performances.

1. Introduction

In the classical linear control design methods nonlinear components are approximated by linear characteristics, which are usually valid near the operating point only, and they handle the nonlinear behavior with an uncertainty assumption and a μ analysis/synthesis approach. Since the linear model is not able to describe the behavior of a nonlinear plant in the whole operation region, these methods result in conservative controllers.

The gain-scheduling methods propose solutions to the nonlinear problem. In these methods the design of the nonlinear controller is decomposed into the design of a number of linear controllers. Consequently, any analysis/design based on this theory is generally valid only during near the equilibrium points and not valid between them. Moreover, the switch between the linear controllers may cause stability and controllability problems, see [5]. Another scheme is based on the Linear Parameter Varying (LPV) method, in which the highly nonlinear effects can be taken into consideration, see [6, 8]. The LPV modelling defines the nonlinear model in state space representation form in such a way that the model structure is nonlinear in the parameters, but is linear in the states. Furthermore, this state space representation of the LPV model is valid in the whole operating region of interest.

The actual plant of the suspension system always includes nonlinear components which must be taken into consideration. The dynamic characteristics of suspension components, i.e. dampings and springs, have nonlinear properties, and they are not time-invariant, but change during the vehicle life cycles, see [3, 7]. A possible way of control design is the feedback linearization method, which has been proposed for nonlinear controller design [1]. A full state feedback controller has been designed using a backstepping procedure [4]. However, in this method, signals which are assumed to be measured are deduced in practice from other measured signals. Furthermore, slight model errors or uncertainties can cause a controller to become unstable.

In this paper, an LPV modelling and control design approach is proposed for the active suspension design problem. First, an LPV model is constructed, which handles the nonlinear characteristics of the physical components. Then the LPV model is augmented with the control specifications, i.e. the performance specifications, the trade-off between the performance demands, and the multiplicative uncertainty, which is caused by neglected dynamics. Finally, the control design based on the LPV method is performed.

The structure of the paper is as follows. In Section 2 a nonlinear model of the active suspension system is presented. An LPV model of the active suspension system is developed in which scheduling signals are selected. In Section 3 an augmented LPV model for control design which includes the performance specifications and the multiplicative uncertainty is created. In Section 4 the controlled system is demonstrated. Finally, Section 5 contains some concluding remarks.

2. The LPV Modelling of the Suspension Systems

In *Fig. 1* a two-degree-of-freedom quarter-car model is shown. The body mass m_s represents the sprung mass, which corresponds to one of the corners of the vehicle, and the unsprung mass m_u represents the wheel at one corner. The parameters k_t, k_s, b_s are the tyre stiffness, the suspension stiffness, and the damping rate of the suspension, respectively. The control signal F is generated by the actuator. x_1 and x_2 denote the vertical displacement of the sprung mass and the unsprung mass, respectively. The disturbance w is caused by road irregularities.

In the modelling phase of the control design several specifications are used: a./ The suspension structure is defined by the dynamics of the nonlinear components. b./ The performance demands for ride comfort, road holding, suspension deflection and input force are taken into consideration. c./ The trade-off between performance specifications is defined by a nonlinear function. d./ The model uncertainty is assumed to be in an output multiplicative structure.

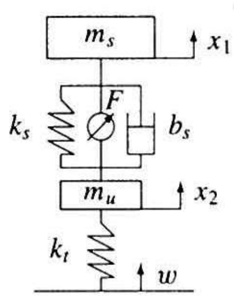

Fig. 1. Quarter-car model

The force equations of the quarter-car model are:

$$F_{m_s} = F_{k_s} + F_{b_s} - F \tag{1}$$

$$F_{m_u} = -F_{k_s} - F_{b_s} - F_{k_t} + F, \tag{2}$$

where the force from the sprung mass acceleration and the unsprung mass acceleration, the suspension damping force, the suspension spring force, the tire force, respectively, are as follows:

$$F_{m_s} = m_s\ddot{x}_1, \tag{3}$$

$$F_{m_u} = m_u\ddot{x}_2, \tag{4}$$

$$F_{b_s} = b_s^l(\dot{x}_2 - \dot{x}_1) - b_s^{sym}|\dot{x}_2 - \dot{x}_1| + b_s^{nl}\sqrt{|\dot{x}_2 - \dot{x}_1|}\,\mathrm{sgn}(\dot{x}_2 - \dot{x}_1) \tag{5}$$

$$F_{k_s} = k_s^l(x_2 - x_1) + k_s^{nl}(x_2 - x_1)^3 \tag{6}$$

$$F_{k_t} = k_t(x_2 - w) \tag{7}$$

Here, parts of the nonlinear suspension damping b_s are b_s^l, b_s^{nl} and b_s^{sym}. The b_s^l coefficient affects the damping force linearly while b_s^{nl} has a nonlinear impact on the damping characteristics. b_s^{sym} describes the asymmetric behavior of the characteristics. Parts of the nonlinear suspension stiffness k_s are a linear coefficient k_s^l and a nonlinear one, k_s^{nl}. The nonlinear model is the following:

$$\begin{aligned} m_s\ddot{x}_1 &= k_s^l(x_2 - x_1) + k_s^{nl}(x_2 - x_1)^3 + b_s^l(\dot{x}_2 - \dot{x}_1) - b_s^{\mathrm{sym}}|\dot{x}_2 - \dot{x}_1| \\ &\quad + b_s^{nl}\sqrt{|\dot{x}_2 - \dot{x}_1|}\,\mathrm{sgn}(\dot{x}_2 - \dot{x}_1) - F \end{aligned} \tag{8}$$

$$\begin{aligned} m_u\ddot{x}_2 &= -k_s^l(x_2 - x_1) - k_s^{nl}(x_2 - x_1)^3 - b_s^l(\dot{x}_2 - \dot{x}_1) + b_s^{\mathrm{sym}}|\dot{x}_2 - \dot{x}_1| \\ &\quad - b_s^{nl}\sqrt{|\dot{x}_2 - \dot{x}_1|}\,\mathrm{sgn}(\dot{x}_2 - \dot{x}_1) - k_t(x_2 - w) + F \end{aligned} \tag{9}$$

The state vector x is selected as follows:

$$x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T \tag{10}$$

in which the components of the state vector x are the vertical displacement of the sprung mass x_1, the vertical displacement of the unsprung mass x_2, their derivatives $x_3 = \dot{x}_1$, $x_4 = \dot{x}_2$.

In the LPV modelling ρ parameters, which are directly measured or can be calculated from the measured signals, must be selected. In the LPV model of the active suspension system two parameters are selected. The relative velocity and the relative displacement are selected as scheduling parameters:

$$\rho_b = \dot{x}_2 - \dot{x}_1 \tag{11}$$

$$\rho_k = x_2 - x_1 \tag{12}$$

Parameter ρ_b depends on the relative velocity, parameter ρ_k is equal to the relative displacement. In practice, the relative displacement is a measured signal. The relative velocity is then determined by numerical differentiation from the measured relative displacement.

The nonlinear damping force equation in (5) can be partitioned in the following way:

$$F_{b_s}(\rho_b) = b_s^l(x_4 - x_3) - b_s^{\mathrm{sym}}\mathrm{sgn}(\rho_b)(x_4 - x_3) + b_s^{nl}\sqrt{|\rho_b|}\,\mathrm{sgn}(\rho_b) \tag{13}$$

in which the first term is the linear part and the second and the third terms are the nonlinear part of the damping force. The linear part and the second part of the damping force can be expressed as a linear combination of the states, however, the third part cannot. It is noted that in the control design procedure the asymmetric behavior defined by (13) is handled, however, the nonlinear part of the damping in equation (5) is taken into consideration as an exogenous disturbance signal. This signal is a fictitious input signal u_{fict}. Thus, the compensator to be designed guarantees the minimization of the nonlinear effect since it provides the disturbance attenuation specification. Similarly, the nonlinear spring force in (6) can be reformulated in the following way:

$$F_{k_s}(\rho_k) = k_s^l(x_2 - x_1) + k_s^{nl}\rho_k^2(x_2 - x_1) \tag{14}$$

This force can be expressed by a linear combination of states allowing the force to have nonlinear ρ dependence.

With the scheduling parameters, the equations of the quarter-car model are:

$$\begin{aligned}\ddot{x}_1 = {} & \frac{k_s^l}{m_s}(x_2 - x_1) + \frac{k_s^{nl}}{m_s}\rho_k^2(x_2 - x_1) + \frac{b_s^l}{m_s}(x_4 - x_3) - \frac{b_s^{sym}}{m_s}|\rho_b| \\ & + \frac{b_s^{nl}}{m_s}\sqrt{|\rho_b|}\,\mathrm{sgn}(\rho_b) - \frac{1}{m_s}F\end{aligned} \tag{15}$$

$$\begin{aligned}\ddot{x}_2 = {} & -\frac{k_s^l}{m_u}(x_2 - x_1) - \frac{k_s^{nl}}{m_u}\rho_k^2(x_2 - x_1) - \frac{b_s^l}{m_u}(x_4 - x_3) + \frac{b_s^{sym}}{m_u}|\rho_b| \\ & - \frac{b_s^{nl}}{m_u}\sqrt{|\rho_b|}\,\mathrm{sgn}(\rho_b) - \frac{k_t}{m_u}(x_2 - w) + \frac{1}{m_u}F\end{aligned} \tag{16}$$

The state space representation of the LPV model is as follows. It is noted that in the control design procedure the nonlinear part of the damping in *Eq.* (5) is considered as an exogenous disturbance signal. In the equation it is denoted by a fictitious input signal u_{fict}.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -\frac{k_s^l}{m_s}-\frac{k_s^{nl}}{m_s}\rho_k^2 & \frac{k_s^l}{m_s}+\frac{k_s^{nl}}{m_s}\rho_k^2 & -\frac{b_s^l}{m_s} & \frac{b_s^l}{m_s}\\ \frac{k_s^l}{m_u}+\frac{k_s^{nl}}{m_u}\rho_k^2 & -\frac{k_s^l}{m_u}+\frac{k_s^{nl}}{m_u}\rho_k^2-\frac{k_t}{m_u} & \frac{b_s^l}{m_u} & -\frac{b_s^l}{m_u}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0\\ \frac{1}{\tau}\end{bmatrix} w$$
$$+ \begin{bmatrix} 0 & 0\\ 0 & 0\\ -\frac{b_s^{\mathrm{sym}}}{m_s}|\rho_b| + \frac{b_s^{nl}}{m_s}\sqrt{|\rho_b|} & -\frac{1}{m_s}\\ \frac{b_s^{\mathrm{sym}}}{m_u}|\rho_b| - \frac{b_s^{nl}}{m_u}\sqrt{|\rho_b|} & \frac{1}{m_u}\end{bmatrix}\begin{bmatrix}u_{\mathrm{fict}}\\ u\end{bmatrix} \tag{17}$$

3. The LPV Modelling for Control Design

Consider the closed-loop system in *Fig. 2*, which includes the feedback structure of the model G and controller K, and elements associated with the uncertainty models and performance objectives. In the diagram, u is the control input, which is generated by actuators, y is the measured output, which contains the perturbed sprung mass acceleration, n is the measurement noise. In the figure, w is the disturbance signal, which is caused by road irregularities. $\tilde{z}$ represents the performance outputs: the passenger comfort (heave acceleration) ($z_a = \ddot{x}_1$), the suspension deflection ($z_s = x_s - x_u$), the wheel relative displacement ($z_t = x_u$) and the control input (z_u).

The uncertainties of the model are represented by W_r and Δ_m. W_r is assumed to be known, and Δ_m is assumed to be unknown with $\|\Delta_m\|_\infty < 1$. Design models used for active suspension control typically exhibit high fidelity at lower frequencies ($\omega < 10$ Hz), but they degrade rapidly at higher frequencies due to such poorly modelled or neglected effects as flexibility. Thus, W_r is selected as

$$W_r = 2.25\frac{\mathrm{s}+20}{\mathrm{s}+450}. \tag{18}$$

The purpose of weighting functions W_{p_a}, W_{p_d}, W_{p_t} and W_{p_u} is to keep the heave acceleration, suspension deflection, wheel travel, and control input small over the desired frequency range. These weighting functions chosen for performance outputs can be considered as penalty functions, i.e. weights should be large in a frequency range where small signals are desired and small where larger performance

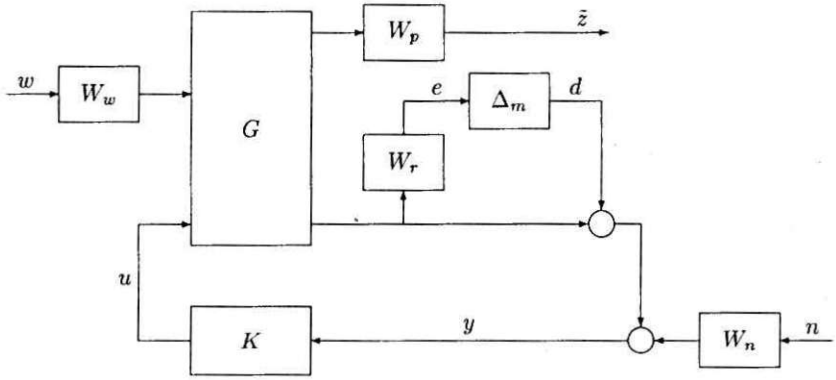

Fig. 2. The closed-loop interconnection structure

outputs can be tolerated. Thus, W_{p_a} and W_{p_d} are selected as

$$W_{p_a} = \phi_a \frac{(s/0.01 + 1)(s/2000 + 1)}{(s/8 + 1)(s/12 + 1)}, \tag{19}$$

$$W_{p_d} = \phi_d \frac{(s/0.01 + 1)(s/2000 + 1)}{(s/8 + 1)(s/12 + 1)} \tag{20}$$

Here it is assumed that in the low frequency domain disturbances at the heave accelerations of the body should be rejected by a factor of ϕ_a and at the suspension deflection by a factor of ϕ_d. The other W_{p_i}'s are selected as $W_{p_t} = 1$ and $W_{p_u} = 1 \cdot 10^{-3}$.

The trade-off between passengers' comfort and suspension deflection is due to the fact that it is not possible to keep them simultaneously together. A large gain ϕ_a and a small gain ϕ_d correspond to a design that emphasizes passenger comfort. On the other hand, choosing ϕ_a small and ϕ_d large corresponds to a design that focuses on suspension deflection. The LPV controller schedules on suspension deflection, and focuses on minimizing either the heave acceleration or suspension deflection response, depending on the magnitude of the vertical suspension deflection. In order to achieve the shift in focus from vertical acceleration to suspension deflection the weights associated with these signals are chosen to be parameter-dependent. In the mechanism two parameters are defined: ρ_1 and ρ_2. When the suspension deflection is below ρ_1, the gain ϕ_a is selected constant 15 and the gain ϕ_d is zero. When the deflection is between ρ_1 and ρ_2 the gains change linearly, i.e. the gain ϕ_a decreases to zero, while the gain ϕ_d increases to 100. When the value of the suspension deflection is greater than ρ_2, the gain ϕ_d is constant 100 and the gain ϕ_a is zero. In the example, ρ_1 and ρ_2 are selected as 0.05 and 0.08, respectively. This corresponds to an LPV controller that minimizes only the vertical acceleration

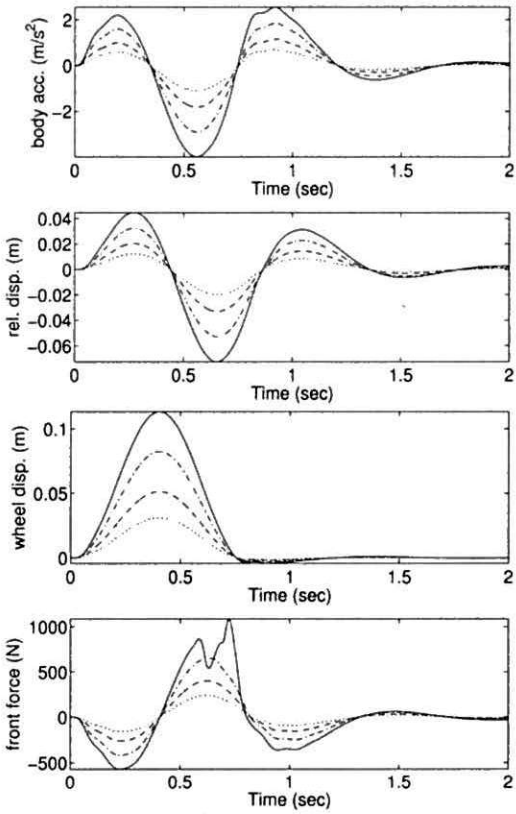

Fig. 3. Time responses of the nonlinear controlled system using LPV controllers for different bumps

when the suspension travel is less than 5 cm, and which gradually begins focusing on the suspension deflection when the travel is greater than 5 cm. Above 8 cm it minimizes only the suspension deflection.

4. Simulation Examples

In the first demonstration example the LPV controller, in which a balance between the minimization of the heave acceleration and the suspension deflection are taken into consideration, is analyzed. The controlled systems are tested by using bumps of different magnitude, i.e. 3 cm, 5 cm, 8 cm, and 11 cm maximal values. The time responses of the heave accelerations, the relative displacements, the wheel travels, and the control forces are illustrated in *Fig. 3*. The nominal parameters of the quarter-car model is in *Table 1*.

Table 1. Parameters of the car model

Parameters (symbols)	Value
sprung mass (m_s)	290 kg
unsprung mass (m_u)	40 kg
suspension stiffness (k_s^l, k_s^{nl})	$235 \cdot 10^2$ N/m, $235 \cdot 10^4$ N/m
tire stiffness (k_t)	$190 \cdot 10^3$ N/m
damping (b_s^l, b_s^{nl}, b_s^{sym})	700 N/m/s, 400 N/m/s, 400 N/m/s
time constant (τ)	$\frac{1}{30}$ s

In the second example, the LPV synthesis is performed in three different approaches. They differ from each other in the weighting strategy applied. The solid line corresponds to the LPV synthesis, in which a balance between the minimization of the vertical acceleration and the suspension deflection are taken into consideration. The dashed line illustrates the result of the synthesis, in which the control design focuses only on the minimization of the vertical acceleration, while the dashed-dotted line illustrates the case when only the suspension deflection is minimized. The time responses are illustrated in *Fig. 4*.

In the case of the LPV-based controlled system, in which a balance between the different optimization criteria is taken into consideration, the effects of the disturbance both on the sprung mass acceleration and the suspension deflection are perceived as relatively small oscillations with short duration. In the case of the LPV control which only takes the body mass acceleration into consideration, the effects of the disturbance on the suspension deflection are seen as long duration. In the case of the LPV control which minimizes only the suspension deflection, the effects of the disturbance on the sprung mass acceleration are seen as large overshoot with long duration.

Finally, the controlled system based on the LPV controller is compared with the gain scheduling controllers. In the latter the operational region of the suspension is segmented into a large number of operational intervals for which linear controllers based on the H_∞-norm are designed. The controllers switch according to the current operation. The controlled systems are tested by using a bump with 11 cm maximal value. The time responses are illustrated in *Fig. 5*. The switching of the gain scheduling controllers shows in the signal of the control force. Note that at the same time the switching-off effects cannot be seen in the performance signals. The effects of the disturbances on the sprung mass acceleration and the suspension deflection show smaller oscillations and shorter duration in the LPV control case.

5. Conclusion

This paper is concerned with the nonlinear modelling of a vehicle suspension system. An LPV technique is applied in the construction of the nonlinear model, in

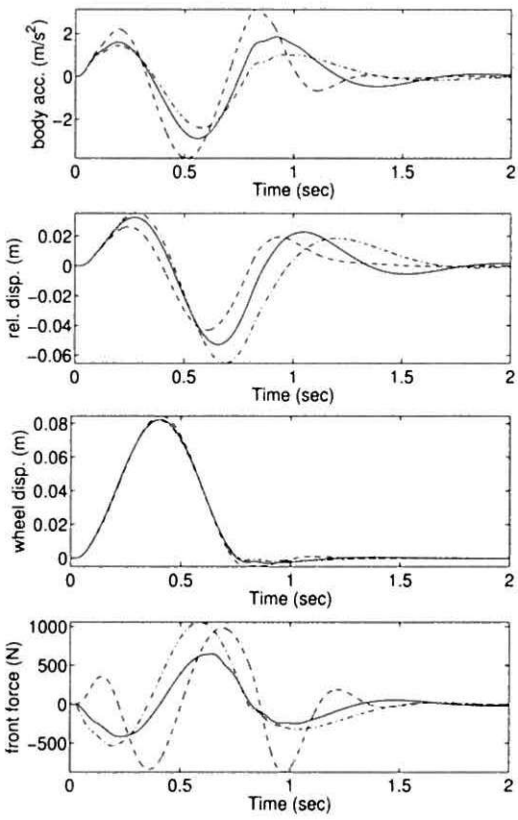

Fig. 4. Time responses of the controlled system using LPV controllers

which the highly nonlinear effects in the state space description can be taken into consideration. Two scheduling parameters, which are linked to the measured signals, are chosen. The trade-off between the performance demands is guaranteed by using parameter dependent gains. Thus, the designed controlled system meet the performance demand, i.e. in the case of small suspension travel the vertical acceleration is minimized, in the case of large suspension travel the suspension deflection is minimized, otherwise a balance between these performances is ensured by using weighting functions.

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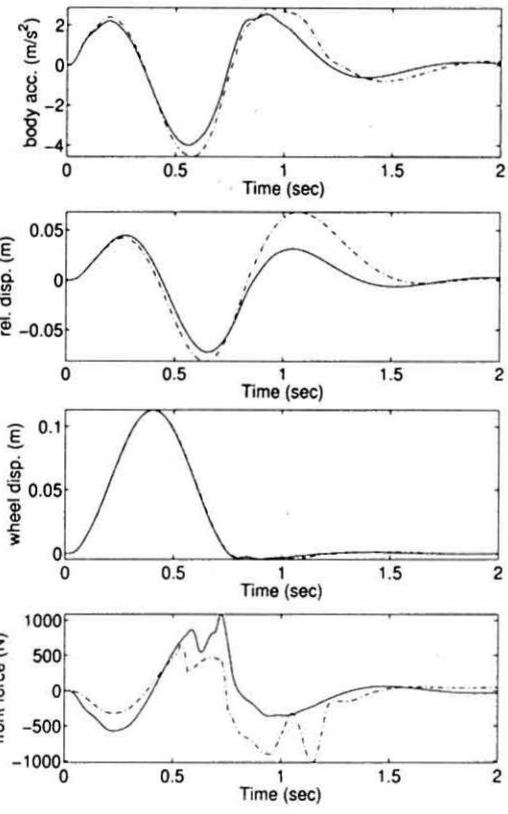

Fig. 5. Time responses of the controlled system using an LPV and a $\mathcal{H}_\infty$ control

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